Outside options, reputations, and the partial success of the Coase conjecture

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Abstract

A buyer and seller bargain over a good's price in continuous time, the buyer has a private value $v \in [\underline{v}, \overline{v}]$ and a positive outside option $w \in [\underline{w}, \overline{w}]$. Additionally, bargainers can either be rational or committed to some fixed price. When the sets of buyer values and commitment types are rich and the probability of commitment vanishes, outcomes are approximately equivalent to those when the can seller make an ultimatum at any price below $\max\{\underline{w}, \underline{v}/2\}$. Although there is minimal delay, outcomes need not be efficient as the buyer sometimes chooses her outside option.

Keywords: Bargaining, reputation, coase conjecture

1 Introduction

What effect do outside options have on bargaining with incomplete information? The existing literature suggests a surprisingly dramatic impact. Most notably, consider an infinite horizon game where in every period the seller proposes a price for a good to a buyer with private information about her value $v \in [\underline{v}, \overline{v}]$. If there are no outside options, then the Coase conjecture holds: when agents are sufficiently patient (offers are frequent) the seller proposes a price of $\underline{v}$ almost immediately if there is "gap", $\underline{v} > 0$, or buyer strategies are stationary (Fudenberg and Tirole (1985), Gul et al. (1986)). The idea is that if today's offer $p > \underline{v}$ is rejected, the seller will update her beliefs and cut her price tomorrow, but in which case even a high value buyer would not accept today unless the price is already low $p \approx \underline{v}$. However, if the buyer can get a strictly positive outside option $w \in [\underline{w}, \overline{w}]$ by exiting the market at the end of the period, then Board and

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Pycia (2014) show the seller acts as if she had commitment: she can choose any price in the first period, and the buyer either accepts it or exits the market.[1] The logic is that if bargaining continues into period 2, the seller will never charge a price below the lowest net value buyer remaining, $v - w$, giving that buyer a continuation payoff below w, so she would prefer to exit in period 1 instead to avoid discounting.

How robust is Board and Pycia (2014)'s result? It relies heavily on only the seller being able to make offers, leaving the lowest net value type with no gains from trade. Paradoxically, if the buyer knew her type, and could take her outside option before the start of bargaining (period 0), she would never enter negotiations, as the lowest net value who did would get a discounted continuation payoff below her outside option.[2] If instead the buyer could sometimes make counteroffers, she might hope for a continuation payoff higher than her outside option when the game continued into period 2.[3] However, introducing offers by the informed party introduces signalling. Off-path buyer offers can then be "punished with beliefs", interpreted as coming from the highest net value buyer, $\bar{v} - \underline{w}$, and so support a wide variety of on path play.[4,5]

One way to help mitigate the power of belief punishments is to introduce a small probability of commitment into the model, types which always propose some fixed price and never back down (another form of incomplete information). Abreu and Gul (2000) first introduced these types on both sides of a bargaining problem with fixed surplus to divide (so there is only one type of rational bargainer); they showed discrete time outcomes converge to the unique equilibrium of a concession game when offers are frequent, independent of almost all details of the bargaining protocol.

When rational agents have private information, a small probability of commitment types can provide clear predictions despite the potential for signalling, because rational agents who imitate commitment types cannot be arbitrarily punished with beliefs as their behavior is on path. Abreu et al. (2015) consider a continuous time concession game with a fixed surplus, when one party's discount rate is private information (high or low) and

[1] Chang (2021) shows that if outside options are dispersed and positively related to values, then the seller's optimal dynamic mechanism may feature declining prices over time.

[2] A similar problem arises in auctions when there must be at least two bids and bids have a sunk cost.

[3] Certainly, if the buyer made all offers instead, she could get the good for free. An alternative assumption to stop unravelling is that buyers don't know their value for good until bargaining starts.

[4] For similar reasons there are multiple equilibria in Board and Pycia (2014) if the buyer sometimes has no outside option. In some the seller chooses any ultimatum and the buyer is believed to have a high outside option if she remains in period 2. Others have a Coasean structure, with low and declining prices.

