AFFIRMATIVE ACTION IN INDIA VIA VERTICAL, HORIZONTAL, AND OVERLAPPING RESERVATIONS

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ABSTRACT. Sanctioned by its constitution, India is home to an elaborate affirmative action program for allocation of public jobs, where historically discriminated groups are protected with vertical reservations implemented as "set asides," and other disadvantaged groups are protected with horizontal reservations implemented as "minimum guarantees." Concurrent implementation of these two policies with overlapping beneficiaries makes this program more complex than others elsewhere. An allocation mechanism mandated by the Supreme Court judgement *Anil Kumar Gupta vs. Uttar Pradesh (1995)* suffers from a number of anomalies, including disadvantaged candidates losing positions to privileged candidates of lower merit, triggering countless litigations and disarray in the country. Foretelling a recent reform in India, we propose an alternative mechanism that resolves all anomalies, and uniquely characterize it with desiderata reflecting the laws of India. Subsequently reinvented with an August 2020 High Court judgement and mandated for the state of Gujarat, our mechanism is endorsed for India with a December 2020 judgement of the Supreme Court.

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1. Introduction

Sanctioned by the country's constitution, one of the world's most comprehensive affirmative action programs is implemented in India. Allocation of government positions and seats at publicly funded educational institutions have to comply with mandates outlined by the landmark Supreme Court judgement *Indra Sawhney and others v. Union of India (1992)*,[1] widely known as the *Mandal Commission Case*. Under these mandates, an allocation mechanism that relies on an objective merit list of candidates is integrated with two types of affirmative action policies referred to as *vertical reservations* and *horizontal reservations*. Of the two types of policies formulated by the constitution bench of the Supreme Court, vertical reservations are intended as the higher-level protection policy, and as such they are mandated to be implemented on a "set aside" (or equivalently "over-and-above") basis. This means that a position awarded to an individual who deserves an unreserved position solely on the basis of her merit score does not count towards a vertically reserved position if the individual belongs to a protected class. In the past these higher level protections were exclusively intended for classes that faced historical discrimination such as *Scheduled Castes (SC), Scheduled Tribes (ST),* and *Other Backward Castes (OBC)*, although with a January 2019 amendment in the constitution their scope now includes *Economically Weaker Sections (EWS)* of the rest of the society. Horizontal reservations, on the other hand, are intended as the lower-level protection policy, and as such they are mandated to be implemented on a "minimum guarantee" basis. This means that a position awarded to a member of a beneficiary class for this lower-level protection policy always counts towards horizontally reserved positions, even if the individual receives this position solely on the basis of her merit score. As of January 2021, *persons with disabilities* is the only group in India that is eligible for horizontal reservations at the federal level.[2] In several states, however, there are additional beneficiary groups for horizontal reservations. For example, horizontal reservations for *women* is mandated in several states including in Bihar with 35% (of the positions), Andhra Pradesh and Gujarat with $33\frac{1}{3}$% each, and Madhya Pradesh, Uttarakhand, Chhattisgarh, Rajasthan, and Sikkim with 30% each. Other groups who are eligible for horizontal reservations in various applications include *ex-servicemen, sportsmen,* and *speakers of the local language*.

In India, beneficiary groups for vertical reservations do not overlap. An individual can be a member of at most one *(vertical) reserve-eligible category*. This structure results in a straightforward implementation of vertical reservations in the absence of horizontal

[1] The case is available at `https://indiankanoon.org/doc/1363234/` (last accessed on 01/19/2021).

[2] See the Supreme Court judgement *Union Of India & Anr vs National Federation Of The Blind & ... on 8 October, 2013*, available at `https://indiankanoon.org/doc/178530295/` (last accessed on 01/23/2021).

reservations: First open (i.e., unreserved) positions are allocated based on merit scores, and next for each of the (mutually exclusive) groups eligible for vertical reservation, positions that are set aside are allocated to the remaining members of the protected group again based on their merit scores. Let us refer this simple allocation mechanism as the *over-and-above choice rule*. Assuming that beneficiary groups for horizontal reservations do not overlap either, the implementation of horizontal reservations is equally straightforward: First horizontally protected positions are allocated to their intended beneficiaries based on merit scores for each protected group, and next all unfilled positions (i.e., open positions and unfilled horizontally protected positions) are allocated to remaining individuals with highest merit scores. When implemented *individually*, these two basic types of reservation policies are widespread throughout the world in a wide range of applications including affirmative action policies in K-12 education or college admissions, H-1B visa allocation in the U.S., and most recently Covid-19 pandemic medical resource allocation, although in some of these applications policymakers do not seem to appreciate the distinction between the two policies. In that sense the clear distinction made between the vertical and horizontal reservation policies in India at the Supreme Court level is rather extraordinary. But despite the clear formulation and distinction of the these affirmative action policies in the Indian legal framework, the country has endured thousands of litigations on their real-life implementation. One of the driving forces for that disruption is the following additional complexity in India pertaining to implementation of these policies:

- Unlike most applications of reservation policies in the rest of the world where these policies are implemented individually, vertical and horizontal reservation policies are implemented *concurrently* in India.

There is one other potential complexity that further complicates the implementation of horizontal reservations:

- In some of the applications in India, beneficiary groups for horizontal reservations do *overlap*.

Even though the principles that govern implementation of vertical and horizontal reservation policies were clearly formulated in *Indra Sawhney (1992)*, this judgement has not addressed how to implement them. This was subsequently done in another judgement *Anil Kumar Gupta v. State of U.P. (1995)*, where the Supreme Court formulated a choice rule for this complex version of the problem by augmenting the basic over-and-above choice rule with a *horizontal adjustment subroutine* that carries out the necessary corrections to accommodate the horizontal reservations.[3] Through this subroutine, however, *Anil Kumar Gupta (1995)* also introduced a number of anomalies to the resulting choice rule. We refer this allocation rule, federally mandated in India for twenty five years, as the *SCI-AKG choice rule*.

While the aspect of overlapping horizontal reservations has received some attention in both theoretical (Kurata et al. (2017)), and applied literature (Correa et al. (2019)), to the best of our knowledge the concurrent implementation of vertical and horizontal reservation policies has never been studied before. Moreover, the theoretical solution offered in Kurata et al. (2017) for implementation of overlapping horizontal reservations is mainly intended for a different variant of the problem where individuals have strict preferences for whether and which protection is invoked in securing a position. This modeling choice allows formulating the problem as a special case of the celebrated *matching with contracts* model (Hatfield and Milgrom, 2005). Although Correa et al. (2019) apply this theoretical solution to the real-life K-12 school choice application in Chile, the resulting allocation mechanism suffers from three shortcomings presented in Section 3 largely due to the mismatch between the application and the theory developed by Kurata et al. (2017).

In this paper we make contributions to microeconomic theory and also to the applied field of market design. Our main contributions to the field of market design are:

(1) formulation of the legal framework for implementation of vertical and horizontal reservation policies in India,
(2) formulation of the SCI-AKG choice rule along with its shortcomings, and documentation of the the scale of its disruption in India due to some of these shortcomings,
(3) introduction of a simple modification of the SCI-AKG choice rule that escapes all these shortcomings, and
(4) formulation of a number of related shortcomings of the Chilean choice rule for allocation of K-12 public school seats, and introduction of an alternative choice rule that escapes these shortcomings.

Our main contributions to microeconomic theory, which in part overlap with our contributions to market design, are:

(5) formulation of a simpler version of the model with overlapping horizontal reservations (but no vertical reservations), and in this context

[3]The case is available at `https://indiankanoon.org/doc/1055016/`. See also *Rajesh Kumar Daria vs Rajasthan Public Service (2007)* for a more detailed description of the procedure, available at `https://indiankanoon.org/doc/698833/`. (Both cases last accessed on 01/29/2021).

   (a) introduction of an allocation mechanism, the *meritorious horizontal choice rule*, that differs from all its predecessors in its "smart" processing of reserved positions,
   (b) characterization of the meritorious horizontal choice rule as the unique rule that satisfies three basic axioms, and

(6) formulation of a general version of the problem allowing for concurrent implementation of vertical reservations with overlapping horizontal reservations, and in this context
   (a) introduction of an allocation mechanism, the *two-step meritorious horizontal (2SMH) choice rule*, that escapes all shortcomings of the SCI-AKG choice rule, and
   (b) characterization of the 2SMH choice rule as the unique choice rule that satisfies four axioms motivated by the legal framework in India as well as the challenges faced in the country due to the limitations of the SCI-AKG choice rule.

It is worthwhile to emphasize that, even though our proposed 2SMH choice rule escapes all deficiencies of the SCI-AKG choice rule formulated and documented in our paper, its mechanics is surprisingly similar to this faulty mechanism.[4] Our proposed 2SMH choice rule can be seen as a modification of the SCI-AKG choice rule in two aspects, each modification eliminating some of the shortcomings.

As we have highlighted earlier, the culprit behind the shortcomings of the SCI-AKG choice rule is its *horizontal adjustment subroutine*, or more specifically how this subroutine is implemented. To illustrate the mechanics of this subroutine, consider the horizontal reservations for women. Whenever these protections are not satisfied for a vertical category v under the over-and-above choice rule, the subroutine replaces the lowest merit-score men admitted to category-v positions with the highest merit-score unselected women who are eligible for category v. The description of the subroutine is given in the legal framework in a similar way for a single beneficiary group (such as women), but the procedure is also well-defined and well-behaved when it is sequentially applied to multiple beneficiary groups provided that group memberships do not overlap. The first set of shortcomings of the SCI-AKG choice rule, presented in detail in Section 3.1, emerge only

[4]This aspect of an allocation mechanism is generally considered to be highly plausible in the field of market design, since it may potentially increase the likelihood of its acceptance in the field. Indeed, following the March 2019 circulation of the first draft of this paper and while it was under revision for this journal, this prospect became increasingly more likely after two important developments, first a mandate of a simpler version of this mechanism (defined in the absence of overlapping horizontal reservations) in the State of Gujarat in August 2020, and subsequently the endorsement of this simpler version by the Supreme Court in December 2020. We present a detailed account of these developments in the Epilogue in Section 9.

when the beneficiary group memberships for horizontal reservations overlap. We refer to this case as *overlapping horizontal reservations*. In this case, the sequential implementation of the subroutine

- may differ in its outcome depending on the sequence of horizontal adjustments,
- may provide an "under adjustment" of horizontal reservation protections, and
- may result in "ineffective" adjustments by admitting needlessly low merit score individuals.

Fortunately, there is a clear solution to this conundrum that relies on enhancing the basic mechanics of the horizontal adjustment subroutine with a "maximal matching" procedure. In order to provide an intuition for this innovation, consider the following analogy.

There is a gathering where lunch is served. One of the two guests, Violet, is a vegetarian, whereas the second guest, Charlie, is flexible in his diet. Suppose there is one vegetarian and one chicken sandwich available for the two guests. If Charlie is served his lunch prior to Violet, it would be a blunder to offer him the only vegetarian sandwich, since that would mean Violet has to skip her lunch. Charlie is flexible in his diet, whereas Violet is not, and serving the only vegetarian sandwich to Charlie results in wasting this valuable flexibility. A more careful server would have planned ahead taking into Violet's dietary restriction into consideration, and thus would have offered Charlie the chicken sandwich utilizing the flexibility in his diet. Now consider the accommodation of horizontal reservations in India. Suppose there is a female candidate Freya, a disabled female candidate Devi, one horizontally reserved position for female candidates, and one horizontally reserved position for disabled candidates. Just as it is implausible to offer the vegetable sandwich to Charlie in the above scenario, it is implausible to assign Devi the horizontally reserved position for female candidates and consequently deny Freya a position since she is not qualified for the horizontally reserved position for disabled candidates. Both horizontal reservation protections can be granted if Devi is instead assigned the horizontally reserved position for disabled candidates. Ironically, many real-life applications do not utilize the flexibility generated by candidates who qualify for multiple types of horizontal reservations, and thus suffer from the very shortcoming we illustrate in these two scenarios. In addition to our main application in India, the school choice system in Chile also suffers from the same shortcoming, precisely for the reason illustrated here. A better design would not give up the flexibility generated by candidates who qualify for multiple horizontal reservations, and instead it would capitalize on it. This is the basic idea under our proposed meritorious horizontal choice rule. Indeed, not only this choice rule eliminates all three shortcomings of the horizontal adjustment subroutine described above, it is also the only choice rule to do so (Theorem 2).

The above-described limitations of the SCI-AKG rule may be viewed as of second order importance in the field, because they are are hard to verify in practice and they likely affect the outcome rarely. The SCI-AKG choice rule, however, has two additional shortcomings that have been visibly disruptive in India in the last two decades. This time, the source of the anomaly has to do with how the horizontal adjustment subroutine is implemented rather than its mechanics. When the subroutine is applied for the open-category positions, individuals from reserve-eligible categories are ruled out for potential horizontal adjustments and only individuals from the general category are deemed eligible for this role. That is, by announcing their eligibility for vertical reservation protections, individuals risk losing their open-category horizontal reservation protections. Therefore, contrary to the objectives of affirmative action policies, the SCI-AKG choice rule can generate outcomes where a candidate from a disadvantaged group, despite being more meritorious, may lose a position to a candidate from a more privileged group. We refer to this irregularity as a failure to satisfy *no justified envy* (or a potential for *justified envy*). In addition to this highly implausible possibility, the SCI-AKG choice rule may also penalize candidates for reporting their eligibility for a vertical reservation protection, since it risks them to lose their open-category horizontal reservation protections. In that sense, the procedure is not *incentive compatible*. These two anomalies, first formally introduced by (Aygün and Bó, 2016) in the context of Brazilian college admissions, are clearly against the philosophy of affirmative action. Not only they result in countless lawsuits, but they also provide a loophole in the procedure that can be used to discriminate against members of backward classes. In Section 8.1, we provide ample evidence that these shortcomings are responsible for widespread confusion in India, often resulting in legal action, inconsistent rulings, and even defiance in some states through the implementation of better-behaved versions of the mandated procedure. We also provide evidence in Section 8.1.2 that, in some jurisdictions these shortcomings are exploited by local officials to discriminate against members of backward classes. These litigations often result in the interruption of the recruitment process, as well as reversals of recruitment decisions. Reporting a judgement by the Gujarat High Court, an article in *The Times of India* highlights this very issue:[5]

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The advertisement was issued in 2010 and recruitment took place in 2016
amid too many litigations over the issue of reservation ...
With the recent observation by the HC, the merit list will now be changed
for the third time.  Those already selected and at present under training
might lose their jobs, and half a dozen new candidates might find their
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[5] *The Times of India* story is available at `https://timesofindia.indiatimes.com/city/ahmedabad/general-seat-vacated-by-quota-candidate-remains-general-hc/articleshowprint/57658109.cms` (last accessed on 04/12/2019).

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names on the new list. However, all appointments have been made by the HC
conditionally and subject to final outcome of these multiple litigations.
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A simple search of the phrase "horizontal reservation" via Indian Kanoon, a free search engine for Indian Law, reveals the scale of the litigations relating to this concept. Excluding cases at lower courts, as of 01/19/2021 there are 1961 cases at the Supreme Court and State High Courts related to the implementation of horizontal reservations.[6] The potential for justified envy under the SCI-AKG choice rule is the primary culprit for a significant fraction of these cases. There is, however, reason to be optimistic that this impasse may be finally coming to an end. That is because, just as this paper was under revision, there has been a major breakthrough on this very issue; one that has a strong potential to bring an end to this predicament.

Prior to the March 2019 circulation of the first draft of our paper, the inability of the SCI-AKG choice rule to eliminate justified envy was never directly addressed by the highest court of India, despite the large scale disarray it has created for over two decades in lower courts. This situation has recently changed in a decisive way with its December 2020 judgement *Saurav Yadav & Ors v. State of Uttar Pradesh & Ors (2020)*,[7] where the Supreme Court mantated elimination of justified envy in allocation of all public positions, bringing and end to the 25-years tenure of its SCI-AKG choice rule. While the Supreme Court has not mandated an alternative choice rule, it has endorsed the *two-step minimum guarantee choice rule*, a rule that is mandated in the State of Gujarat through its August 2020 High Court judgement *Tamannaben Ashokbhai Desai v. Shital Amrutlal Nishar (2020)*.[8] Importantly, this rule that is given for the basic case of non-overlapping horizontal reservations in the High Court's judgement, is equivalent to our proposed 2SMH choice rule in this simple environment. We report a detailed account of these recent developments, and how they relate to our paper in Section 9.

The rest of our paper is organized as follows. After formulating the model in Section 2, we present a single-category analysis with overlapping horizontal reservations in Section 3 and our analysis for the most general version of the model in Section 4. In Section 5 we present a theoretical comparison of the SCI-AKG choice rule with the 2SMH choice

[6]Not all cases on "horizontal reservation" is about disputes related to elimination of justified envy or incentive compatibility. However during our search, we observed that the terminology of "migration" was used is some of the cases to indicate the situations where members of reserved categories were allowed for horizontal adjustments of open-category positions, and the more refined search of "horizontal reservation, migration" generated 256 cases at the Supreme Court and High Courts. As far as we can tell, a vast majority of these cases relate to the shortcomings on justified envy.

[7]Judgement available in https://main.sci.gov.in/supremecourt/2019/44789/44789_2019_34_1501_25207_Judgement_18-Dec-2020.pdf, last accessed 01/26/2021.

[8]Judgement available in https://www.livelaw.in/pdf_upload/pdf_upload-380856.pdf, last accessed 01/26/2021.

rule. We conclude the theoretical part of our paper with Section 6 where an extension of our model is presented allowing for heterogeneity of positions across multiple institutions and a detailed literature review is presented. We devote Section 7 to our application in Chile, and Section 8 to our primary application from India, documenting elaborate evidence on the practical relevance of our findings. We conclude with an epilogue in Section 9 and relegate all proofs and the institutional background on vertical and horizontal reservations to an Online Appendix.

2. Model and the Primitives

There exist a finite set of individuals $\mathcal{I}$ and q identical positions. Each individual is in need of a single position and has a distinct merit score.[9] Let $\sigma(i) \in \mathbb{R}_+$ denote the merit score of individual $i \in \mathcal{I}$. While individuals with higher merit scores have higher claims for a position in the absence of affirmative action policies, special provisions are provided for the members of various disadvantaged populations through two types of affirmative action policies, called **vertical reservations (VR)** and **horizontal reservations (HR)**. We refer these policies as **VR protections** and **HR protections**.