[5] Chatterjee et al. (2022) get clear predictions in stationary equilibria of a coalitional bargaining game where a veto player has a privately known outside option and can sometimes make offers. The outside option is sufficiently large that the veto player either accepts the first offer she receives, or exits. Again, if she could exit before the start of bargaining, she always would.

there are no outside options. They show that if the set of demands made by commitment types is sufficiently rich, then outcomes must be Coasean as the probability of commitment vanishes: there is immediate agreement on the same terms as would have been agreed if the informed party were known to be her most patient type.[6,7]

On the other hand, Compte and Jehiel (2002) suggest commitment types can have little effect on rational agents' behavior when they have outside options. In an alternating offer protocol with a fixed surplus, and commitment types that aggressively offer an opponent less than her outside option, rational agents never imitate commitment types. The authors' don't consider what happens with more generous commitment types.

In this paper, I introduce commitment types into a model where the seller and buyer sequentially announce prices, before playing a continuous time concession game. A rational buyer has private information about her value and positive outside option. For each possible buyer value v there is some probability of the lowest outside option, $\underline{w} > 0$.

Equilibria have a similar structure to existing reputational models but need not be unique. Rational players always imitate commitment types' prices, however, if the seller's price is unacceptable to a rational buyer, $v - p_s < w$, she only imitates the lowest such price $\underline{p} > 0$. After demand announcements, concession and exit behavior ensure the seller and the rational buyer with the highest remaining value are indifferent between conceding at one instant or the next (the skimming property holds: lower value buyers concede later if at all). Eventually both players reaches a probability 1 reputation for commitment at the same time $T^* < \infty$.

My main result shows that when the sets of buyer values and commitment types are rich and the probability of commitment vanishes, bargaining outcomes are partially Coasean: they are approximately those which would arise if the seller could issue an ultimatum at a price below $p^* = \{\underline{v}/2, \underline{w}\}$ which the buyer must either immediately accept, or exit. Loosely, the set of buyer values is rich if for any $d \in [\underline{v}, \overline{v}]$ there is a possible buyer value close to d, while the set of commitment types is rich if for any $d' \in [0, \overline{v}]$ there is some type which makes a demand close to d'.

The result suggests that the classical Coasean prediction (from Fudenberg and Tirole (1985), and Gul et al. (1986)) of minimal bargaining delays is robust to the presence of

[6] This also matches the alternating offer division when the agent is known to be the patient type. By contrast, Rubinstein (1985) identifies a wide variety of equilibria in an alternating offers game without commitment types, which he selects between on the basis of axioms.

[7] They also show that if commitment types sometimes delay making their fixed demand, non-Coasean limit outcomes are possible as patient agents try to signal their type. Analyzing such types in the current setting, as well as types that vary their demands over time in history contingent ways (see Abreu and Pearce (2007) and Fanning (2016)) has the potential to be very challenging and is left for future work.

outside options, and so is the prediction of low prices when $\underline{v}$ and $\underline{w}$ are both small in contrast to the potential for high prices in Board and Pycia (2014).

However, the result also diverges from some features of the classical Coase conjecture in a similar direction to Board and Pycia (2014). Notably, prices can be high because $\underline{v}$ or $\underline{w}$ are high, even when net values are not. The seller has some market power and outcomes are inefficient (due to exit of some positive net value buyers). For instance, a seller may know her product (e.g. bespoke software) provides substantial value to the buyer (e.g. in cost savings), $\underline{v} >> 0$, even though she doesn't know what competitors can offer. Or the seller may know a competitor's product (e.g. off-the-shelf software with a nationally set price) offers a high outside option, $\underline{w} >> 0$, without knowing the buyer's value for her own product. In either case, $p^* >> 0$ even if $\min\{v - w > 0\} \approx 0$.