2.1. Vertical Reservations. Qualification for VR protections is determined through a category membership. Let $\mathcal{R}$ denote the set of **reserve-eligible categories** and g denote the **general category**. Each individual belongs to a single category given by a category membership function $\rho : \mathcal{I} \to \mathcal{R} \cup \emptyset$, where, for any individual $i \in \mathcal{I}$,

- $\rho(i) = \emptyset$ indicates that i is a member of the general category g, and
- $\rho(i) = c$ indicates that i is a member of the reserve-eligible category $c \in \mathcal{R}$.

Members of the general category do not receive any special provisions under the VR policies. In contrast, q^c positions are "set aside" exclusively for the members of category $c \in \mathcal{R}$ under the VR policies. These positions are referred to as **category-c positions**. Since no position is earmarked for the members of the general category, the remaining $q^o = q - \sum_{c \in \mathcal{R}} q^c$ positions are open for all individuals. These positions are referred to as **open-category positions** (or **category-o positions**). Observe that,

- in contrast to category-c positions that are exclusively reserved for the members of the reserve-eligible category $c \in \mathcal{R}$,
- open-category positions are *not* exclusively reserved for the members of the general category g.

[9] While students can have the same merit score in practice, tie-breaking rules are used to strictly rank them. For example, the Union Public Service Commission uses age and exam scores to break ties. See `https://www.upsc.gov.in/sites/default/files/TiePrinciplesEngl-26022020-R.pdf` (last accessed on 6/7/2020).

Let $\mathcal{V} = \mathcal{R} \cup \{o\}$ denote the set of **vertical categories for positions**.

Definition 1. *Given a category $v \in \mathcal{V}$, an individual $i \in \mathcal{I}$ is **eligible** for positions in category v if*

$$v = o \ \text{ or } \ \rho(i) = v.$$

That is, while all individuals are eligible for open-category positions, only individuals who are members of category c are eligible for category-c positions for any reserve-eligible category $c \in \mathcal{R}$. Let $\mathcal{I}^v \subseteq \mathcal{I}$ denote the set of individuals eligible for category $v \in \mathcal{V}$.

Given a category $v \in \mathcal{V}$, a **single-category choice rule** is a function $C^v : 2^{\mathcal{I}} \to 2^{\mathcal{I}^v}$ such that, for any $I \subseteq \mathcal{I}$,

$$C^v(I) \subseteq I \cap \mathcal{I}^v \quad \text{and} \quad |C^v(I)| \leq q^v.$$

A **choice rule** is a profile of single-category choice rules $C = (C^v)_{v \in \mathcal{V}}$ such that, for any set of individuals $I \subseteq \mathcal{I}$ and two distinct categories $v, v' \in \mathcal{V}$,

$$C^v(I) \cap C^{v'}(I) = \emptyset.$$

In addition to the specification of the recipients of the positions, a choice rule also specifies the categories of these positions.

Given a choice rule $C = (C^v)_{v \in \mathcal{V}}$, define the **aggregate choice rule** $\widehat{C} : 2^{\mathcal{I}} \to 2^{\mathcal{I}}$ as

$$\widehat{C}(I) = \bigcup_{v \in \mathcal{V}} C^v(I) \qquad \text{for any } I \subseteq \mathcal{I}.$$

For any set of individuals, the aggregate choice rule gives the set of chosen individuals across all categories.

In the absence of horizontal reservations introduced in Section 2.2, the following three federally-mandated principles uniquely define a choice rule, making the implementation of VR policies straightforward. First, an allocation must respect *inter se* merit: Given two individuals from the same category, if the individual with the lower merit score is given a position, then the individual with the higher merit score must also be given a position. Next, when an individual from a reserve-eligible category receives an open-category position on the basis of his merit score alone, this assignment does not count against the vertical reservations for his reserve-eligible category. This is the sense in which VR positions are "set aside" for members of reserve-eligible categories, regardless of who receives open-category positions. Finally, subject to eligibility requirements, all positions has to be filled to the extent the two principles above would allow. It is easy to see that these three principles uniquely imply the following choice rule: First, individuals with the highest merit scores are allocated the open-category positions. Next, positions reserved for the reserve-eligible categories are allocated to the remaining members of these categories, again based on their merit scores.

2.2. Horizontal Reservations within Vertical Reservations. In addition to a possible membership of a category, each individual also has a (possibly empty) set of traits. Each trait represents a disadvantage in the society, and the government provides individuals who have this trait with easier access to positions. The set of traits is finite and denoted by $\mathcal{T}$, and the (possibly empty) set of traits of individual $i \in \mathcal{I}$ is given by $\tau(i) \subseteq \mathcal{T}$.

For any reserve-eligible category $c \in \mathcal{C}$ and any trait $t \in \mathcal{T}$, let q_t^c denote the number of category-c positions that are HR-protected for individuals from category-c with trait-t. Similarly, let q_t^o denote the number of open-category positions that are HR-protected for individuals with trait-t .

For each vertical category $v \in \mathcal{V}$, we assume that the aggregate number of HR-protected positions for category v is no more than the number of positions in this category, i.e., for every category $v \in \mathcal{V}$,

$$\sum_{t \in \mathcal{T}} q_t^v \leq q^v.$$

Unlike the VR protections which are provided on a "set-aside" basis, HR protections are provided within each vertical category on a "minimum guarantee" basis. Importantly, while an individual can qualify for multiple HR protections through different traits, upon admission she counts towards the minimum guarantee only for one of them. For example a woman with disabilities can count towards either the minimum guarantee for the HR protections for women or the minimum guarantee for the HR protections for persons with disabilities, but not both. We refer this convention of implementing HR protections as **one-to-one HR matching**. Under an alternative **one-to-all HR matching** convention, an individual can accommodate the minimum guarantee for all her traits. There are two reasons for this important modeling choice. First of all, while either convention appears to be allowed by Indian laws, the former is more widespread in the field.[10] The second reason is technical. The analysis of HR policies becomes both computationally hard and also less elegant under the one-to-all HR matching convention.[11]

3. Single-Category Analysis with Overlapping Horizontal Reservations

Since HR policies are implemented within vertical categories, we start our analysis with the case of a single category, and thus with a special case of our model with HR protections only. Throughout this section, we fix a category $v \in \mathcal{V}$.

[10]This is sometimes explicitly indicated by the allocation rule and sometimes implicitly implied by the practice of assigning individuals to category-trait pairs.

[11]See Sönmez and Yenmez (2020) for an analysis with two traits.

While each individual can benefit from VR protections through at most one reserve-eligible category, she can potentially benefit from HR protections through multiple traits even within a single vertical category. This results in a potentially overlapping structure for HR policies, thus making it technically more involved than the analysis of VR policies. This technical aspect of the overlapping HR protections is also the source of three shortcomings of the SCI-AKG choice rule. To motivate our axioms and the meritorious horizontal choice rule that we introduce later in this section, we first present the mechanics and shortcomings of the SCI subroutine responsible for handling the HR protections.

3.1. AKG Horizontal Adjustment Subroutine and Its Shortcomings. The mechanics for implementing HR protections is described in the two Supreme Court judgements *Anil Kumar Gupta (1995)* and *Rajesh Kumar Daria (2007)*. Both judgements describe the procedure for a single trait, although the procedure can be repeated sequentially for each trait. In our description below, we adhere to this straightforward extension of the procedure.[12]

As we argue in Section 2.1, implementing VR protections is straightforward in the absence of HR protections. First open-category positions are allocated to the highest merit score candidates (across all categories), followed by the positions at each reserve-eligible category to the highest merit score remaining candidates from these categories. This is indeed the first step of the SCI-AKG choice rule. Once a tentative assignment is determined, the necessary adjustments are subsequently made to implement HR protections, first for the open-category positions, and subsequently for positions at each reserve-eligible category. The adjustment process is repeated for each trait.

Formally, for a given category $v \in \mathcal{V}$ of positions, let a set of individuals $J \subseteq \mathcal{I}^v$ who are tentatively assigned to category-v positions and a set of individuals $K \subseteq \mathcal{I}^v \setminus J$ who are eligible for horizontal adjustments at category v be such that

(1) $|J| = q^v$ and
(2) $\sigma(i) > \sigma(i')$ for any $i \in J$ and $i' \in K$.

Then for a given processing sequence $t^1, t^2, \ldots, t^{|\mathcal{T}|}$ of traits, the horizontal adjustment process is carried out with the following procedure.

AKG Horizontal Adjustment Subroutine (AKG-HAS)

Step 1 (Trait-t^1 adjustments): Let r_1 be the number of individuals in J with trait t^1.

Case 1. $r_1 \geq q^v_{t^1}$

[12]The description of this mechanics in the Supreme Court judgements *Anil Kumar Gupta (1995)* and *Rajesh Kumar Daria (2007)* can be seen in Section C.3 of the Online Appendix.

Let J^1 be the set of $q^v_{t^1}$-highest merit score individuals in J with trait t^1. Finalize their assignments as the recipients of trait-t^1 HR-protected positions within category v. Proceed to Step 2.

Case 2. $r_1 < q^v_{t^1}$

Let J^1_m be the set of all individuals in J with trait t^1. Let s_1 be the number individuals in K who have trait t^1. Let J^1_h be

- the set of $(q^v_{t^1} - |J^1_m|)$ highest merit score individuals in K who have trait t^1 if $s_1 \geq q^v_{t^1} - |J^1_m|$, and
- the set of all individuals in K who have trait t^1 if $s_1 < q^v_{t^1} - |J^1_m|$.

Let $J^1 = J^1_m \cup J^1_h$ and finalize their assignments as the recipients of trait-t^1 HR-protected positions within category v. Proceed to Step 2.

Step $k \in \{2, \ldots, |\mathcal{T}|\}$ (Trait-t^k adjustments): Let r_k be the number of individuals in $J \setminus \bigcup_{\ell=1}^{k-1} J^\ell$ with trait t^k.

Case 1. $r_k \geq q^v_{t^k}$

Let J^k be the set of $q^v_{t^k}$ highest merit score individuals in $J \setminus \bigcup_{\ell=1}^{k-1} J^\ell$ with trait t^k. Finalize their assignments as the recipients of trait-t^k HR-protected positions within category v. Proceed to Step 2.

Case 2. $r_k < q^v_{t^k}$

Let J^k_m be the set of all individuals in $J \setminus \bigcup_{\ell=1}^{k-1} J^\ell$ with trait t^k. Let s_k be the number individuals in $K \setminus \bigcup_{\ell=1}^{k-1} J^\ell$ with trait t^k. Let J^k_h be

- the set of $(q^v_{t^1} - |J^k_m|)$ highest merit score individuals in $K \setminus \bigcup_{\ell=1}^{k-1} J^\ell$ who have trait t^k if $s_k \geq q^v_{t^k} - |J^k_m|$, and
- the set of all individuals in $K \setminus \bigcup_{\ell=1}^{k-1} J^\ell$ who have trait t^k if $s_k < q^v_{t^k} - |J^k_m|$.

Let $J^k = J^k_m \cup J^k_h$ and finalize their assignments as the recipients of trait-t^k HR-protected positions within category v. Proceed to Step (k+1).

Step $(|\mathcal{T}|+1)$ (Finalization of category-v no-trait assignments): Let J^0 be the set of $\left(q^v - \sum_{\ell=1}^{|\mathcal{T}|} |J^\ell|\right)$ highest merit score individuals in $J \setminus \bigcup_{\ell=1}^{|\mathcal{T}|} J^\ell$

The procedure selects the set of individuals in $\bigcup_{\ell=0}^{|\mathcal{T}|} J^\ell$. Here J^0 is the set of individuals from the original group J who have survived the horizontal adjustment process without invoking any HR protection, and J^k is the set of individuals who accommodate trait-t^k HR protections for any trait t^k.[13]

[13] While all individuals in J^k accommodate trait-t^k HR protections, only those who are in the set $J^k \setminus J$ owe their assignments to trait-t^k HR protections.

When each individual has at most one trait, it is easy to see that the processing sequence of traits becomes immaterial under the AKG-HAS, and it produces the same outcome as the following category-v *minimum guarantee choice rule* C^v_{mg} (Echenique and Yenmez (2015)) applied to the set of individuals $J \cup K$.

Minimum Guarantee Choice Rule C^v_{mg}

For every $I \subseteq \mathcal{I}^v$,
Step 1: For each trait $t \in \mathcal{T}$,

- choose all individuals in I with trait t if the number of trait-t individuals in I is no more than q^v_t, and
- q^v_t highest merit-score individuals in I with trait t otherwise.

Step 2: For positions unfilled in Step 1, choose unassigned individuals in I with highest merit scores.

Our first result formulates this observation.

Proposition 1. *Suppose that each individual has at most one trait. Let $v \in \mathcal{V}$ be any category of positions, $J \subseteq \mathcal{I}^v$ be a set of individuals who are tentatively assigned category-v positions, and $K \subseteq \mathcal{I}^v \setminus J$ be a set of unmatched individuals who are eligible for category-v positions. If $|J| = q^v$ and every individual in J has a higher merit score than every individual in K, then $C^v_{mg}(J \cup K)$ is the set of individuals who are assigned to category-v positions under the AKG-HAS.*

While well-behaved when each individual has no more than one trait, this procedure suffers from three shortcomings when individuals have multiple traits. We next present these shortcomings in two examples.

Example 1. There is one category (say open category), four individuals i_1, i_2, i_3, i_4 and two positions. There are two traits t_1, t_2 with one HR-protected position for each trait. Individual i_1 has both traits, individuals i_2, i_3 have no trait and individual i_4 has trait t_1 only. Individuals are merit ranked as

$$\sigma(i_1) > \sigma(i_2) > \sigma(i_3) > \sigma(i_4).$$

Prior to horizontal adjustments, the positions are tentatively awarded to individuals i_1 and i_2. Since only one of the minimum guarantees can be accommodated by this group, the AKG-HAS is invoked. Nonetheless, if trait-t_1 adjustments are carried out prior to trait-t_2 adjustments, this allocation does not change through the AKG-HAS: Once the highest merit score individual i_1 accounts for the trait-t_1 minimum guarantee, no one else has trait t_2. Hence the final set of awardees remains unaltered from the tentative one as $\{i_1, i_2\}$. If, however, trait-t_2 adjustments are carried out prior to trait-t_1 adjustments, then the allocation changes through the AKG-HAS: This time the highest merit score individual i_1 accounts for the trait-t_2 minimum guarantee instead, and the trait-t_1 minimum guarantee can be accommodated subsequently by individual i_4. Hence in this second scenario, the final set of awardees is $\{i_1, i_4\}$.

Two shortcomings of the AKG-HAS is revealed by Example 1:

(1) The outcome of this adjustment procedure depends on the processing sequence of the traits, and hence the single-category choice rule induced through this subroutine is not well-defined.
(2) Fewer than the maximum feasible number of HR protections may be accommodated under this procedure.

Essentially, Example 1 shows that the mechanical processing sequence of traits may result in an "under adjustment" under the AKG-HAS. Our next example reveals that, it can also result in adjustments by admitting needlessly low merit score individuals.

Example 2. There is one category (say open category), four individuals i_1, i_2, i_3, i_4 and two positions. There are two traits t_1, t_2 with one HR-protected position for each trait. Individual i_1 has both traits, individual i_2 has no trait, individual i_3 has only trait t_2, and individual i_4 has only trait t_1. Individuals are merit ranked as

$$\sigma(i_1) > \sigma(i_2) > \sigma(i_3) > \sigma(i_4).$$

Prior to horizontal adjustments, the positions are tentatively awarded to individuals i_1 and i_2. Since only one of the minimum guarantees can be accommodated by this group, the AKG-HAS is invoked. If trait-t_1 adjustments are carried out prior to trait-t_2 adjustments, then the AKG-HAS replaces individual i_2 with individual i_3: The highest merit score individual i_1 already accounts for the trait-t_1 minimum guarantee, and subsequently individual i_3 is the only remaining individual with trait t_2. Hence the final set of awardees is $\{i_1, i_3\}$. If, however, trait-t_2 adjustments are carried out prior to trait-t_1 adjustments, then the AKG-HAS instead replaces individual i_2 with individual i_4: The highest merit score individual i_1 this time accounts for the trait-t_2 minimum guarantee, and subsequently individual i_4 is the only remaining individual with trait t_1. Hence the final set of awardees is $\{i_1, i_4\}$.

Example 2 reveals a third shortcoming of the AKG-HAS: Depending on the processing sequence of traits, this procedure may carry out its adjustments in an "ineffective" way by admitting lower merit-score individuals than it is necessary. In this example, it is not necessary to admit the lowest merit-score individual i_4 to accommodate the HR protections.

These two examples not only motivate the axioms formulated in Section 3.2, but also the *meritorious horizontal choice rule* introduced in Section 3.3.

3.2. HR Graph and Single-Category Axioms. To formulate our single-category axioms, we use the following construction.

Given a category $v \in \mathcal{V}$ and a set of individuals $I \subseteq \mathcal{I}^v$, construct the following two-sided **category-v HR graph**. On one side of the graph, there are individuals in I. On the other side, there are HR-protected positions for category v. Let H^v_t denote the set of trait-t HR-protected positions for category v and let $H^v = \bigcup_{t\in\mathcal{T}} H^v_t$. There are q^v_t positions in H^v_t and $\sum_{t\in\mathcal{T}} q^v_t$ positions in H^v. An individual $i \in I$ and a position $p \in H^v_t$ are **connected** in this graph if and only if individual i has trait t.

Definition 2. *Given a category $v \in \mathcal{V}$ and a set of individuals $I \subseteq \mathcal{I}^v$, a **trait-matching** of individuals in I with HR-protected positions in H^v is a function $\mu : I \to H^v \cup \{\emptyset\}$ such that*

(1) for any $i \in I$ and $\mu(i) \in H^v$, individual i is connected with position $\mu(i)$, and
(2) for any $i, j \in I$,

$$\mu(i) = \mu(j) \neq \emptyset \implies i = j.$$

Definition 3. *Given a category $v \in \mathcal{V}$ and a set of individuals $I \subseteq \mathcal{I}^v$, a trait-matching of individuals in I with HR-protected positions in H^v **has maximum cardinality in (category-v) HR graph** if there exists no other trait-matching that assigns a strictly higher number of HR-protected positions to individuals.*

Let $\boldsymbol{n^v(I)}$ denote the maximum number of category-v HR-protected positions that can be assigned to individuals in I.[14]

Definition 4. *Given a category $v \in \mathcal{V}$ and a set of individuals $I \subseteq \mathcal{I}^v$, an individual $i \in \mathcal{I}^v \setminus I$ **increases (category-v) HR utilization of I** if*

$$n^v(I \cup \{i\}) = n^v(I) + 1.$$

We are ready to formulate our single-category axioms. Our first axiom is motivated by Example 1, and it precludes underutilization of HR protections.

Definition 5. *Given a category $v \in \mathcal{V}$ and a set of individuals $I \subseteq \mathcal{I}^v$, a set of individuals $J \subseteq I$ **maximally accommodates (category-v) HR protections for I** if*

$$n^v(J) = n^v(I).$$

[14]This number can be found through a number of polynomial time algorithms such as *Edmonds' Blossom Algorithm* (Edmonds, 1965).

Given a category $v \in \mathcal{V}$, a single-category choice rule C^v ***maximally accommodates HR protections****, if for every set of individuals $I \subseteq \mathcal{I}^v$, the set of selected individuals $C^v(I)$ maximally accommodates HR protections for I.*

Our second axiom is motivated by Example 2, and it precludes unnecessary rejection of higher merit score individuals at the expense of lower merit score ones due to suboptimal accounting of individuals for HR-protected positions.

Definition 6. *Given a category $v \in \mathcal{V}$, a single-category choice rule C^v satisfies* ***no justified envy****, if for every $I \subseteq \mathcal{I}^v$, $i \in C^v(I)$, and $j \in I \setminus C^v(I)$,*

$$\sigma(j) > \sigma(i) \implies n^v\big((C^v(I) \setminus \{i\}) \cup \{j\}\big) < n^v(C^v(I)).$$

In words, if a higher merit score individual j is rejected even though a lower merit score individual i is chosen, then it must be the case that replacing i with j would decrease the number of HR-protected positions that can be filled with intended beneficiaries. When this condition is violated, we say that there is **justified envy**, which means that there exist a set of individuals I and two individuals $i, j \in I$ such that

(1) $\sigma(j) > \sigma(i)$,
(2) $i \in C^v(I)$,
(3) $j \notin C^v(I)$, and
(4) $n^v\big((C^v(I) \setminus \{i\}) \bigcup \{j\}\big) \geq n^v(C^v(I))$.

Therefore, when there is justified envy, a lower merit score individual can be replaced with a higher merit score one without any adverse affect on the intended HR policies.

Our third axiom, standard in the analysis of choice rules, is a weak efficiency property.

Definition 7. *Given a category $v \in \mathcal{V}$, a single-category choice rule C^v is* ***non-wasteful****, if for every $I \subseteq \mathcal{I}^v$,*

$$|C^v(I)| = \min\{|I|, q^v\}.$$

Equivalently, non-wastefulness requires that an individual is rejected only when no position remains.

3.3. Meritorious Horizontal Choice Rule. We are ready to introduce a single-category choice rule that escapes the shortcomings presented in Examples 1 and 2. The main innovation in this choice rule is the optimization it carries out to determine who is to account for each minimum guarantee when some of the individuals can account for one or another due to multiple traits they have. Intuitively, this choice rule exploits the flexibility in trait-matching in order to accommodate the HR protections with higher merit-score individuals.

Given a category $v \in \mathcal{V}$ and a set of individuals $I \subseteq \mathcal{I}^v$, the outcome of this choice rule is obtained with the following procedure.

Meritorious Horizontal Choice Rule $C^v_{Ⓜ}$

Step 1.1: Choose the highest merit-score individual in I with a trait for a HR-protected position. Denote this individual by i_1 and let $I_1 = \{i_1\}$. If there exists no such individual, proceed to Step 2.

Step 1.k ($k \in \{2, \ldots, \sum_{t \in \mathcal{T}} q^v_t\}$): Assuming such an individual exists, choose the highest merit-score individual in $I \setminus I_{k-1}$ who increases the HR utilization of I_{k-1}.[15] Denote this individual by i_k and let $I_k = I_{k-1} \cup \{i_k\}$. If no such individual exists, proceed to Step 2.

Step 2: For unfilled positions, choose unassigned individuals with highest merit scores until either all positions are filled or all individuals are selected.

When the number of individuals is less than q^v, this procedure selects all individuals. Otherwise, if there are more than q^v individuals, then it chooses a set with q^v individuals.

Example 3. Consider the following economy:

- $\mathcal{I} = \{i_1, i_2, i_3, i_4, i_5, i_6, i_7\}$,
- $\mathcal{T} = \{t_1, t_2, t_3\}$,
- $q^v_{t_1} = q^v_{t_2} = q^v_{t_3} = 1$ and $q^v = 5$,
- $\sigma(i_1) > \sigma(i_2) > \sigma(i_3) > \sigma(i_4) > \sigma(i_5) > \sigma(i_6) > \sigma(i_7)$,
- $\tau(i_1) = \emptyset$, $\tau(i_2) = \{t_1, t_2, t_3\}$, $\tau(i_3) = \emptyset$,
  $\tau(i_4) = \{t_1, t_2\}$, $\tau(i_5) = \{t_1\}$, $\tau(i_6) = \{t_3\}$, $\tau(i_7) = \{t_2\}$.

The HR graph for this allocation problem has one HR-protected position for each trait and three HR-protected positions in total. An individual is connected with a position if the individual has the corresponding trait. The HR graph is depicted in Figure 1.

Let $\mathcal{I}$ be the set of individuals in consideration. The meritorious horizontal choice rule works as follows. Having at least one trait each, only individuals i_2, i_4, i_5, i_6, and i_7 are qualified to receive an HR-protected position at the first step. At Step 1.1, individual i_2 is selected because she is the highest merit-score individual who qualifies for a HR-protected position. At Step 1.2, individual i_4 is selected because she is the highest merit-score individual who can simultaneously be trait-matched to a HR-protected position along with i_2. For example, i_2 can be trait-matched with s_1 and i_4 can be trait-matched with s_2 (see the dashed matching in Figure 2).

[15] This can be done with various computationally efficient algorithms, see, for example, the bipartite cardinality matching algorithm (Lawler, 2001, Page 195).

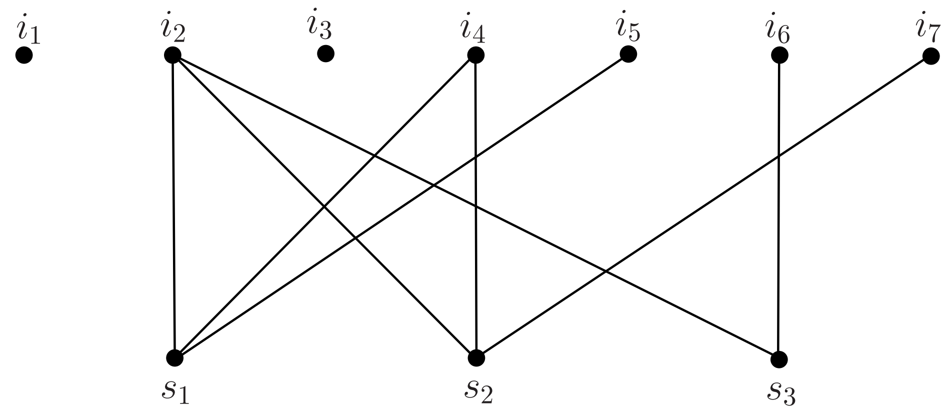

FIGURE 1. The HR graph of the allocation problem in Example 3. For each $k \in \{1, 2, 3\}$, node s_k represents the HR-protected position for trait t_k.

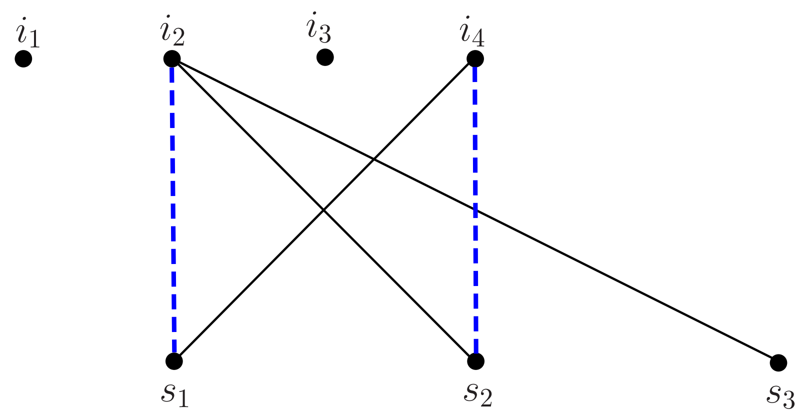

FIGURE 2. At Step 1.2, subject to matching individual i_2 with an HR-protected position, individual i_4 can also be matched with an HR-protected position, thus increasing HR utilization of $\{i_2\}$.

At Step 1.3, individual i_5 is selected because she is the highest merit-score individual who can be trait-matched to a HR-protected position together with i_2 and i_4. However, the implementation of such a trait matching requires that i_2 and i_4 are trait-matched with different positions than the dashed matching shown in Figure 2. To be more precise, i_2 can be trait-matched with s_3, i_4 can be trait-matched with s_2, and i_5 can be trait-matched with s_1 (see the dotted matching in Figure 3).[16]

No remaining individuals can be trait-matched with a HR-protected position together with i_2, i_4, and i_5, so we proceed to Step 2. At Step 2, individuals i_1 and i_3 are selected

[16]This step illustrates the necessity of keeping trait-matching flexible until the end of Step 1.

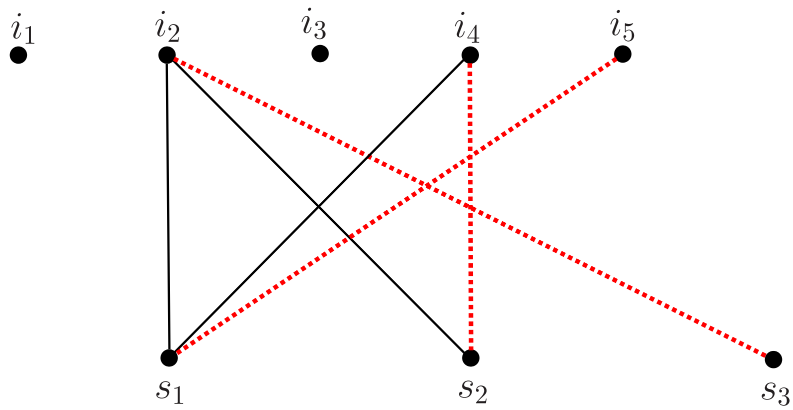

FIGURE 3. At Step 1.3, subject to matching individuals i_2 and i_4 with HR-protected positions, individual i_5 can also be matched with an HR-protected position, thus increasing HR utilization of $\{i_2, i_4\}$. However, matching individual i_5 requires changing the assignment of individual i_2 from position s_1 (i.e. from her assignment in Figure 2) to position s_3, since position s_1 is the only position individual i_5 is connected to.

because there are two vacant positions and they have the highest merit scores among the remaining individuals. Therefore, $C^v_{Ⓜ}(I) = \{i_1, i_2, i_3, i_4, i_5\}$. □

3.4. Single-Category Results. We next present our single-category results, which suggest that the case for the meritorious horizontal choice rule is especially strong in this framework.

Justifying the naming of this choice rule, our first main result shows that the meritorious horizontal choice rule $C^v_{Ⓜ}$ always selects higher merit-score individuals than any other choice rule that maximally accommodates HR protections.

Theorem 1. *Given a category $c \in \mathcal{V}$, let C^v be any single-category choice rule that maximally accommodates HR protections. Then, for every set of individuals $I \subseteq \mathcal{I}^v$,*

(1) $|C^v(I)| \leq |C^v_{Ⓜ}(I)|$, and
(2) for every $k \leq |C^v(I)|$, if i is the k-th highest merit-score individual in $C^v_{Ⓜ}(I)$ and j is the k-th highest merit-score individual in $C^v(I)$, then

$$i = j \quad \text{or} \quad \sigma(i) > \sigma(j).$$

We next present the main result of our single-category analysis, a characterization of the meritorious horizontal choice rule $C^v_{Ⓜ}$.

Theorem 2. *Given a category $v \in \mathcal{V}$, a single-category choice rule*

(1) maximally accommodates HR protections,

(2) satisfies no justified envy, and
(3) is non-wasteful

if, and only if, it is the meritorious horizontal choice rule $C^v_{Ⓜ}$.

Observe that, the algorithm for the meritorious horizontal choice rule $C^v_{Ⓜ}$ simplifies to the algorithm of the minimum guarantee choice rule C^v_{mg} when each individual has at most one horizontal trait.[17] Therefore, an immediate corollary to Theorem 2 is the following characterization of the minimum guarantee choice rule C^v_{mg}.

Corollary 1. *Assume that each individual has at most one horizontal trait and fix a category* $v \in \mathcal{V}$*. A single-category choice rule*

(1) maximally accommodates HR protections,
(2) satisfies no justified envy, and
(3) is non-wasteful

if, and only if, it is the minimum guarantee choice rule C^v_{mg}.

4. Multi-Category Analysis with Overlapping Horizontal Reservations

We are ready to analyze our model in its full generality.

4.1. Multi-Category Axioms. Our first three multi-category axioms are the immediate counterparts of their single-category versions, applied separately to each category.

Definition 8. *A choice rule* $C = (C^\nu)_{\nu \in \mathcal{V}}$ ***maximally accommodates HR protections****, if for every* $I \subseteq \mathcal{I}$, $v \in \mathcal{V}$, *and* $j \in I \setminus \widehat{C}(I)$ *who is eligible for category* v,

$$n^v(C^v(I)) = n^v(C^v(I) \cup \{j\}).$$

In words, an individual who is not awarded a position (at any category) should not be able to increase the utilization of HR protections for any category where she has eligibility.

Definition 9. *A choice rule* $C = (C^\nu)_{\nu \in \mathcal{V}}$ *satisfies* ***no justified envy****, if for every* $I \subseteq \mathcal{I}$, $v \in \mathcal{V}$, $i \in C^v(I)$*, and* $j \in I \setminus \widehat{C}(I)$ *who is eligible for category* v,

$$\sigma(j) > \sigma(i) \implies n^v\big((C^v(I) \setminus \{i\}) \cup \{j\}\big) < n^v(C^v(I)).$$

In words, for any category v,

- if an individual i receives a position at category v,
- while a higher merit-score and category-v eligible individual j is declined a position from all categories (including category v),

[17] More precisely, Step 1 of both algorithms produce the same outcome when each individual has at most one horizontal trait.

- then it must be the case that replacing individual j with individual i results in a decreased utilization of HR protections at category v.

Definition 10. *A choice rule $C = (C^\nu)_{\nu \in \mathcal{V}}$ is **non-wasteful** if, for every $I \subseteq \mathcal{I}$, $v \in \mathcal{V}$, and $j \in I \setminus \widehat{C}(I)$ who is eligible for category v,*

$$|C^v(I)| = q^v.$$

In words, if there is an idle position at a category v while an individual j remains unmatched (thus being declined a position from all categories), then it must be the case that individual j is not eligible for category v.

Our next axiom has no counterpart in the single-category framework for it regulates the relation between the recipients of open category positions and reserved-category positions. Since reserved-category positions are mandated to be allocated on a "set aside" basis under *Indra Sawhney (1992)*, in the absence of HR protections this axiom would simply require the merit score of each recipient of an open-category position to be higher than the merit score of any recipient of a reserved-category position. We extend this condition to our more general model as follows:

Definition 11. *A choice rule $C = (C^\nu)_{\nu \in \mathcal{V}}$ **complies with VR protections** if, for every $I \subseteq \mathcal{I}$, $c \in \mathcal{R}$, and $i \in C^c(I)$,*

(1) $|C^o(I)| = q^o$,

(2) for every $j \in C^o(I)$,

$$\sigma(j) < \sigma(i) \quad \implies \quad n^o\big(C^o(I)\big) > n^o\big((C^o(I) \setminus \{j\}) \cup \{i\}\big), \text{ and}$$

(3) $n^o\big(C^o(I) \cup \{i\}\big) = n^o\big(C^o(I)\big)$.

Here the first two conditions formulate the idea of a vertical reservation à la Indra Sawhney (1992), and they are directly suggested by the concept of "set aside." For an individual to receive a position set aside for a reserve-eligible category, it must be the case that each open-category position is either allocated to a higher merit-score individual or to an individual who accommodates an HR protection. The third condition, while natural, is extra, and it further requires that

- not only open-category positions should be allocated to higher merit-score individuals than the recipients of VR-protected positions,
- but also the open-category horizontal adjustments must be carried out with the highest merit-score individuals who are eligible for these adjustments.

4.2. Two-Step Meritorious Horizontal Choice Rule & Its Characterization. We are ready to formulate and propose a choice rule for our model in its full generality. The following choice rule uses the meritorious horizontal choice rule first to allocate open-category positions, and next in parallel for each reserve-eligible category to allocate vertically-reserved positions.

Two-Step Meritorious Horizontal (2SMH) Choice Rule $C^{2s}_{Ⓜ} = (C^{2s,\nu}_{Ⓜ})_{\nu\in\mathcal{V}}$

For every $I \subset \mathcal{I}$,

$$C^{2s,o}_{Ⓜ}(I) = C^{o}_{Ⓜ}(I), \text{ and}$$
$$C^{2s,c}_{Ⓜ}(I) = C^{c}_{Ⓜ}\big(\{i \in I \setminus C^{o}_{Ⓜ}(I) \ : \ \rho(i) = c\}\big) \quad \text{for any } c \in \mathcal{R}.$$

We now present the main result for our multi-category analysis, a characterization of the 2SMH choice rule.

Theorem 3. *A choice rule*

(1) *maximally accommodates HR protections,*
(2) *satisfies no justified envy,*
(3) *is non-wasteful, and*
(4) *complies with VR protections*

if, and only if, it is the 2SMH choice rule $C^{2s}_{Ⓜ}$.