An implication of the result if there is a single rational buyer type (v, w), is that there will be immediately agreement at a price $p_s \approx \hat{p}^{v,w} := \min\{v/2, v - w\}$. When $w > v/2$ then $\hat{p}^{v,w} < p^*$; the seller offers the buyer a payoff $v - p_s \approx w$ so that she prefers to accept rather than exit. This matches Binmore et al. (1989)'s finding in an alternating offers game under complete information (without commitment types), although unlike in Compte and Jehiel (2002) rational bargainers always imitate (vanishingly unlikely) commitment types. A natural Coasean benchmark price would then seem to be $\min_{(v,w)} \hat{p}^{v,w}$, the efficient price the seller would charge if she knew she was facing the toughest buyer's type $\arg\min_{(v,w)} \hat{p}^{v,w}$. However, my result shows that the seller can potentially charge a much higher price, $p^* = \max\{\underline{v}/2, \underline{w}\}$ given private information. For example, if the buyer's value is known to be $v = 1$ and her outside options are approximately uniformly distributed on $[0, 1]$, then so are her net values $v - w$, and the seller will charge approximately $p^* = 1/2 > \min_{(v,w)} \hat{p}^{v,w} \approx 0$ for a profit of 1/4.

What's special about p^*? The seller's offer is more generous than the buyer's if the buyer gets greater utility from conceding, $v - p_s > p_b$. With a rich set of buyer values and commitment types, a seller's offer is more generous than any counteroffer of the lowest value buyer who eventually concedes, $v^{1,p_s} = \min\{v > p_s + \underline{w}\}$, if and only if $p_s \leq p^*$. For an agent to make her opponent indifferent between conceding at one instant or the next, she must concede at a rate proportional to the generosity of her offer (which determines his cost of delaying his concession). This rate determines how fast her reputation for commitment grows. When the prior probability of commitment is arbitrarily small, so are updated reputations when only buyers with value greater than v^{1,p_s} have conceded, after which reputations must still grow a lot to reach probability 1. For both agents' reputations to reach probability 1 at the same time, therefore, the buyer must immediately (at time 0) concede or exit with probability approaching 1 if $p_s \leq p^*$, and if $p_s > p^*$, the seller must immediately concede to a counterdemand $p_b \approx p^* < p_s$.

The limit outcome's dependence only on the generosity of the seller compared to a value v^{1,p_s} buyer highlights the model's Coasean force. Eventually the seller realizes she faces such a buyer and then what matters is which player has the greatest incentive to give in (the less generous player). A greater willingness to eventually concede translates into immediate concession. Sometimes, when $p_s \leq p^*$, there are low value buyers who won't concede $v - p_s < \underline{w}$ who could make more generous demands than the seller, $p_b > v - p_s$, but instead make the least generous demand $\underline{p} \approx 0$, which ensures the seller's reputation subsequently grows much faster, so the buyer must immediately concede or exit.

In section 5, I present additional implications and extensions of the model. In particular, I show: my results extend to a discrete-time alternating offers game; the seller's profits can increase in the buyer's outside option or in a sunk cost the buyer must pay to initiate bargaining (allowing endogenous participation); and the seller may successfully charge higher prices ($p_s >> p^*$) if buyer values are sparse (not rich).

The remainder of this section highlights additional literature, then section 2 outlines the model, section 3 characterizes equilibria, the main results for vanishing commitment are in Section 4, and section 5 presents extensions. Unless explicitly stated otherwise, proofs are in the Appendix.

1.1 Additional Literature

Hwang and Li (2017) show that the Coase conjecture may hold if a buyer's outside option arrives stochastically. The seller makes all offers, and the buyer's outside option arrives publicly at the end of each period with some probability (after the seller's offer). With frequent offers, the seller almost immediately offers the buyer a price that would make her lowest value type indifferent to waiting for the outside option. The logic driving the result is that buyers must immediately take an outside option when it arrives. Otherwise the lowest buyer type which did not, would receive a continuation payoff equal to that outside option in subsequent periods (as in Board and Pycia (2014)) and so she would prefer to avoid delay. If the stochastic arrival is not publicly observable multiple equilibria exist, some of which are Coasean and some not.