Observe that, since the two choice rules C^{v}_{mg} and $C^{v}_{Ⓜ}$ are equivalent for any category $v \in \mathcal{V}$ when each individual has at most one horizontal trait, our proposed 2SMH choice rule is equivalent to the following *two-step minimum guarantee (2SMG) choice rule* under this condition.[18]

Two-Step Minimum Guarantee (2SMG) Choice Rule $C^{2s}_{mg} = (C^{2s,\nu}_{mg})_{\nu\in\mathcal{V}}$

For every $I \subseteq \mathcal{I}$,

$$C^{2s,o}_{mg}(I) = C^{o}_{mg}(I), \text{ and}$$
$$C^{2s,c}_{mg}(I) = C^{c}_{mg}\big(\{i \in I \setminus C^{o}_{Ⓜ}(I) \ : \ \rho(i) = c\}\big) \quad \text{for any } c \in \mathcal{R}.$$

Therefore, an immediate corollary of Theorem 3 is the following result:

Corollary 2. *Suppose each individual has at most one horizontal trait. A choice rule*

(1) *maximally accommodates HR protections,*
(2) *satisfies no justified envy,*
(3) *is non-wasteful, and*

[18]With the August 2020 judgement *Tamannaben Ashokbhai Desai (2020)* of the High Court of Gujarat, the 2SMH choice rule has been recently mandated in the state of Gujarat. Moreover, with its December 2020 judgement *Saurav Yadav (2020)*, this rule is endorsed by the the Supreme Court for the entire country. Our introduction and advocacy of this rule predates both of these important judgements.

(4) complies with VR protections

if, and only if, it is the 2SMG choice rule C^{2s}_{mg}.

All four axioms that uniquely characterize the 2SMH choice rule (or the 2SMG choice rule for the case of non-overlapping HR protections) are motivated by the legal framework in India along with the shortcomings of the SCI-AKG choice rule. Our first three axioms are fairly benign, and they are formulated to implement HR protections in the most meritorious way (as implied by our Theorem 1). The first two conditions of our last axiom *compliance with VR protections* are also both necessary due to the federally mandated "set aside" nature of the VR protections. The third condition of this axiom further requires carrying out the necessary horizontal adjustments with the highest merit-score individuals eligible for these adjustments, and while natural, it is not federally mandated. Hence, in our view it is the only condition that can be dropped while still staying true to the essence of *Indra Sawhney (1992)*.[19]

5. SCI-AKG Choice Rule vs. 2SMH Choice Rule

In Section 3.1 we show that the AKG horizontal adjustment subroutine is the source of three shortcomings of the SCI-AKG choice rule when an individual can qualify for multiple HR protections, and we introduce the meritorious horizontal choice rule in Section 3.3 as a remedy. We also show that, the AKG-HAS escapes these shortcomings when an individual can qualify for no more than one HR protection. This special case of the of the allocation problem where AKG-HAS is well-behaved is important in practice, because the only federally mandated HR protections in India is for *persons with disabilities*. The more general case where an individual can qualify for multiple HR protections is of practical relevance in some of the states only.[20]

The SCI-AKG choice rule, however, suffers from two additional shortcomings even when there is a single HR protection. Moreover, unlike the limitations of the AKG-HAS that largely escape public scrutiny, these two shortcomings are highly visible and they are responsible from thousands of litigations disrupting recruitment processes throughout India as presented in Section 8.1. To formulate these shortcomings, we need some additional notation.

[19]For the simpler version of the problem with non-overlapping HR protections, the choice rule C^{hor}_{ite} given in an earlier draft of this paper (Sönmez and Yenmez, 2019) fails the third condition of compliance with VR protections, but otherwise it satisfies all other conditions that uniquely characterize the 2SMG choice rule.

[20]While affirmative action in India is our main application due to its concurrent implementation of VR and HR policies, it is not the only real-life application where an individual can qualify for multiple HR protections. For example Section 7 for K-12 school choice in Chile, where a student can qualify for any subset of the HR protections for *financially disadvantaged, special needs,* and *high-achieving* students.

Given a set of individuals $I \in \mathcal{I}$, let

$$I^g = \{i \in I : \rho(i) = \emptyset\}$$

be the set of individuals in I who are members of the general category, and

$$I^c = \{i \in I : \rho(i) = c\}$$

be the set of individuals in I who are members of category c for a given reserve-eligible category $c \in \mathcal{R}$.

5.1. SCI-AKG Choice Rule.

We are ready to formulate the choice rule that is responsible from thousands of litigations in India in the last two decades.[21]

SCI-AKG Choice Rule C^{SCI}

For every $I \subseteq \mathcal{I}$,

Step 1 (Open-category tentative assignment):

- If $|I| \leq q^o$ then assign all individuals in I to open-category positions and terminate the procedure. In this case $C^{SCI,o}(I) = I$ and $C^{SCI,c}(I) = \emptyset$ for any reserve-eligible category $c \in \mathcal{R}$.
- Otherwise, if $|I| > q^o$ then tentatively assign the highest merit-score q^o individuals in I to open-category positions. Let J^o denote the set of individuals who are tentatively assigned to open-category positions in this case.

Step 2 (Finalization of open-category positions): The set of individuals eligible for open-category horizontal adjustments is $I^g \setminus J^o$. Apply the AKG-HAS

- to the set J^o of tentative recipients of open-category positions
- with the set of individuals in $I^g \setminus J^o$ who are eligible for open-category horizontal adjustments

to finalize the set of recipients $C^{SCI,o}(I)$ of open-category positions.

Step 3 (Reserve-eligible category tentative assignment): For any reserve-eligible category $c \in \mathcal{R}$,

- If $|I^c \setminus C^{SCI,o}(I)| \leq q^c$ then assign all individuals in $I^c \setminus C^{SCI,o}(I)$ to category-c positions and terminate the procedure. In this case $C^{SCI,c}(I) = I^c \setminus C^{SCI,o}(I)$.
- Otherwise, if $|I^c \setminus C^{SCI,o}(I)| > q^c$ then tentatively assign the highest merit-score q^c individuals in $I^c \setminus C^{SCI,o}(I)$ to category-c positions. Let J^c denote the set of individuals who are tentatively assigned to category-c positions in this case.

[21] The description of the SCI-AKG choice rule in the Supreme Court judgements *Anil Kumar Gupta (1995)* and *Rajesh Kumar Daria (2007)* can be seen in Section C of the Online Appendix.

Step 4 (Finalization of reserve-eligible category positions): For any reserve-eligible category $c \in \mathcal{R}$, the set of individuals eligible for category-c horizontal adjustments is $I^c \setminus (C^{SCI,o}(I) \cup J^c)$. For any reserve-eligible category $c \in \mathcal{R}$, apply the AKG-HAS

- to the set J^c of tentative recipients of category-c positions
- with the set of individuals in $I^c \setminus (C^{SCI,o}(I) \cup J^c)$ who are eligible for category-c horizontal adjustments

to finalize the set of recipients $C^{SCI,c}(I)$ of category-c positions.

The outcome of the SCI-AKG choice rule is $C^{SCI}(I) = \big(C^{SCI,v}(I)\big)_{v \in \mathcal{V}}$.

5.2. Deeper Anomalies of the SCI-AKG Choice Rule: Potential of Justified Envy and Failure of Incentive Compatibility. We have already presented in Section 3.1 that horizontal adjustments through AKG-HAS is problematic when an individual can qualify for multiple HR protections, an issue which can be corrected by using the meritorious horizontal choice rule for each vertical category. If we go though this adjustment, the native SCI-AKG choice rule transforms into an amended choice rule that is closely related to our proposed 2SMH choice rule. We need the following terminology to present this association.

Given a set of individuals $I \in \mathcal{I}$, define the set of **meritorious reserved candidates** $\boldsymbol{I^m}$ as the set of individuals in I, each of whom

(1) is a member of a reserve-eligible category in $\mathcal{R}$, and
(2) has a merit score among the q^o-highest merit scores of all individuals in I.

The SCI-AKG choice rule takes the following form when its horizontal adjustment process is amended:

Choice Rule $\boldsymbol{C^{SCI}_{\circledR{M}} = (C^{SCI,v}_{Ⓜ})_{v \in \mathcal{V}}}$

For every $I \subseteq \mathcal{I}$,

$$C^{SCI,o}_{Ⓜ}(I) = C^{o}_{Ⓜ}(I^m \cup I^g), \text{ and}$$
$$C^{SCI,c}_{Ⓜ}(I) = C^{c}_{Ⓜ}\big(\{i \in I \setminus C^{o}_{Ⓜ}(I) \; : \; \rho(i) = c\}\big) \quad \text{for any } c \in \mathcal{R}.$$

Observe that the only difference between the two choice rules $C^{SCI}_{Ⓜ}$ and $C^{2s}_{Ⓜ}$ is,

- while all individuals with relevant traits are considered for open-category HR protections under $C^{2s}_{Ⓜ}$,
- only the general category individuals and meritorious reserved candidates are considered for open-category HR protections under $C^{SCI}_{Ⓜ}$.

This observation immediately reveals an important conflict for individuals who both qualify for VR protections through their reserve-eligible categories and also for HR protections through their traits: With the exception of meritorious reserved candidates, these individuals lose their qualification for open-category HR protections by claiming their VR protections. It is important to emphasize that this conflict exists regardless of how many horizontal traits each individual can have, and therefore it is prevalent under both the native and also the amended version of the SCI-AKG choice rule.

This conflict reflects itself in the following two deficiencies that go against the philosophy of affirmative action under both versions of the SCI-AKG choice rule:

(1) ***Failure to satisfy no justified envy***: For example, a woman from the VR-protected category Scheduled Castes may remain unassigned while a lower merit-score woman from the higher-privilege general category receives a position through open-category HR protections for women.
(2) ***Failure to satisfy incentive compatibility***: For example, a woman from Scheduled Castes may remain unassigned by declaring her membership for Scheduled Castes, but she can receive an open-category HR-protected position for women by withholding her Scheduled Castes membership, thus benefitting from not declaring this information.

We have already formulated a more general version of the first property in Section 4.1. We next formulate the second one, first introduced by (Aygün and Bó, 2016) in their analysis of the affirmative action policies in Brazilian college admissions:

Definition 12. *An individual* ***withholds some of her reserve-eligible privileges*** *if she does not declare either her reserve-eligible category membership or some of her traits (or both).*

In India, individuals are not required to declare their reserve-eligible privileges.

Definition 13. *A choice rule C is* ***incentive compatible*** *if, for every $I \subseteq \mathcal{I}$ and $i \in I$, if individual i is selected from I under the aggregate choice rule $\widehat{C}$ by withholding some of her reserve-eligible privileges, then individual i is also selected from I under the aggregate choice rule $\widehat{C}$ by declaring all her reserve-eligible privileges.*

In other words, privileges that are intended to protect an individual do not instead hurt her upon declaring eligibility for these privileges (as one would normally expect) under an incentive compatible choice rule.

Failure of incentive compatibility is implausible both from a normative perspective since it is against the philosophy of affirmative action, and also from a strategic perspective since it forces individuals to chose between their VR protections and open-category

HR protections. As we present clear evidence in Section 8.1.2, it also creates an additional issue in our main application in India. Eligibility for VR protections typically depends on applicant's caste membership. While this information is supposed to be private information, it often can be inferred by the central planner due to indications such as the applicant's last name. Central planner can also obtain this information through documents such as graduation diploma. Hence eligibility for VR protections may not be truly private information, and the lack of incentive compatibility of a choice rule may enable a malicious central planner to use this information to deny an applicant her open-category HR protections. As documented in Section 8.1.2, this type of misconduct not only has been widespread in parts of India, but it even appears to be centrally organized by the local governing bodies in some of its jurisdictions.

We have already explained that the native version of the SCI-AKG choice rule fails to satisfy both no justified envy and also incentive compatibility in a rather visible way. Since an amendment via meritorious horizontal choice rule addresses completely independent shortcomings, the amended version too fails both desiderata. As for our proposed 2SMH choice rule $C^{2s}_{Ⓜ}$, we know from Theorem 3 that it satisfies no justified envy. Our next result shows that this choice rule also satisfies incentive compatibility.

Proposition 2. *The 2SMH choice rule $C^{2s}_{Ⓜ}$ satisfies incentive compatibility.*

5.3. The Case for the 2SMH Choice Rule. In light of our characterization in Theorem 3 and Proposition 2, we believe a compelling case can be made for the 2SMH choice rule $C^{2s}_{Ⓜ}$ as a better alternative to the SCI-AKG choice rule C^{SCI}. Not only $C^{2s}_{Ⓜ}$ escapes all five shortcomings of C^{SCI} presented in Sections 3.1 and 5.2, it does so through two simple modifications on the mechanics and implementation of the technical horizontal adjustment subroutine AKG-HAS, thereby keeping the main ideas of the SCI-AKG choice rule intact.

Naturally, the outcomes of the two choice rules may be different in general. We next show that, compared to $C^{SCI}_{Ⓜ}$

(1) the outcome of the $C^{2s}_{Ⓜ}$ is less favorable for individuals from the general category, and
(2) assuming there are at least as many individuals from each reserve-eligible category as the number of positions they are eligible for, the outcome of the $C^{2s}_{Ⓜ}$ is also more favorable overall for individuals from reserve-eligible categories.

The comparison is made with the amended version of the SCI-AKG choice rule rather than its native version, because the latter is not well-defined when individuals potentially qualify for multiple HR protections.

Proposition 3. *For every $I \subseteq \mathcal{I}$,*

$$\widehat{C}^{2s}_{Ⓜ}(I) \cap I^g \subseteq \widehat{C}^{SCI}_{Ⓜ}(I) \cap I^g,$$

and assuming $|I^c| \geq q^o + q^c$ for each reserve-eligible category $c \in \mathcal{R}$,

$$\sum_{c \in \mathcal{R}} \big|\widehat{C}^{2s}_{Ⓜ}(I) \cap I^c\big| \geq \sum_{c \in \mathcal{R}} \big|\widehat{C}^{SCI}_{Ⓜ}(I) \cap I^c\big|.$$

6. Extension to Centralized Allocation of Positions Across Multiple Institutions and the Related Literature

Mainly motivated by the shortcomings of the SCI-AKG choice rule, our main focus has been allocation of identical positions at a single institution. Over the last fifteen years, the celebrated *individual-proposing deferred acceptance* algorithm (Gale and Shapley, 1962) has become the mechanism of choice for priority-based allocation with heterogenous positions across multiple institutions, where the policies of the institutions are captured through the choice rules that are used in conjunction with this algorithm.[22] For this joint implementation to be well-defined, it is sufficient that each individual has strict preferences over the institutions (but otherwise indifferent between positions of any given institution) and the choice rule at each institution satisfies the following two properties:

Definition 14. *(Kelso and Crawford, 1982) A choice rule C satisfies the* ***substitutes*** *condition, if, for every $I \subseteq \mathcal{I}$,*

$$i \in \widehat{C}(I) \text{ and } j \in I \setminus \{i\} \implies i \in \widehat{C}(I \setminus \{j\}).$$

Definition 15. *(Aygün and Sönmez, 2013) A choice rule C satisfies the* ***irrelevance of rejected individuals*** *condition, if, for every $I \subseteq \mathcal{I}$,*

$$i \in I \setminus \widehat{C}(I) \implies \widehat{C}(I \setminus \{i\}) = \widehat{C}(I).$$

Our next result states that the 2SMH choice rule $C^{2s}_{Ⓜ}$ satisfies both properties.

Proposition 4. *The 2SMH choice rule $C^{2s}_{Ⓜ}$ satisfies the substitutes condition and the irrelevance of rejected individuals condition.*

Therefore, a natural (and straightforward) extension of our model involves a joint implementation of 2SMH choice rule with the individual-proposing deferred acceptance algorithm when there are multiple institutions introducing heterogeneity in positions.

[22] This is also the case for our K-12 school choice application in Chile presented in Section 7.

6.1. Related Literature. Our theoretical analysis of reservation policies differs from its predecessors in the following two ways:

(1) *concurrent* implementation of vertical and horizontal reservation policies and
(2) potentially *overlapping* structure of horizontal reservations.

While there is a rich literature on affirmative action policies in India and elsewhere, our paper is the first one to formally analyze vertical and horizontal reservation policies when they are implemented concurrently. The aspect of overlapping horizontal reservations has received some attention in the literature (Kurata et al. (2017)), albeit for a different variant of the problem where individuals have strict preferences for whether and which protection is invoked in securing a position. When applied in an environment where individuals are indifferent between all positions, choice rules recommended in Kurata et al. (2017) result in all three shortcomings presented in Section 3. This observation is the basis of our proposed reform for the K-12 school choice application in Chile, presented in Section 7.

There are a number of recent papers on reservation policies, most in the context of school choice. Abdulkadiroğlu and Sönmez (2003) study affirmative action policies that limit the number of admitted students of a given type through hard quotas. Kojima (2012) shows that a policy of limiting the number of majority students through hard quotas can hurt minority students, the intended beneficiaries. To overcome the detrimental effect of affirmative action policies based on majority quotas, Hafalir et al. (2013) introduce affirmative action policies based on *minority reserves*. In the absence of overlapping reservations, Echenique and Yenmez (2015) present the first axiomatic characterization of the minimum guarantee choice rule C^v_{mg}. Most recently Pathak et al. (2020b) consider a general model of reservation policies to balance various ethical principles for pandemic medical resource allocation, although their model is not equipped to analyze concurrent implementation of vertical and overlapping horizontal reservation policies.

A few papers study implementation of vertical or (non-overlapping) horizontal reservations individually in various real-life applications. These include Dur et al. (2018) for school choice in Boston, Dur et al. (2020) for school choice in Chicago, and Pathak et al. (2020a) for H-1B visa allocation in the US. All these models are applications of the more general model in Kominers and Sönmez (2016), where the authors introduce a matching model with *slot-specific priorities*. In contrast, our model is independent than Kominers and Sönmez (2016). Three additional papers on reservation policies include Aygün and Turhan (2016, 2017), where the authors study admissions to engineering colleges in India, and Aygün and Bó (2016), where the authors study admissions to Brazilian public universities. While the application in Aygün and Turhan (2016, 2017) is closely related to ours, their analysis is independent because not only horizontal reservations are assumed away altogether in these papers, but also analysis in these papers largely abstract away from the legal requirements in India.[23] In contrast, our model and analysis completely build on Indian laws on reservation policies, and all shortcomings we formulate disappear in the absence of horizontal reservations. The Brazilian affirmative action application studied by Aygün and Bó (2016) relates to ours in that it also includes multi-dimensional reservation policies, but unlike our models their application is a special case of Kominers and Sönmez (2016). There is, however, one important element in our paper that directly builds on Aygün and Bó (2016). Not only the two desiderata that play an important role in our proposed reform in India, *no justified envy* and *incentive compatibility* are originally introduced by Aygün and Bó (2016), but also evidence from aggregate data is presented in this paper that the presence of justified envy is widespread in Brazil. As in Aygün and Bó (2016), we also present extensive evidence of justified envy in the field, but in addition we also document the large scale disruption this anomaly creates in the field. Other less related papers on reservation policies include Westkamp (2013), Ehlers et al. (2014), Kamada and Kojima (2015), and Fragiadakis and Troyan (2017).

More broadly, our paper contributes to the field of market design, where economists are increasingly taking advantage of advances in technology to design new or improved allocation mechanisms in applications as diverse as entry-level labor markets (Roth and Peranson, 1999), school choice (Balinski and Sönmez, 1999; Abdulkadiroğlu and Sönmez, 2003), spectrum auctions (Milgrom, 2000), kidney exchange (Roth et al., 2004, 2005), internet auctions (Edelman et al., 2007; Varian, 2007), course allocation (Sönmez and Ünver, 2010; Budish, 2011), cadet-branch matching (Sönmez and Switzer, 2013; Sönmez, 2013), assignment of airline arrival slots (Schummer and Vohra, 2013; Schummer and Abizada, 2017), and refugee matching (Jones and Teytelboym, 2017; Delacrétaz et al., 2016; Andersson, 2017).

7. Application: School Inclusion Law in Chile

With the promulgation of the *School Inclusion Law* in Chile in 2015, a centralized school choice system has been adopted in Chile, following a similar series of reforms throughout the world (Correa et al., 2019). The system is the product of an ongoing collaboration between the Chilean Ministry of Education and a team of researchers from economics and operations research, and it covers all grades prior to higher education (i.e., Pre-K to grade 12). The system was first implemented in 2016 as a pilot program in the smallest of

[23]See also the discussion of Indian college admissions in (Echenique and Yenmez, 2015, Appendix C.1).

the sixteen regions of Chile, and it has been adopted in all regions but the Metropolitan Area of Santiago by 2019, where over 274,000 students applied to more than 6400 schools.

As many of its predecessors, the Chilean school choice system is based on the celebrated individual-proposing deferred acceptance algorithm, and the following three features in its design make it a perfect application of our model:

(1) To promote diversity, the School Inclusion Law includes affirmative action policies for financially disadvantaged students and children with special needs. Under the new system, these policies are implemented through reserved seats at each school. In addition, a number of schools are allowed to reserve seats for high-achieving students. Hence, using our terminology there are three traits, *Financially disadvantaged, Special needs, High-achieving*, where a student potentially can have any subset of these traits, possibly including none of them. Students with none of the three traits are called *Regular*.
(2) While a student with multiple traits (say a financially disadvantaged student who is also high-achieving) is eligible for reserved seats for each of her traits, parallel to our modeling choice of overlapping horizontal reservations she "consumes" only one of the reserved seats in case she receives a seat. This feature in Chilean design eliminates potential complementarities between the regular students and students with multiple traits.
(3) Reserved seats at each school are implemented in the form of a minimum guarantee.

As also emphasized in the Introduction, a subtle implication of the second design feature is that it allows the model to be interpreted as an application of the *matching with contracts* model of Hatfield and Milgrom (2005), where the contractual term between a school and a student specifies which of the four types of seats (i.e., open seats, reserved seats for financially disadvantaged students, reserved seats for special needs students, and reserved seats for high-achieving students) the student receives, an approach that is taken in Kurata et al. (2017). However, the theory of matching with contracts is developed under the assumption that students have strict preferences over all their contracts, which in this context corresponds to them having strict preferences on the specific type of seats they receive at each school. Since students have preferences over only schools, a fixed tie-breaking rule is used to construct student preferences over specific type of seats at each school. In Correa et al. (2019), the designers emphasize that the choice of a tie-breaking rule is not straightforward, and it has distributional consequences. In order to implement the reserves in the form of a minimum guarantee, they break ties in a way each student is assumed to prefer reserved seats for any of their traits to open seats. When each student has at most one trait, this construction assures that the reserves are implemented as a minimum guarantee (Hafalir et al. (2013), Sönmez and Yenmez (2019)). However, analogous to phenomena presented in Examples 1 and 2 in Section 3.1, interpreting this problem as an application of matching with contracts and relying on a fixed tie-breaking between reserved seats results in a number of shortcomings including an "underutilization" of reserved positions as well as their "ineffective" implementation through admitting needlessly low baseline priority students. Therefore, a better approach would be using the meritorious horizontal choice rule for each school.

8. Application: Affirmative Action in India

Background on the legal framework for implementation of vertical and horizontal reservation policies in India is presented in Section C of the Online Appendix, and a formal comparison of the SCI-AKG choice rule with our proposed 2SMH choice rule is presented in Section 5. In this section we present extensive evidence on the disarray caused by the shortcomings of the SCI-AKG choice rule in India, and articulate how the potential disruptive effects of this allocation rule can be expected to be amplified in the presence of a new vertical reservation category introduced by a 2019 amendment of the Constitution of India.

8.1. Litigations on the SCI-AKG Choice Rule. As we have argued in Section 5.2, the SCI-AKG choice rule allows for justified envy. Moreover, it also fails incentive compatibility due to backward class candidates losing their open-category HR protections upon claiming their VR protections by declaring their backward class status.

The failure of SCI-AKG choice rule to satisfy no justified envy is fairly straightforward to observe. All it takes is a rejected backward class candidate to realize that her merit score is higher than an accepted general-category candidate, even though she qualifies for all the HR protections the less-deserving (but still accepted) candidate qualifies for. Since the primary role of the reservation policy is positive discrimination for candidates with more vulnerable backgrounds, this situation is very counterintuitive, and it often results in legal action. Focusing on complications caused by either anomaly, we next present several court cases to document how they handicap concurrent implementation of vertical and horizontal reservation policies in India.

8.1.1. ***High Court Cases Related to Justified Envy.*** [24]

[24] Much of our analysis and the High Court judgements we present in this section parallels the arguments and the decision of the December 2020 Supreme Court case *Saurav Yadav v State of Uttar Pradesh (2020)*. Our analysis predates this important judgement, and it was already presented in an earlier draft of this paper in Sönmez and Yenmez (2019).

The possibility of justified envy under the SCI-AKG choice rule has resulted in numerous court cases throughout India for more than two decades, and since the presence of justified envy in the system is highly implausible, these legal challenges often result in controversial rulings. In addition, there are also cases where authorities who implement a better-behaved version of the choice rule, one that does not suffer from this shortcoming, are nonetheless challenged in court, on the basis that their adopted choice rules differ from the one mandated by the Supreme Court. These court cases are not restricted to lower courts, and include several cases in state high courts. Even at the level of state high courts, the judgements on this issue are highly inconsistent, largely due to the disarray created by the possibility of justified envy under the SCI-AKG choice rule. We next present four representative cases from high courts, each from a different state:

(1) ***Ashish Kumar Pandey And 24 Others vs State Of U.P. And 29 Others on 16 March, 2016,* Allahabad High Court**.[25] This lawsuit was brought to Allahabad High Court by 25 petitioners, disputing the mechanism employed by the State of Uttar Pradesh—the most populous state in India with more than 200 million residents—to apply the provisions of horizontal reservations in their allocation of more than 4000 civil police and platoon commander positions. Of these positions, 27%, 21%, 2% are each vertically reserved for backward classes OBC, SC, and ST, respectively, and 20%, 5%, and 2% are each horizontally reserved for women, ex-servicemen, and dependents of freedom fighters, respectively. While only 19 women are selected for open-category positions based on their merit scores, the total number of female candidates is less than even the number of open-category horizontally reserved positions for women, and as such all remaining women are selected. However, instead of assigning them positions from their respective backward class categories (as it is mandated under the SCI-AKG choice rule), all of them are assigned positions from the open category. Similarly, backward class candidates are deemed eligible to use horizontal reservations for dependents of freedom fighters and ex-serviceman as well. The counsel for the petitioners argues that not only did the State of U.P. make an error in their implementation of horizontal reservations, but also that the error was intentional. The following quote is from the court case:

> `Per contra, learned counsel appearing for the petitioners would submit that fallacy was committed by the Board deliberately, and with malafide intention to deprive the meritorious candidates their rightful placement in the open category. The candidates seeking horizontal reservations belonging to OBC and SC category were wrongly`

[25]The case is available at `https://indiankanoon.org/doc/74817661/` (last accessed on 03/07/2019).

> adjusted in the open category, whereas, they ought to have been adjusted in their quota provided in respective social category. The action of the Board is not only motivated, but purports to take forward the unwritten agenda of the State Government to accommodate as many number of OBC/SC candidates in the open category.

The judge of the case sides with the petitioners, and rules that the State of Uttar Pradesh must correct their erroneous application of the provisions of horizontal reservations. The judge further emphasizes that the State has played foul, stating:

> There is merit in the submission of the learned counsel for the petitioners that the conduct of the members of the Board appears not only mischievous but motivated to achieve a calculated agenda by deliberately keeping meritorious candidates out of the select list. The Board and the officials involved in the recruitment process were fully aware of the principle of horizontal reservations enshrined in Act, 1993 and Government Orders which were being followed by them in previous selections of SICP and PC (PAC), but in the present selection they chose to adopt a principle against their own Government Orders and the statutory provisions which were binding upon them...
> I am constrained to hold that both the State and the Board have played fraud on the principles enshrined in the Constitution with regard to public appointment.

What is especially surprising is, despite the heavy tone of this judgement, the State goes on to appeal in another Allahabad High Court case *State Of U.P. And 2 Ors. vs Ashish Kumar Pandey And 58 Ors, 29 July, 2016*,[26] in an effort to continue using its preferred method for implementing horizontal reservations. Perhaps unsurprisingly, this appeal was denied by the High Court.

This particular case clearly illustrates that there is a strong resistance in at least some of the states for implementing the provisions of horizontal reservations as mandated under the SCI-AKG choice rule. While this resistance most likely reflects the political nature of this debate, the arguments of the counsel for the state to maintain their preferred mechanism are mostly based on the presence of justified envy under the SCI-AKG choice rule.

[26]The case is available at `https://indiankanoon.org/doc/71146861/` (last accessed on 03/07/2019).

(2) ***Asha Ramnath Gholap vs President, District Selection Committee & Ors. on March 3rd, 2016*, Bombay High Court**.[27] In this case, there are 23 pharmacist positions to be allocated; 13 of these positions are vertically reserved for backward classes and the remaining ten are open for all candidates. In the open category, eight of the ten positions are horizontally reserved for various groups, including three for women. The petitioner, Asha Ramnath Gholap, is an SC woman, and while there is one vertically reserved position for SC candidates, there is no horizontally reserved position for SC women. Under the SCI-AKG choice rule, she is not eligible for any of the horizontally reserved women positions at the open category. Nevertheless, she brings her case to the Bombay High Court based on an instance of justified envy, described in the court records as follows:

> `It is the contention of the petitioner that Respondent Nos. 4 & 5 have received less marks than the petitioner and as such, both were not liable to be selected. The petitioner has, therefore, approached this court by invoking the writ jurisdiction of this court under Article 226 of the Constitution of India, seeking quashment of the select list to the extent it contains the names of Respondent Nos.4 and 5 against the seats reserved for the candidates belonging to open female category.`

Under the federal law, there is no merit to this argument, because the SCI-AKG choice rule allows for justified envy. However, the judges side with the petitioner on the basis that a candidate cannot be denied a position from the open category based on her backward class membership, essentially ruling out the possibility of justified envy under a Supreme Court-mandated choice rule, which is designed to allow for positive discrimination for the vulnerable groups in the society. Their justification is given in the court records as follows:

> `We find the argument advanced as above to be fallacious. Once it is held that general category or open category takes in its sweep all candidates belonging to all categories irrespective of their caste, class or community or tribe, it is irrelevant whether the reservation provided is vertical or horizontal. There cannot be two interpretations of the words ‘open category’ ...`

(3) ***Smt. Megha Shetty vs State Of Raj. & Anr on 26 July, 2013*, Rajasthan High Court**.[28] In contrast to *Asha Ramnath Gholap (2016)* where the judges have been

[27] The case is available at `https://indiankanoon.org/doc/178693513/` (last accessed on 03/08/2019).
[28] The case is available at `https://indiankanoon.org/doc/78343251/` (last accessed on 10/08/2019).

erroneous siding with petitioners whose lawsuits are based on instances of justified envy, in this case a petitioner who is a member of the general category seeks legal action against the state on the basis that several horizontally reserved open-category women positions are allocated to women from OBC who are not eligible for these positions (unless they receive it without invoking the benefits of horizontal reservation). While all these OBC women have higher merit scores than the petitioner and the state have apparently used a better behaved procedure, the petitioner's case has merit because SCI-AKG choice rule allows for justified envy in those situations. In an earlier lawsuit, the petitioner's lawsuit was already declined by a single judge of the same court based on an erroneous interpretation of *Indra Sawhney (1992)*. The petitioner subsequently appeals this erroneous decision and brings the case to a larger bench of the same court. However, the three judges side with the earlier judgement, thus erroneously dismissing the appeal. Their decision is justified as follows:

> `The outstanding and important feature to be noticed is that it is not the case of the appellant-petitioner that she has obtained more marks than those 8 OBC (Woman) candidates, who have been appointed against the posts meant for General Category (Woman), inasmuch as, while the appellant is at Serial No.184 in the merit list, the last OBC (Woman) appointed is at Serial No.125 in the merit list. The controversy raised by the appellant is required to be examined in the context and backdrop of these significant factual aspects.`

As seen from this argument, many judges have difficulty perceiving that the Supreme Court-mandated procedure could possibly allow for justified envy.

(4) ***Arpita Sahu vs The State Of Madhya Pradesh on 21 August, 2012*** **Madhya Pradesh High Court**.[29] The petitioner files a lawsuit based on an instance of justified envy, however in contrast to *Asha Ramnath Gholap (2016)*, the judges have correctly dismissed the petition in this case.

8.1.2. ***Wrongful Implementation and Possible Misconduct.*** It is bad enough that the Supreme Court-mandated SCI-AKG choice rule is not incentive compatible, forcing some candidates to risk losing their open-category HR protections by claiming their VR protections. To make matters worse, in some cases candidates are denied access to open-category HR protections even when they do not submit their backward class status, giving up their VR protections. Therefore, even when the candidate applies for a position as a general-category candidate without claiming the benefits of VR protections, the central

[29] The case is available at `https://indiankanoon.org/doc/102792215/` (last accessed on 10/10/2019).

planner processes the application as if the backward class status was claimed, denying the candidate's eligibility for open-category HR protections. The central planners are often able to do this, because last names in India are, to a large extent, indicative of a caste membership. This type of misconduct seems to be fairly widespread in some jusistictions, and it is the main cause of the lawsuit in dozens of cases such as the two Bombay High Court cases *Shilpa Sahebrao Kadam vs The State Of Maharashtra (2019)* and *Vinod Kadubal Rathod vs Maharashtra State Electricity (2017)*.[30] Indeed, this type of misconduct is sometimes intentional and systematic. The following statement is from *Shilpa Sahebrao Kadam (2019)*:

> `According to Respondent - Maharashtra Public Service Commission, in view of the Circular dated 13.08.2014, only the candidates belonging to open (Non-reserved) category can be considered for open horizontally reserved posts meaning thereby, the reserved category candidates cannot be considered for open horizontally reserved post. Reference is made to a communication issued by the Additional Chief Secretary (Service) of the State of Maharashtra dated 26.07.2017, whereunder it is prescribed that a female candidate belonging to any reserved category, even if tenders application form seeking employment as an open category candidate, the name of such candidate shall not be recommended for employment against a open category seat.`

Moreover, not all decisions in these lawsuits are made in accordance with the SCI-AKR choice rule, which allows candidates to forego their VR (or HR) protections. This is the case both for the first lawsuit and the last one listed above. For example, in the last lawsuit given above, two petitioners each applied for a position without declaring their backward class membership, with an intention to benefit from open-category HR protections. Following their application, these petitioners were requested to provide their school leaving certificates, which provided information on their backward class status. Upon receiving this information, the petitioners were declined eligibility for open-category HR protections, even though they never claimed their VR protections. Hence, they filed the fourth lawsuit given above. Remarkably, their petition was declined on the basis of their backward class membership. Here we have a case where the authorities not only go to great lengths to obtain the backward class membership of the candidates, and wrongfully decline their eligibility for open category HR protections, but they also manage to get their

[30]The cases are available at `https://indiankanoon.org/doc/89017459/` and `https://indiankanoon.org/doc/162611497/` (last accessed on 03/09/2019).

lawsuits dismissed. The mishandling of this case is consistent with the concerns indicated in the February 2006 issue of *The Inter-Regional Inequality Facility* policy brief:[31]

> Another issue relates to the access of SCs and STs to the institutions of justice in seeking protection against discrimination. Studies indicate that SCs and STs are generally faced with insurmountable obstacles in their efforts to seek justice in the event of discrimination. The official statistics and primary survey data bring out this character of justice institutions. The data on Civil Rights cases, for example, shows that only 1.6% of the total cases registered in 1991 were convicted, and that this had fallen to 0.9% in 2000.

8.1.3. *Loss of Access to HR protections without any Access to VR protections*. The main justification offered in various Supreme Court cases for denying backward class members their open-category HR protections is avoiding a situation where an excessive number of positions are reserved for members of these classes. In several cases, however, members of these classes are denied access to open-category HR protections even when the number of VR-protected positions is zero for their reserve-eligible vertical category. This is the case in the following two court cases:

(1) *Tejaswini Raghunath Galande v. The Chairman, Maharashtra Public Service Commission and Ors. on 23 January 2019*, Writ Petition Nos. 5397 of 2016 & 5396 of 2016, High Court of Judicature at Bombay.[32]
(2) Original Application No. 662/2016 dated 05.12.2017, Maharashtra Administrative Tribunal, Mumbai.[33]

In both cases, while the petitioners claimed their VR protections, there was no VR-protected position for their class. Yet in both cases petitioners lost their open-category HR protections. In the first case, the petitioners' lawsuit to benefit from open-category HR protections was initially declined by a lower court, resulting in the appeal at the High Court. The lower court's decision was overruled in the High Court, and her request was granted. On the other hand, the second petitioner's similar request was declined by the Maharashtra Administrative Tribunal. What is more worrisome in the second case is that, while initially three positions were VR-protected for the petitioner's backward class, after the petitioners application these VR-protected positions were withdrawn. Therefore, the

[31]The policy brief is available at `https://www.odi.org/sites/odi.org.uk/files/odi-assets/publications-opinion-files/4080.pdf` (last accessed 03/09/2019).