Nava and Schiraldi (2019) highlight what they call a robust Coase conjecture, when the seller can offer the buyer differentiated goods. The seller makes all offers and after purchasing one variety, the consumer receives no value from buying a second variety. When offers are frequent, the market clears instantaneously, with the buyer purchasing one of the varieties offered, however, the seller retains some market power and the outcome is not efficient. The seller offers a low price (possibly 0) for one variety, and a high price for the other and allows consumers to select between them. The low price for the first variety, effectively creates a consumer outside option, which (by Board and Pycia (2014)) allows her to charge a monopolistic price for the second variety. The authors suggest Board and Pycia (2014)'s result is similarly consistent with a properly understood Coase conjecture. However, seemingly many prices can clear the market with outside options, and the low prices identified in my analysis when $\underline{v} \approx \underline{w} \approx 0$ seem "more Coasean" than Board and Pycia (2014)'s. If the minimum outside option created by low priced varieties is small with differentiated goods, then my results suggest seller profits are likely to be similarly small.

In addition to outside options, the existing literature has identified many reasons the Coase conjecture may fail: a monopolist may rent rather than sell, or under-invest in capacity (Bulow (1982)), or use best-price provisions (Butz (1990)), or buyers may use non-stationary strategies if there is no "gap" between their values and the seller's (Ausubel and Deneckere (1989)). However, other factors that might be thought to interfere with the Coasean logic, merely see it confirmed in different guises. For instance, if a second buyer may arrive to compete with the first, the seller's profit is driven down to what she would get from waiting for that buyer's arrival (Fuchs and Skrzypacz (2010)).

Inderst (2005) and Kim (2009) identify Coasean results when a seller has a vanishing probability of being a commitment type, and the buyer has private information about her value but no outside option and is never a commitment type. Peski (2021) also identifies a Coasean result when dividing two pies, one bargainer has private information about her relative value of the pies, and there is a vanishingly small probability that each bargainer is committed to some menu of divisions.[8]

In a contemporary paper, Pei and Vairo (2022) investigate a related reputational bargaining model, where a rational seller and rational buyer with a private value (but no outside option) simultaneously announce any price demands before becoming committed to them with positive probability (similar to the protocol of Kambe (1999)). There is always a Coasean equilibrium with immediate agreement (all agents announce the same price $p_s = p_b$ which approaches $\underline{v}/2$ as commitment vanishes), but if buyer values are sparse (not rich), there can also be non-Coasean equilibria (the seller charges $p_s > \underline{v}$, as do buyers with $v > p_s$, while buyers with $v \leq p_s$ counterdemand $p_b = 0$ and never

[8]Normalizing an agent's payoff from receiving all of both pies to equal one, in the limit the uninformed bargainer proposes a menu of all divisions that give her a payoff of 1/2 (a lower bound on her payoff facing any known type), and the informed party selects among these. The Coasean logic is similar to that in my paper and Abreu et al. (2015).

concede); this mirrors the (unique limit) equilibrium I highlight in section 5 with high seller prices ($p_s >> p^*$) when buyer values are sparse.[9]

Atakan and Ekmekci (2013) show that reputational bargaining with outside options endogenously determined by a search market can lead to inefficiency.[10] Endogenous outside options are also central to Özyurt (2015), who shows that even vanishingly small reputational concerns allow a wide range of prices in Bertrand-competition like setting with two sellers and a single buyer.[11]

2 The model

In this section I outline a simple baseline model; I show how results extend to a discrete time alternating offers game in Section 5.