[32]The case is available at `https://www.casemine.com/judgement/in/5c713d919eff4312dfbb5900` (last accessed on 03/09/2019).

[33]The case is available at `https://mat.maharashtra.gov.in/Site/Upload/Pdf/O.A.662%20of%202016.pdf` (last accessed on 03/09/2019).

candidate declared her backward class status, giving up her open-category HR protection, presumably to gain access to VR-protected positions set aside her reserve-eligible class, only to learn that she had given up her eligibility for nothing.

8.2. The Discord between the SCI-AKG Choice Rule and the 103rd Amendment of the Constitution of India. In a highly debated reform on the reservation system, the January 2019 *One Hundred and Third Amendment of the Constitution of India* provides up to 10% VR protections to the economically weaker sections (EWS) in the general category.

In a case that is pending as of January 2021, the One Hundred and Third Amendment was immediately challenged at the Supreme Court and it was referred to a larger five-judge bench of the Supreme Court in August 2020.[34] Despite the challenge at the Supreme Court, the EWS reservation has already been adopted by federal institutions throughout India as well as by most states at their state-run public institutions. If implemented jointly with the SCI-AKG choice rule, the EWS reservation can be expected to amplify the legal challenges formalized in Section 5.2 and documented in Section 8.1. Especially in states with a strong presence of horizontal reservations (such as states with 30-35% horizontal women reservation), legal challenges based on justified envy may become the norm rather than an exception if the SCI-AKG choice rule is implemented with a 10% vertical EWS reservation. That is because, any candidate who applies both for the vertical EWS reservation and any HR protections lose access to open-category HR protections under the SCI-AKG choice rule. To weight in what this would mean in the field, let us make some simple "back-of-the-envelope" calculations.

It is estimated that around 26% of the population in India do not belong to the Other Backward Classes (OBC), Scheduled Castes (SC), and Scheduled Tribes (ST) categories.[35] Therefore, prior to the January 2019 amendment of the Constitution, approximately 26% of the population belonged to the general category. While the amendment is intended only for the economically weaker sections of the general category, according to most estimates more than 80% of the members of this group satisfy the eligibility criteria for the EWS reservation.[36] This means, with the introduction of the EWS reservation, the fraction

[34]See `https://www.scobserver.in/court-case/reservations-for-economically-weaker-sections` for the pending Supreme Court case *Youth for Equality v. Union of India*.

[35]See the 01/07/2017-dated *Hindustan Times* story "Quota for economically weak in general category could benefit 190 mn," available at `https://www.hindustantimes.com/india-news/quota-for-economically-weak-in-general-category-could-benefit-190-mn/story-6vvfGmXBohmLrCYkgM1NYJ.html`, last accessed on 04/14/2019.

[36]See the 01/08/2019 dated *Business Today* story "In-depth: Who is eligible for the new reservation quota for general category?" available at `https://www.businesstoday.in/current/economy-politics/in-depth-who-is-eligible-for-the-new-reservation-quota-for-general-category/story/308062.html`, (last accessed on 04/14/2019).

of the population who are ineligible for any VR protections reduces to a mere 5-6% of the population of India. Therefore, the "new general category," those members of the society who are ineligible for any VR protections, shrinks to approximately 5-6% of the whole population. A key implication of this observation is the following: Under the SCI-AKG choice rule, only this "elite" 5-6% of the population qualifies for the adjustments for open-category HR protections, which could easily be more than 10% of all positions in states with extensive provision of HR protections. Thus, had the Supreme Court not abandoned the SCI-AKG choice rule in an important December 2020 judgement, maintaining the EWS reservation would have likely increased litigations due to justified envy considerably throughout India, especially in states such as Bihar, Gujarat, Andhra Pradesh, Madhya Pradesh, Rajasthan, Uttarakhand, Chhattisgarh, Sikkim, all with $30 - 35\%$ HR protections for women.

9. Epilogue: December 2020 Supreme Court of India Resolution on Elimination of Justified Envy and the Demise of the SCI-AKG Choice Rule

As our paper was under revision for this journal, a December 2020 Supreme Court judgement in *Saurav Yadav v State of Uttar Pradesh (2020)* became headline news in India. Using arguments parallel to our analysis presented in Sections 5.2 and 8.1.1, a three-judge bench of the highest court reached some of the same conclusions we have reached in this paper. Most notably, similar to our policy recommendations, with this judgement

(1) all allocation rules for public recruitment are federally mandated to eliminate *justified envy*, and thereby
(2) the SCI-AKG choice rule, mandated for 25 years, loses its legality.

Using several of the same judgements we present in Section 8.1, the judges have also highlighted the inconsistencies between several High Court judgements in relation to their approach to possibility of justified envy in allocation. The Supreme Court judges also declared that while the "first view" that enforces *no justified envy* by the High Court judgements of Rajasthan, Bombay, Gujarat, and Uttarakhand is "correct and rational," the "second view" that allows for *justified envy* by the High Court judgements of Allahabad and Madhya Pradesh is not.[37]

While the axiom of no justified envy is federally enforced with *Saurav Yadav v State of Uttar Pradesh (2020)*, unlike in *Anil Kumar Gupta (1995)* no explicit procedure is federally mandated with this judgement. However, through its August 2020 judgement

[37] It is important to emphasize that, prior to this ruling, the second view—now deemed incorrect and irrational—was the one that is in line with the SCI-AKG choice rule, whereas the first view—now deemed correct and rational—deviated from the previously mandated choice rule.

Tamannaben Ashokbhai Desai v. Shital Amrutlal Nishar (2020), the High Court of Gujarat mandated the use of the *two-step minimum guarantee choice rule* for the state of Gujarat. The mandated choice rule in Gujarat is described for a single group of beneficiaries (women) for horizontal reservations under this High Court ruling,[38] and therefore not only it is equivalent to our proposed 2SMH choice rule in this special case as explained in Section 4.2, but also it is the only choice rule that satisfies our four axioms as presented in Corollary 2. While the Supreme Court has not enforced any specific rule in its December 2020 judgement, it has endorsed the *two-step minimum guarantee choice rule* given in *Tamannaben Ashokbhai Desai v. Shital Amrutlal Nishar (2020).*

> 36. Finally, we must say that the steps indicated by the High Court of Gujarat in para 56 of its judgment in Tamannaben Ashokbhai Desai contemplate the correct and appropriate procedure for considering and giving effect to both vertical and horizontal reservations. The illustration given by us deals with only one possible dimension. There could be multiple such possibilities. Even going by the present illustration, the first female candidate allocated in the vertical column for Scheduled Tribes may have secured higher position than the candidate at Serial No.64. In that event said candidate must be shifted from the category of Scheduled Tribes to Open / General category causing a resultant vacancy in the vertical column of Scheduled Tribes. Such vacancy must then enure to the benefit of the candidate in the Waiting List for Scheduled Tribes - Female.
>
> The steps indicated by Gujarat High Court will take care of every such possibility. It is true that the exercise of laying down a procedure must necessarily be left to the concerned authorities but we may observe that one set out in said judgment will certainly satisfy all claims and will not lead to any incongruity as highlighted by us in the preceding paragraphs.

Since neither the Supreme Court's nor the Gujarati High Court's judgement involves issues that pertain to overlapping horizontal reservations, these decisions are parallel to our recommendation, albeit in a simpler environment. While the primary objective of these judgements are eliminating justified envy, they also restored the incentive compatibility of the system and eliminated a major discord with the One Hundred and Third Amendment of the Constitution of India.

[38]See Section C.4 in the Online Appendix for the description of the procedure in *Tamannaben Ashokbhai Desai (2020)*

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Online Appendix

Appendix A. Mathematical Preliminaries

In this appendix, we provide some preliminary results that we use in our proofs. First, we introduce some graph-theoretic terminology.

Consider a category $v \in \mathcal{V}$ and a set of individuals $I \subseteq \mathcal{I}^v$. Let G be the category-v HR graph for I. The vertices of G are individuals in I and reserved positions in H^v. There exists an edge between an individual $i \in I$ and a position reserved for trait $t \in \mathcal{T}$ if i has trait t. A **matching** is a set of edges without common vertices. A matching **covers** a vertex if it has an edge adjacent to that vertex.

Lemma 1 (Dulmage-Mendelsohn Theorem)**.** *Consider the HR graph for $v \in \mathcal{V}$ and $I \subseteq \mathcal{I}^v$. Suppose that there exist a matching that covers individuals in $J \subseteq I$ and a matching that covers reserved positions in $S \subseteq H^v$. Then there exists a matching that covers both J and S.*

See Theorem 4.1 in Lawler (2001, Page 191) for a proof of this lemma.

An **alternating path** between matching M_1 and matching M_2 is a path of connected edges that starts at a vertex covered by M_1 but not by M_2 and ends at a vertex covered by M_2 but not by M_1 such that edges of the path belong alternately to M_1 and M_2.

Lemma 2 (Alternating Path)**.** *Let M_1 and M_2 be two distinct matchings that cover the same set of positions in a HR graph. Suppose that there exists a vertex i covered by M_1 but not by M_2. Then there exists an alternating path between matching M_1 and matching M_2 that starts at i.*

Proof. Let $i_1 \equiv i$ and (i_1, s_1) be the edge that covers i_1 in M_1. Since M_1 and M_2 cover the same set of positions, there exists an edge (i_2, s_1) in M_2. If i_2 is not covered by M_1, then we are done. Otherwise, i_2 is covered by both M_1 and M_2. Let (i_2, s_2) be the edge in M_1 that covers i_2. Since M_1 and M_2 cover the same set of positions, there exists an edge (i_3, s_2) in M_2. If i_3 is not covered by M_1, then we are done. Otherwise, i_3 is covered by both M_1 and M_2. Continue this construction. Since there exists a finite number of vertices, this construction ends in finite time at a vertex i_k covered by M_2 but not by M_1. This finishes the construction of an alternating path starting at i. See Figure 4 for an illustration of the alternating path that is constructed. □

For the next result, we extend the definition of the substitutes condition and the irrelevance of rejected individuals condition to single-category choice rules.

Lemma 3. *Let $v \in \mathcal{V}$. $C^v_{\circledR}$ satisfies the substitutes condition and the irrelevance of rejected individuals condition.*

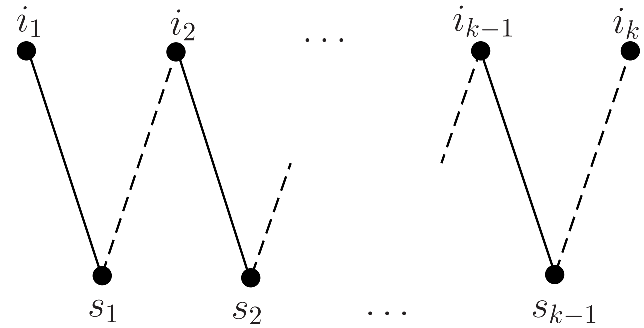

FIGURE 4. The alternating path between M_1 and M_2 constructed in the proof of Lemma 2. The edges in M_1 are solid and the edges in M_2 are dashed.

Proof. The irrelevance of rejected individuals condition is satisfied trivially by the construction of $C^v_{Ⓜ}$. We show that $C^v_{Ⓜ}$ also satisfies the substitutes condition.

Let $I \subseteq \mathcal{I}^v$, $i \in C^v_{Ⓜ}(I)$, and $j \in I \setminus i$. To prove the substitutes condition, we show that $i \in C^v_{Ⓜ}(I \setminus \{j\})$. If $j \notin C^v_{Ⓜ}(I)$, then $C^v_{Ⓜ}(I \setminus \{j\}) = C^v_{Ⓜ}(I)$ because $C^v_{Ⓜ}$ satisfies the irrelevance of rejected individuals condition, so $i \in C^v_{Ⓜ}(I \setminus \{j\})$.

For the rest of the proof assume that $j \in C^v_{Ⓜ}(I)$. If i is chosen before j in the construction of $C^v_{Ⓜ}(I)$, then $i \in C^v_{Ⓜ}(I \setminus \{j\})$ since in the construction of $C^v_{Ⓜ}(I \setminus \{j\})$ all the steps will be the same as in the construction of $C^v_{Ⓜ}(I)$ until individual j is considered. Now assume that j is chosen before i in the construction of $C^v_{Ⓜ}(I)$. Before we consider two separate cases below, we introduce the following notation. For any $I' \subseteq \mathcal{I}^v$, let $C^1(I') \subseteq I'$ denote the set of individuals chosen at Step 1 in the construction of $C^v_{Ⓜ}(I')$ and $C^2(I') \subseteq I'$ denote the set of individuals chosen at Step 2 in the construction of $C^v_{Ⓜ}(I')$. If $j \in C^2(I)$, then $i \in C^v_{Ⓜ}(I \setminus \{j\})$ follows because at the second step individuals with the highest merit scores are chosen. For the following cases, assume that $j \in C^1(I)$.

Case 1 ($n^v(I \setminus \{j\}) = n^v(I) - 1$): We claim that

(1) $C^1(I \setminus \{j\}) = C^1(I) \setminus \{j\}$ and
(2) $C^2(I \setminus \{j\}) \supseteq C^2(I) \setminus \{j\}$.

To prove the first displayed equation note that by Dulmage-Mendelsohn Theorem $C^1(I) \setminus \{j\}$ and $C^1(I \setminus \{j\})$ can be matched with the same set of positions in the HR reservation graph. Therefore, $n^v(C^1(I) \setminus \{j\}) = n^v(I) - 1$ and $n^v(C^1(I \setminus \{j\}) \cup \{j\}) = n^v(I)$. When Theorem 1 is applied to the set of individuals $I \setminus \{j\}$ when the number of positions for category v is $n^v(I) - 1$, we get that the individual with the k-th highest merit score in $C^1(I \setminus \{j\})$ has a weakly higher merit score than the individual with the k-th highest merit score in $C^1(I) \setminus \{j\}$ for every $k \in \{1, \ldots, n^v(I) - 1\}$. Likewise, when Theorem 1 is applied to the set of individuals I when the number of positions for category v is $n^v(I)$, we get that the individual with the k-th highest merit score in $C^1(I)$ has a weakly higher merit score than the individual with the k-th highest merit score in $C^1(I \setminus \{j\}) \cup \{j\}$ for every $k \in \{1, \ldots, n^v(I)\}$. The last two sets of inequalities imply that $C^1(I \setminus \{j\}) = C^1(I) \setminus \{j\}$ since individuals have distinct merit scores. The second displayed equation follows from the first one since at the second step unassigned individuals with the highest merit scores are chosen.

The first displayed equation implies that if $i \in C^1(I)$, then $i \in C^1(I \setminus \{j\})$. The second displayed equation implies that if $i \in C^2(I)$, then $i \in C^2(I \setminus \{j\})$. Therefore, $i \in C^v_{Ⓜ}(I \setminus \{j\})$.

Case 2: $(n^v(I \setminus \{j\}) = n^v(I))$: As in the previous case, we get that $C^1(I \setminus \{j\}) = (C^1(I) \setminus \{j\}) \cup \{j'\}$ where $j' \in I \setminus C^1(I)$ by Dulmage-Mendelsohn Theorem and Theorem 1. Therefore, if $i \in C^1(I)$, then $i \in C^1(I \setminus \{j\})$. Furthermore, if $j' \notin C^2(I)$, then $C^2(I \setminus \{j\}) = C^2(I)$. Otherwise, if $j' \in C^2(I)$, then $C^2(I \setminus \{j\}) \supseteq C^2(I) \setminus \{j'\}$. Therefore, regardless of whether j' is in $C^2(I)$ or not, $i \in C^2(I)$ implies $j' = i$ or $i \in C^2(I \setminus \{j\})$. As a result, $i \in C^v_{Ⓜ}(I \setminus \{j\})$. □

Appendix B. Proofs

In this section, we present the main proofs.

Proof of Proposition 1. Let $I = J \cup K$ and I' be the set of individuals assigned to category-v positions by AKG-HAS. We first show that

(1) $|I'| = q^v$,
(2) there exists no instance of justified envy involving an individual in I' and an individual in $I \setminus I'$,
(3) I' maximally accommodates category-v HR protections for I.

Then the proof follows from Corollary 1.

Proof of (1): $|I'| = q^v$ follows because at Step $|\mathcal{T}| + 1$ of AKG-HAS all positions are filled.

Proof of (2): Let $i \in I'$ and $j \in I \setminus I'$ such that $\sigma(j) > \sigma(i)$. Since $j \notin I'$, either j does not have a trait or there are at least q^v_t individuals in I' where t is j's only trait. If j does not have a trait, then i must have a trait t' such that the number of individuals in I' who has trait t' is $\min\{q^v_{t'}, |\{i' \in I : t' \in \tau(i')\}|\}$. Then $n((I' \setminus \{i\}) \cup \{j\}) = n(I') - 1$, which means that there is no instance of justified envy involving j and i. If j has trait t, then it must be that i does not have trait t, there are at least q^v_t individuals with trait t in I', and i must have a trait $t' \neq t$ such that the number of individuals in I' who have trait t' is

$\min\{q^v_{t'}, |\{i' \in I : t' \in \tau(i')\}|\}$. Then, as before, $n((I' \setminus \{i\}) \cup \{j\}) = n(I') - 1$, which means that there is no instance of justified envy involving j and i.

Proof of (3): For every trait t, there is a corresponding step of AKG-HAS so that the number of individuals in I' who has trait t is $\min\{q^v_t, |\{i' \in I : t \in \tau(i')\}|\}$. Since each individual has at most one trait, this implies that $n^v(I') = n^v(I)$. □

Proof of Theorem 1. Let $I \subseteq \mathcal{I}^v$ be a set of individuals. To show part (1), note that $|C^v_{Ⓜ}(I)| = \min\{q^v, |I|\}$. Furthermore, for single-category choice rule C^v, $C^v(I) \subseteq I$ and $|C^v(I)| \leq q^v$. Therefore,

$$|C^v(I)| \leq \min\{q^v, |I|\} = |C^v_{Ⓜ}(I)|.$$

We show part (2) by mathematical induction on parameters $(q^v, (q^v_t)_{t \subseteq \mathcal{T}})$. We show the claim that for an ordering of agents in $C^v_{Ⓜ}(I) \setminus C^v(I)$ and $C^v(I) \setminus C^v_{Ⓜ}(I)$ that the k-th agent in $C^v_{Ⓜ}(I) \setminus C^v(I)$ has a higher priority than the k-th agent in $C^v(I) \setminus C^v_{Ⓜ}(I)$, which implies part (2).

For the base case when there are no reserved positions, statement (2) holds because $C^v_{Ⓜ}$ chooses all individuals at Step 2 according to the merit score ranking. Now suppose that the claim holds for all parameters bounded above by $(q^v, (q^v_t)_{t \subseteq \mathcal{T}})$. Consider parameters $(q^v, (q^v_t)_{t \subseteq \mathcal{T}})$. If all individuals in $C^v_{Ⓜ}(I) \setminus C^v(I)$ are chosen at Step 2, then the claim holds as in the base case because individuals in $C^v(I) \setminus C^v_{Ⓜ}(I)$ are available at Step 2 in the construction of $C^v_{Ⓜ}(I)$.

Consider the situation when there exists at least one individual in $C^v_{Ⓜ}(I) \setminus C^v(I)$ chosen at Step 1. Let i be the individual with the highest priority in $C^v_{Ⓜ} \setminus C^v(I)$ chosen at Step 1 and t be the trait of the position that she is matched with. By Lemma 4, $C^v_{Ⓜ}$ maximally accommodates HR protections, so in the HR graph, there exists a matching M_1 that matches $C^v_{Ⓜ}(I)$ to a set of reserved positions S with maximum cardinality $n^v(I)$. Since C^v also maximally accommodates HR protections, by Dulmage-Mendelsohn Theorem (see Lemma 1) there exists another matching M_2 that matches $C^v(I)$ to S both of which have cardinality $n^v(I)$. By Lemma 2, there exists an alternating path that starts at i and ends at an individual $i' \in C^v(I) \setminus C^v_{Ⓜ}(I)$. Therefore, individual i can be replaced with individual i' in $C^v_{Ⓜ}(I)$ without changing the set of positions covered in the HR graph for I. Hence, by construction of $C^v_{Ⓜ}(I)$, $\sigma(i) > \sigma(i')$ because i' is available when i is chosen at Step 1.

Now consider the reduced market when capacity q^v and trait-t reservation q^v_t are both reduced by one and the set of individuals is $I \setminus \{i, i'\}$. In this reduced market, $C^v_{Ⓜ}(I \setminus \{i, i'\})$ is equal to $C^v_{Ⓜ}(I) \setminus \{i\}$ for the original market because $i' \notin C^v_{Ⓜ}(I)$ and the construction of $C^v_{Ⓜ}(I \setminus \{i, i'\})$ chooses individuals in the same order as they are chosen in $C^v_{Ⓜ}(I)$. In particular, the set of individuals chosen before i at $C^v_{Ⓜ}(I)$ are chosen in the same order in $C^v_{Ⓜ}(I \setminus \{i, i'\})$. Furthermore, after i is chosen the set of updated parameters are exactly the same. Therefore, the same set of individuals are chosen in the same order after i is chosen in $C^v_{Ⓜ}(I)$. In addition, $C^v(I) \setminus \{i'\}$ maximally accommodates HR protections and $i \notin C^v(I) \setminus \{i'\}$. By the induction hypothesis, the individuals in $C^v_{Ⓜ}(I \setminus \{i, i'\})$ and $C^v(I) \setminus \{i'\}$ can be ordered with the required property, which implies the hypothesis. Therefore, the hypothesis holds for every set of parameters $(q^v, (q^v_t)_{t \subseteq \mathcal{T}})$. □

Proof of Theorem 2. We first show that $C^v_{Ⓜ}$ satisfies the stated properties in several lemmas and then show that the unique category-v choice rule satisfying these properties is $C^v_{Ⓜ}$.

Lemma 4. *$C^v_{Ⓜ}$ maximally accommodates HR protections.*

Proof. Suppose, for contradiction, that $C^v_{Ⓜ}$ does not maximally accommodate HR protections. Hence, there exists $I \subseteq \mathcal{I}^v$ such that $C^v_{Ⓜ}(I)$ does not maximally accommodate HR protections for I. Therefore, in the HR graph for $C^v_{Ⓜ}(I)$, the maximum cardinality that can be attained by a matching is strictly less than $n^v(I)$. Let $\bar{I} \subseteq C^v_{Ⓜ}(I)$ be the set of individuals who are chosen at Step 1 in the construction of $C^v_{Ⓜ}(I)$. By assumption, $|\bar{I}| = n^v(C^v_{Ⓜ}(I)) < n^v(I)$. Now consider a maximum matching for the HR graph for I. Let S be the set of positions matched, so $|S| = n^v(I)$. By Dulmage-Mendelsohn Theorem (see Lemma 1), there exists a matching that assigns every individual in $\bar{I}$ and every reserved position in S in the HR graph of I. But this is a contradiction to the construction of $C^v_{Ⓜ}(I)$, as there exists an individual who increases HR utilization of $\bar{I}$. ■

Lemma 5. *$C^v_{Ⓜ}$ satisfies no justified envy.*

Proof. Suppose, for contradiction, that $C^v_{Ⓜ}$ has justified envy. Therefore, there exist a set of individuals $I \subseteq \mathcal{I}^v$, individuals $i \in C^v_{Ⓜ}(I)$, $j \in I \setminus C^v_{Ⓜ}(I)$ with $\sigma(j) > \sigma(i)$ and $n^v\left((C^v_{Ⓜ}(I) \setminus \{i\}) \cup \{j\}\right) \geq n^v(C^v_{Ⓜ}(I))$. Consider category-$v$ choice rule C^v such that

$$C^v(I') = \begin{cases} C^v_{Ⓜ}(I'), & \text{if } I' \neq I \\ (C^v_{Ⓜ}(I) \setminus \{i\}) \cup \{j\}, & \text{if } I' = I. \end{cases}$$

Since $C^v_{Ⓜ}$ maximally accommodates HR protections, $C^v(I')$ maximally accommodates HR protections for I' whenever $I' \neq I$ because $C^v(I') = C^v_{Ⓜ}(I')$. Furthermore,

$$n^v(C^v(I)) = n^v\left((C^v_{Ⓜ}(I) \setminus \{i\}) \cup \{j\}\right) \geq n^v(C^v_{Ⓜ}(I))$$

and the fact that $C^v_{Ⓜ}$ maximally accommodates HR protections by Lemma 4 (i.e., $n^v(C^v_{Ⓜ}(I)) = n^v(I)$) imply that $n^v(C(I)) = n^v(I)$ because of the fact that $n^v(I)$ is the maximum cardinality. Hence, $C^v(I)$ maximally accommodates HR protections for I. By

Theorem 1, for every $k \leq |C^v(I)|$, the individual with the k-th highest priority in $C^v_{Ⓜ}(I)$ has a weakly higher priority than the individual with the k-th highest priority in $C^v(I)$. This is a contradiction to the construction of C^v because

$$C^v(I) = (C^v_{Ⓜ}(I) \setminus \{i\}) \cup \{j\}$$

and $\sigma(j) > \sigma(i)$. ■

Lemma 6. *$C^v_{Ⓜ}$ is non-wasteful.*

Proof. $C^v_{Ⓜ}$ is non-wasteful because at the second step all the unfilled positions are filled with the unmatched individuals until all positions are filled or all individuals are assigned to positions. ■

Lemma 7. *Let $v \in \mathcal{V}$. If a category-v choice rule maximally accommodates HR protections, satisfies no justified envy, and is non-wasteful, then it has to be $C^v_{Ⓜ}$.*

Proof. Let C^v be a category-v choice rule that maximally accommodates category-v HR protections, satisfies no justified envy, and is non-wasteful. Suppose, for contradiction, that $C^v \neq C^v_{Ⓜ}$. Therefore, there exists $I \subseteq \mathcal{I}^v$ such that $C^v(I) \neq C^v_{Ⓜ}(I)$. Since both choice rules are non-wasteful

$$|C^v(I)| = |C^v_{Ⓜ}(I)|.$$

Since $C^v(I) \neq C^v_{Ⓜ}(I)$, this equation implies that

$$\left|C^v_{Ⓜ}(I) \setminus C^v(I)\right| = \left|C^v(I) \setminus C^v_{Ⓜ}(I)\right| > 0.$$

We consider two cases depending on the value of $n^v(I)$.

Case 1: If $n^v(I) = 0$, then no individual in I has a trait that has a positive reservation. Therefore, $C^v_{Ⓜ}(I)$ consists of $\min\{|I|, q^v\}$ individuals with the highest merit score in I. This is a contradiction to the assumption that $C^v(I)$ satisfies no justified envy because any individual $i \in C^v_{Ⓜ}(I) \setminus C^v(I) \neq \emptyset$ has a higher merit score than any individual $j \in C^v(I) \setminus C^v_{Ⓜ}(I) \neq \emptyset$ and $n^v((C^v(I) \setminus \{j\}) \cup \{i\}) \geq n^v(C^v(I)) = 0$. Hence, there is an instance of justified envy for $C^v(I)$ involving $i \in I \setminus C^v(I)$ and $j \in C^v(I)$, which is a contradiction.

Case 2: Let $n^v(I) = n > 0$. Therefore, there are n individuals chosen at Step 1 of $C^v_{Ⓜ}(I)$. For $1 \leq k \leq n$, let i_k be the k-th individual chosen at Step 1 of $C^v_{Ⓜ}(I)$. Consider a maximum matching M_1 in the HR graph for $C^v_{Ⓜ}(I)$ that matches $I_1 \equiv \{i_1, \ldots, i_n\}$. We show that $C^v(I) \supseteq I_1$. Let S be the set of positions that are matched in M_1. Since C^v maximally accommodates HR protections, there exists a maximum matching in the HR graph for $C^v(I)$ that has cardinality n. Furthermore, by Dulmage-Mendelsohn Theorem (see Lemma 1), there exists a matching of a subset of $C^v(I)$ to positions in S. Let $I_2 \subseteq C^v(I)$

be the set of these individuals and M_2 be this matching. Suppose, for contradiction, that $I_1 \setminus C^v(I) \neq \emptyset$. Let i_k be the individual with the lowest index in $I_1 \setminus C^v(I)$. By Lemma 2, there exists an alternating path between M_1 and M_2 that starts at i_k and ends at a vertex j covered by M_2 but not by M_1. Therefore, i_k and j can be replaced with each other in both M_1 and M_2 without decreasing the maximum cardinality. By construction of $C^v_{Ⓜ}$, $\sigma(i_k) > \sigma(j)$ because j is available when i_k is chosen. But this is a contradiction to the assumption that $C^v(I)$ satisfies no justified envy because $j \in C^v(I)$, $i_k \in I \setminus C^v(I)$, $\sigma(i_k) > \sigma(j)$, and $n^v(C^v(I)) = n^v((C^v(I) \setminus \{j\}) \cup \{i_k\})$. Therefore, $I_1 \subseteq C^v(I)$.

By construction of $C^v_{Ⓜ}(I)$, every individual in $C^v_{Ⓜ}(I) \setminus I_1$ is chosen at Step 2. Therefore, these individuals have a higher merit score than any individual in $I \setminus C^v_{Ⓜ}(I)$. Let $j \in C^v(I) \setminus C^v_{Ⓜ}(I)$, which is non-empty by assumption. Therefore, $j \in I \setminus C^v_{Ⓜ}(I)$, which means that any individual $i \in C^v_{Ⓜ}(I) \setminus C^v(I)$ has a strictly higher merit score than j. This is a contradiction to the assumption that $C^v(I)$ satisfies no justified envy because $j \in C^v(I)$, $i \in I \setminus C^v(I)$, $\sigma(i) > \sigma(j)$, and $n^v(C^v(I)) = n = n^v((C^v(I) \setminus \{j\}) \cup \{i\})$ where the last equation follows from $I_1 \subseteq (C^v(I) \setminus \{j\}) \cup \{i\}$ and the fact that $n^v(I_1) = n$. ■

This finishes the proof of Theorem 2. □

Proof of Theorem 3. Let $C = (C^v)_{v \in \mathcal{V}}$ be a choice rule that complies with VR protections, maximally accommodates HR protections, satisfies no justified envy, and is non-wasteful. We show this result using the following lemmas.

Lemma 8. $C^o = C^{2s,o}_{Ⓜ}$.

Proof. We prove that C^o maximally accommodates category-o HR protections, satisfies no justified envy, and is non-wasteful.

First, we show that C^o maximally accommodates category-o HR protections. Suppose, for contradiction, that $n^o(C^o(I)) < n^o(I)$ for some $I \subseteq \mathcal{I}$. Then there exists $i \in I \setminus C^o(I)$ such that $n^o(C^o(I) \cup \{i\}) = n^o(C^o(I)) + 1$. If $i \in I \setminus \widehat{C}(I)$, then we get a contradiction with the assumption that C maximally accommodates HR protections. Otherwise, if $i \in C^c(I)$ where $c \in \mathcal{R}$, then we get a contradiction with the assumption that C complies with VR protections. Therefore, C^o maximally accommodates category-o HR protections.

Next, we show that C^o satisfies no justified envy. Let $i \in C^o(I)$ and $j \in I \setminus C^o(I)$ such that $\sigma(j) > \sigma(i)$. If $j \in I \setminus \widehat{C}(I)$, then

$$n^o\left((C^o(I) \setminus \{i\}) \cup \{j\}\right) < n^o\left(C^o(I)\right)$$

because C satisfies no justified envy. However, if $i \in C^c(I)$ for category $c \in \mathcal{R}$, then

$$n^o\left((C^o(I) \setminus \{i\}) \cup \{j\}\right) < n^o\left(C^o(I)\right)$$

because C complies with VR protections. Therefore, C^o satisfies no justified envy.

Now, we show that C^o is non-wasteful, which means that $|C^o(I)| = \min\{|I|, q^o\}$ for every $I \subseteq \mathcal{I}$. If there exists an individual $i \in I$ such that $i \notin \widehat{C}(I)$, then $|C^o(I)| = q^o$ because C is non-wasteful. If there exists an individual $i \in I$ such that $i \in C^c(I)$ where $c = \rho(i) \in \mathcal{R}$, then $|C^o(I)| = q^o$ because C complies with VR protections. If these two conditions do not hold, then all the individuals are allocated open-category positions, i.e., $I = C^o(I)$. Therefore, under all possibilities, we get $|C^o(I)| = \min\{|I|, q^o\}$, which means that C^o is non-wasteful.

Since C^o maximally accommodates category-o HR protections, satisfies no justified envy, and is non-wasteful, we get $C^o = C^o_{Ⓜ}$ (Theorem 2), and hence $C^o = C^{2s,o}_{Ⓜ}$. ■

Let $c \in \mathcal{R}$, $I \subseteq \mathcal{I}$, and $\bar{I}^c = \{i \in I \setminus C^o_{Ⓜ}(I) | \rho(i) = c\}$.

Lemma 9. *$C^c(I)$ maximally accommodates category-c HR protections for $\bar{I}^c$.*

Proof. Suppose, for contradiction, that $n^c(C^c(I)) < n^c(\bar{I}^c)$. This is equivalent to

$$n^c(C^c(I)) < n^c(\bar{I}^c) = n^c\Big(C^c(I) \cup \{i \in I \setminus \widehat{C}(I) | \rho(i) = c\}\Big),$$

which implies that there exists $i \in I \setminus \widehat{C}(I)$ who is eligible for category c such that

$$n^c(C^c(I \cup \{i\})) = n^c(C^c(I)) + 1.$$

This equation contradicts the assumption that C maximally accommodates HR protections. Therefore, $C^c(I)$ maximally accommodates category-c HR protections for $\bar{I}^c$. ■

Lemma 10. *$C^c(I)$ satisfies no justified envy for $\bar{I}^c$.*

Proof. Let $i \in C^c(I)$ and $j \in \bar{I}^c \setminus C^c(\bar{I}^c)$ be such that $\sigma(j) > \sigma(i)$. Note that $i \in \bar{I}^c$. Since C satisfies no justified envy, we have

$$n^c\left(C^c(I)\right) > n^c\left((C^c(I) \setminus \{j\}) \cup \{i\}\right).$$

Hence, C^c satisfies no justified envy for $\bar{I}^c$. ■

Lemma 11. $|C^c(I)| = \min\{|\bar{I}^c|, q^c\}$.

Proof. We consider two cases. First, if $C^c(I) = \bar{I}^c$, then $|C^c(I)| = \min\{|\bar{I}^c|, q^c\}$ because $|C^c(I)| \leq q^c$. Otherwise, if $C^c(I) \neq \bar{I}^c$, then there exists $i \in \bar{I}^c \setminus C^c(I)$. Therefore, $i \in I \setminus \widehat{C}(I)$. Since C is non-wasteful, we get $|C^c(I)| = q^c$ Since $i \in \bar{I}^c \setminus C^c(I)$ and $|C^c(I)| = q^c$, $|\bar{I}^c| > q^c$. Therefore, $|C^c(I)| = q^c = \min\{|\bar{I}^c|, q^c\}$. ■

Therefore, $C^c(I)$ maximally accommodates category-c HR protections for $\bar{I}^c$, $C^c(I)$ satisfies no justified envy for $\bar{I}^c$, and $C^c(I)$ is non-wasteful for $\bar{I}^c$. By Theorem 2, $C^c(I) =$

$C^c_{Ⓜ}(\bar{I}^c)$ and, thus,

$$C^c(I) = C^c_{Ⓜ}(\bar{I}^c) = C^c_{Ⓜ}(\{i \in I \setminus C^o_{Ⓜ}(I) | \rho(i) = c\}) = C^{2s,c}_{Ⓜ}(I).$$

□

Proof of Proposition 2. Suppose that i is chosen by $\widehat{C}^{2s}_{Ⓜ}$ when she withholds some of her reserve-eligible privileges. If i is chosen by $C^o_{Ⓜ}$ for an open-category position, then i will still be chosen by declaring all her reserve-eligible privileges because $C^o_{Ⓜ}$ does not use the category information of individuals and an individual can never benefit from not declaring some of her traits under $C^o_{Ⓜ}$ because she will have more edges in the category-o HR graph. Otherwise, if i is chosen by $C^c_{Ⓜ}$ where $\rho(i) = c \in \mathcal{R}$ then she must have declared her reserve-eligible category c. In addition, by declaring all her traits she will still be chosen by $C^c_{Ⓜ}$ if she is not chosen before for the open-category positions because she will have more edges in the HR graph for category-c positions.

□

Proof of Proposition 3. Let $I \subseteq \mathcal{I}$ be a set of individuals and $I^m \subseteq I$ be the set of reserve-eligible individuals considered at Step 1 of $\widehat{C}^{SCI}_{Ⓜ}$ when I is the set of applicants.

Let $i \in \widehat{C}^{2s}_{Ⓜ}(I) \cap I^g$. Then $i \in C^o_{Ⓜ}(I) \cap I^g$ because $\widehat{C}^{2s}_{Ⓜ}(I) \cap I^g = C^o_{Ⓜ}(I) \cap I^g$. Since $C^o_{Ⓜ}$ satisfies the substitutes condition (Lemma 3), $i \in C^o_{Ⓜ}(I^m \cup I^g)$ because $i \in I^g$ and $i \in C^o_{Ⓜ}(I)$. Therefore, $i \in C^o_{Ⓜ}(I^m \cup I^g) \cap I^g$, which implies $i \in \widehat{C}^{SCI}_{Ⓜ}(I) \cap I^g$ because $\widehat{C}^{SCI}_{Ⓜ}(I) \cap I^g = C^o_{Ⓜ}(I^m \cup I^g) \cap I^g$. Therefore, we conclude that $\widehat{C}^{2s}_{Ⓜ}(I) \cap I^g \subseteq \widehat{C}^{SCI}_{Ⓜ}(I) \cap I^g$.

The assumption that $|I^c| \geq q^o + q^c$, for each reserve-eligible category $c \in \mathcal{R}$, implies that all category-c positions are filled under $C^{2s}_{Ⓜ}$ and $C^{SCI}_{Ⓜ}$. In addition, the first part of the proposition implies that there are weakly more individuals with reserved categories assigned to open-category positions under $C^{2s}_{Ⓜ}$ than under $C^{SCI}_{Ⓜ}$. Therefore,

$$\sum_{c \in \mathcal{R}} \left| \widehat{C}^{2s}_{Ⓜ}(I) \cap I^c \right| \geq \sum_{c \in \mathcal{R}} \left| \widehat{C}^{SCI}_{Ⓜ}(I) \cap I^c \right|.$$

□

Proof of Proposition 4. To show the substitutes condition, let $I \subseteq \mathcal{I}$, $i \in \widehat{C}^{2s}_{Ⓜ}(I)$, and $j \in I \setminus \{i\}$. Since $i \in \widehat{C}^{2s}_{Ⓜ}(I)$, either $i \in C^{2s,o}_{Ⓜ}(I)$ or $i \in C^{2s,c}_{Ⓜ}(I)$ where $\rho(i) = c \in \mathcal{R}$. If $i \in C^{2s,o}_{Ⓜ}(I) = C^o_{Ⓜ}(I)$, then $i \in C^o_{Ⓜ}(I \setminus \{j\}) = C^{2s,o}_{Ⓜ}(I \setminus \{j\})$ since $C^o_{Ⓜ}$ satisfies the substitutes condition (Lemma 3). Now consider the other possibility that $i \in C^{2s,c}_{Ⓜ}(I)$ where $\rho(i) = c \in \mathcal{R}$. Let

$$\bar{I}^c = \{i' \in I \setminus C^o_{Ⓜ}(I) : \rho(i') = c\}$$

and

$$\overline{(I \setminus \{j\})}^c = \{i' \in (I \setminus \{j\}) \setminus C^o_{Ⓜ}((I \setminus \{j\})) : \rho(i') = c\}.$$

The assumption that $i \in C^{2s,c}_{Ⓜ}(I)$ and $C^{2s,c}_{Ⓜ}(I) = C^c_{Ⓜ}(\bar{I}^c)$ imply that $i \in C^c_{Ⓜ}(\overline{(I \setminus \{j\})}^c)$ whenever $i \notin C^o_{Ⓜ}(I \setminus \{j\})$ because $C^c_{Ⓜ}$ satisfies the substitutes condition (Lemma 3) and $\bar{I}^c \supseteq \overline{(I \setminus \{j\})}^c$ since $C^o_{Ⓜ}$ satisfies the substitutes condition (Lemma 3). Therefore, $i \in \widehat{C}^{2s}_{Ⓜ}(I \setminus \{j\})$, which means that $C^{2s}_{Ⓜ}$ satisfies the substitutes condition.

To show the irrelevance of rejected individuals condition, let $I \subseteq \mathcal{I}$ and $i \in I \setminus \widehat{C}^{2s}_{Ⓜ}(I)$. Since $i \in I \setminus \widehat{C}^{2s}_{Ⓜ}(I)$, $i \in I \setminus C^o_{Ⓜ}(I)$ which implies that $C^o_{Ⓜ}(I) = C^o_{Ⓜ}(I \setminus \{i\})$ because $C^o_{Ⓜ}$ satisfies the irrelevance of rejected individuals condition (Lemma 3). Fix $c \in \mathcal{R}$ and let

$$\bar{I}^c = \{j \in I \setminus C^o_{Ⓜ}(I) : \rho(j) = c\}$$

and

$$\overline{(I \setminus \{i\})}^c = \{j \in (I \setminus \{i\}) \setminus C^o_{Ⓜ}((I \setminus \{i\})) : \rho(j) = c\}.$$

If $i \notin \bar{I}^c$, then $\bar{I}^c = \overline{(I \setminus \{i\})}^c$ because $C^o_{Ⓜ}(I) = C^o_{Ⓜ}(I \setminus \{i\})$, and hence $C^c_{Ⓜ}(\bar{I}^c) = C^c_{Ⓜ}(\overline{(I \setminus \{i\})}^c)$, which is equivalent to $C^{2s,c}_{Ⓜ}(I) = C^{2s,c}_{Ⓜ}(I \setminus \{i\})$. Otherwise, if $i \in \bar{I}^c$, then $\bar{I}^c = \overline{(I \setminus \{i\})}^c \cup \{i\}$ because $C^o_{Ⓜ}(I) = C^o_{Ⓜ}(I \setminus \{i\})$. Furthermore, $i \notin \widehat{C}^{2s}_{Ⓜ}(I)$ implies that $i \notin C^c_{Ⓜ}(\bar{I}^c)$. As a result, since $C^c_{Ⓜ}$ satisfies the irrelevance of rejected individuals condition (Lemma 3), $C^c_{Ⓜ}(\bar{I}^c) = C^c_{Ⓜ}(\bar{I}^c \setminus \{i\}) = C^c_{Ⓜ}(\overline{(I \setminus \{i\})}^c)$, which is equivalent to $C^{2s,c}_{Ⓜ}(I) = C^{2s,c}_{Ⓜ}(I \setminus \{i\})$. We conclude that $\widehat{C}^{2s}_{Ⓜ}(I) = \widehat{C}^{2s}_{Ⓜ}(I \setminus \{i\})$, so $C^{2s}_{Ⓜ}$ satisfies the irrelevance of rejected individuals condition.

Appendix C. Institutional Background on Vertical and Horizontal Reservations

In this appendix, we present

(1) the description of the concepts of vertical reservation and horizontal reservation as they are quoted in the Supreme Court judgements *Indra Sawhney (1992)* and *Rajesh Kumar Daria (2007)* in Sections C.1 and C.2,
(2) the main quotes from the Supreme Court judgements *Anil Kumar Gupta (1995)* and *Rajesh Kumar Daria (2007)* that allows us to formulate the SCI-AKG choice rule in Section C.3, and
(3) the description of the choice rule that is mandated in the State of Gujarat as it is quoted in the August 2020 High Court of Gujarat judgement *Tamannaben Ashokbhai Desai (2020)* in Section C.4.

C.1. Indra Sawhney (1992): Introduction of Vertical and Horizontal Reservations. The terms *vertical reservation* and *horizontal reservation* are coined by the Constitution bench of the Supreme Court of India, in the historical judgement *Indra Sawhney (1992)*, where

- the former was formulated as a policy tool to accommodate the higher-level protective provisions sanctioned by the Article 16(4) of the Constitution of India, and
- the latter was formulated as a policy tool to accommodate the lower-level protective provisions sanctioned by the Article 16(1) of the Constitution of India.

The description of these two affirmative action policies, and how they are intended to interact with each other is given in the judgement with following quote:

> A little clarification is in order at this juncture: all reservations are not of the same nature. There are two types of reservations, which may, for the sake of convenience, be referred to as 'vertical reservations' and 'horizontal reservations'. The reservation in favour of scheduled castes, scheduled tribes and other backward classes [under Article 16(4)] may be called vertical reservations whereas reservations in favour of physically handicapped [under clause (1) of Article 16] can be referred to as horizontal reservations. Horizontal reservations cut across the vertical reservations -- what is called interlocking reservations. To be more precise, suppose 3% of the vacancies are reserved in favour of physically handicapped persons; this would be a reservation relatable to clause (1) of Article 16. The persons selected against his quota will be placed in the appropriate category; if he belongs to SC category he will be placed in that quota by making necessary adjustments; similarly, if he belongs to open competition (OC) category, he will be placed in that category by making necessary adjustments.

It is further emphasized in the judgement that vertical reservations in favor of backward classes SC, ST, and OBC (which the judges refer to as *reservations proper*) are "set aside" for these classes.

> In this connection it is well to remember that the reservations under Article 16(4) do not operate like a communal reservation. It may well happen that some members belonging to, say Scheduled Castes get selected in the open competition field on the basis of their own merit; they will not be counted against the quota reserved for Scheduled Castes; they will be treated as open competition candidates.

C.2. Rajesh Kumar Daria (2007): The Distinction Between Vertical Reservation and Horizontal Reservation. The distinction between vertical reservations and horizontal reservations, i.e. the "over-and-above" aspect of the former and the "minimum guarantee" aspect of the latter, is further elaborated in the Supreme Court judgement *Rajesh Kumar Daria (2007)*.

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The second relates to the difference between the nature of vertical
reservation and horizontal reservation.  Social reservations in favour of
SC, ST and OBC under Article 16(4) are 'vertical reservations'.  Special
reservations in favour of physically handicapped, women etc., under
Articles 16(1) or 15(3) are 'horizontal reservations'.  Where a vertical
reservation is made in favour of a backward class under Article 16(4), the
candidates belonging to such backward class, may compete for non-reserved
posts and if they are appointed to the non-reserved posts on their own
merit, their numbers will not be counted against the quota reserved for
the respective backward class.  Therefore, if the number of SC candidates,
who by their own merit, get selected to open competition vacancies, equals
or even exceeds the percentage of posts reserved for SC candidates, it
cannot be said the reservation quota for SCs has been filled.  The entire
reservation quota will be intact and available in addition to those
selected under Open Competition category.  [Vide - Indira Sawhney (Supra),
R. K. Sabharwal vs.  State of Punjab (1995 (2) SCC 745), Union of India
vs.  Virpal Singh Chauvan (1995 (6) SCC 684 and Ritesh R. Sah vs.  Dr.
Y. L. Yamul (1996 (3) SCC 253)].  But the aforesaid principle applicable
to vertical (social) reservations will not apply to horizontal (special)
reservations.  Where a special reservation for women is provided within the
social reservation for Scheduled Castes, the proper procedure is first to
fill up the quota for scheduled castes in order of merit and then find out
the number of candidates among them who belong to the special reservation
group of 'Scheduled Castes-Women'.  If the number of women in such list
is equal to or more than the number of special reservation quota, then
there is no need for further selection towards the special reservation
quota.  Only if there is any shortfall, the requisite number of scheduled
caste women shall have to be taken by deleting the corresponding number
of candidates from the bottom of the list relating to Scheduled Castes.
To this extent, horizontal (special) reservation differs from vertical
(social) reservation.  Thus women selected on merit within the vertical
reservation quota will be counted against the horizontal reservation for
women.
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C.3. Anil Kumar Gupta (1995): Implementation of Horizontal Reservations Compartmentalized within Vertical Reservations. While horizontal reservations can be implemented either as *overall horizontal reservations* for the entire set of positions, or as *compartment-wise horizontal reservations* within each vertical category including the open category (OC), the Supreme Court recommended the latter in their judgement of *Anil Kumar Gupta (1995)*:

> We are of the opinion that in the interest of avoiding any complications and intractable problems, it would be better that in future the horizontal reservations are comparmentalised in the sense explained above. In other words, the notification inviting applications should itself state not only the percentage of horizontal reservation(s) but should also specify the number of seats reserved for them in each of the social reservation categories, viz., S.T., S.C., O.B.C. and O.C.

The procedure to implement compartmentalized horizontal reservation is described in *Anil Kumar Gupta (1995)* as follows:

> The proper and correct course is to first fill up the O.C. quota (50%) on the basis of merit: then fill up each of the social reservation quotas, i.e., S.C., S.T. and B.C; the third step would be to find out how many candidates belonging to special reservations have been selected on the above basis. If the quota fixed for horizontal reservations is already satisfied - in case it is an over-all horizontal reservation - no further question arises. But if it is not so satisfied, the requisite number of special reservation candidates shall have to be taken and adjusted/accommodated against their respective social reservation categories by deleting the corresponding number of candidates therefrom. (If, however, it is a case of compartmentalised horizontal reservation, then the process of verification and adjustment/accommodation as stated above should be applied separately to each of the vertical reservations.

The adjustment phase of the procedure for implementation of horizontal reservation is further elaborated in the Supreme Court judgement *Rajesh Kumar Daria (2007)* as follows:

> If 19 posts are reserved for SCs (of which the quota for women is four), 19 SC candidates shall have to be first listed in accordance with merit, from out of the successful eligible candidates. If such list of 19 candidates contains four SC women candidates, then there is no need to disturb the list by including any further SC women candidate. On the other hand, if the list of 19 SC candidates contains only two woman candidates, then the next two SC woman candidates in accordance with merit, will have to be included in the list and corresponding number of candidates from the bottom of such list shall have to be deleted, so as to ensure that the final 19 selected SC candidates contain four women SC candidates. [But if the list of 19 SC candidates contains more than four women candidates, selected on own merit, all of them will continue in the list and there is no question

of deleting the excess women candidate on the ground that 'SC-women' have been selected in excess of the prescribed internal quota of four.]

C.4. Tamannaben Ashokbhai Desai (2020): High Court Mandate on Adoption of the Two-Step Minimum Guarantee Choice Rule in the State of Gujarat. With its August 2020 High Court judgement *Tamannaben Ashokbhai Desai (2020)*, the two-step minimum guarantee choice rule (2SMG) is now mandated for allocation of state public jobs in the State of Gujarat. While the choice rule is given in the judgement only for a single horizontal trait (women), it is also well-defined and well-behaved for multiple (but non-overlapping) traits as presented in Corollary 2. Originally introduced in Sönmez and Yenmez (2019) prior to the judgement of the High Court of Gujarat,[39] in December 2020 the choice rule is endorsed by the Supreme Court judgement *Saurav Yadav (2020)* for the entire country. Paragraph 56 of the High Court of Gujarat judgement *Tamannaben Ashokbhai Desai (2020)* describes the mandated procedure as follows:

For the future guidance of the State Government, we would like to explain the proper and correct method of implementing horizontal reservation for women in a more lucid manner.

PROPER AND CORRECT METHOD OF IMPLEMENTING HORIZONTAL RESERVATION FOR WOMEN

⋮

Step 1: Draw up a list of at least 100 candidates (usually a list of more than 100 candidates is prepared so that there is no shortfall of appointees when some candidates don't join after offer) qualified to be selected in the order of merit. This list will contain the candidates belonging to all the aforesaid categories.

Step 2: From the aforesaid Step 1 List, draw up a list of the first 51 candidates to fill up the OC quota (51) on the basis of merit. This list of 51 candidates may include the candidates belonging to SC, ST and SEBC.

Step 3: Do a check for horizontal reservation in OC quota. In the Step 2 List of OC category, if there are 17 women (category does not matter), women's quota of 33% is fulfilled. Nothing more is to be done. If there is a shortfall of women (say, only 10 women are available in the Step 2 List of OC category), 7 more women have to be added. The way to do this is to, first, delete the last 7 male candidates of the Step 2 List. Thereafter, go down the Step 1 List after item no. 51, and pick the first 7 women (category does not matter). As soon as 7 such women from Step 1

[39] The two-step minimum guarantee choice rule is referred to as C^{hor}_{2s} in Sönmez and Yenmez (2019).

List are found, they are to be brought up and added to the Step 2 List to make up for the shortfall of 7 women. Now, the 33% quota for OC women is fulfilled. List of OC category is to be locked. Step 2 List list becomes final.

Step 4: Move over to SCs. From the Step 1 List, after item no. 51, draw up a list of 12 SC candidates (male or female). These 12 would also include all male SC candidates who got deleted from the Step 2 List to make up for the shortfall of women.

Step 5: Do a check for horizontal reservation in the Step 4 List of SCs. If there are 4 SC women, the quota of 33% is complete. Nothing more is to be done. If there is a shortfall of SC women (say, only 2 women are available), 2 more women have to be added. The way to do this is to, first, delete the last 2 male SC candidates of the Step 4 List and then to go down the Step 1 List after item no. 51, and pick the first 2 SC women. As soon as 2 such SC women in Step 1 List are found, they are to be brought up and added to the Step 4 List of SCs to make up for the shortfall of SC women. Now, the 33% quota for SC women is fulfilled. List of SCs is to be locked. Step 4 List becomes final. If 2 SC women cannot be found till the last number in the Step 1 List, these 2 vacancies are to be filled up by SC men. If in case, SC men are also wanting, the social reservation quota of SC is to be carried forward to the next recruitment unless there is a rule which permits conversion of SC quota to OC.

Step 6: Repeat steps 4 and 5 for preparing list of STs.

Step 7: Repeat steps 4 and 5 for preparing list of SEBCs.