A buyer and seller bargain in a continuous time concession/exit game, where the seller has a single indivisible good. Time 0 is subdivided into 4 times, $0^1 < 0^2 < 0^3 < 0^4$, to allow for sequential decisions to be made with no discounting of payoffs between them. At time 0^1 the seller proposes a price $p_s \in P$ where $P \subset (0, \infty)$ is some finite set. At time 0^2, the buyer can observe the seller's price p_s and can: immediately concede (accept her opponent's price; action c), counterdemand $p_b \in (0, p_s) \cap P$, or exit the market (action e). If the game continues to time 0^3: the seller observes p_b and chooses a stopping time $t^s \in (0, \infty]$ to concede, while the buyer chooses a stopping time $t^b \in (0, \infty]$ and an action $a \in \{c, e\}$, where (t^b, c) denotes a decision to concede at time t^b, and (t^b, e) denotes a decision to exit. If agents choose the same stopping time ($t^s = t^b$), each agent's chosen action occurs with probability 1/2 (concession or possibly exit).

Both buyer and seller can either be rational or a commitment type. A rational seller has no value for the good and no outside option. A rational buyer of type (v, w) has an outside option $w > 0$ and a value $v > w$.[12] If the good is traded at price p at time $t \geq 0$ then a rational seller gets a payoff $e^{-rt}p$, and a rational buyer gets the payoff $e^{-rt}(v - p)$, where r is a common discount rate (without loss of generality $r = 1$). If instead, the

[9]The uniqueness of my limit prediction stems in part from sellers making offers before buyers.

[10]Firms and workers flow into a search market and are randomly matched to bargain over a surplus. Bargainers can be rational or a single commitment type which returns to the search market if convinced her opponent is committed. Given no delay before matching in the search market, steady state equilibria are inefficient with no immediate concession in bargaining, even as commitment vanishes. As in Compte and Jehiel (2002), the effect a richer set of commitment types is unclear.

[11]This occurs because if the buyer observes a seller undercut it's rival's posted price, she uses that as an outside option to obtain an even better price in bargaining with the high priced rival.

[12]An buyer with $v < w$ will never agree to any price for the good and would immediately exit at 0^2; I explore endogenous participation in bargaining more generally in section 5.

rational buyer exits the market at time t she receives her outside option for a payoff $e^{-rt}w$, while a rational seller gets a payoff of 0.

The distribution of rational buyer types has finite support Θ with probability mass function g, so that $\sum_{(v,w)\in\Theta} g(v,w) = 1$. Let $V = \{v : (v,w) \in \Theta\}$ and $W = \{w : (v,w) \in \Theta\}$ so then $\underline{v} = \min V$, $\overline{v} = \max V$ and $\underline{w} = \min W$, and $\overline{w} = \max W$. I assume $g(v,\underline{w}) > 0$ for all $v \in V$, so there is always a chance of the minimum outside option; this is implicitly an assumption about the richness of types (without it, the main results require a more complicated definition of a rich type set). I further assume that $v - w \neq p$ and $(v-p)/(p-p') \neq w'/(v'-p'-w')$ and $v-p \neq p'$ for all $(v,w),(v',w') \in \Theta$ and $p, p' \in P$ with $p > p'$; given the finiteness of Θ and P, equalities are non-generic.

The probability of player i being a commitment type is $z_i \in (0,1)$. There is a finite set $P_i \subset P$ of commitment types for agent i. Conditional of being committed, she is of type $p_i \in P_i$ with probability $\pi_i(p_i) \in (0,1)$. Type p_i demands the price p_i in the bargaining game, concedes only if offered a better price (i.e. if $p_s < p_b$ for the buyer) and never exits the market. To simplify the exposition, I assume $\max P_b \leq \max P_s$, so the most largest seller commitment price is larger than the buyer's. I also assume the price is a buyer commitment demand, $\underline{p} = \min P_b = \min P$, which may be substantive, but is not needed in the alternating offer game of Section 5 (which admits a continuum of prices).

Let $\mu_s(p_s)$ be the probability that a rational seller proposes a price $p_s \in P$ at 0^1, and given p_s let $\mu_b^{p_s,v,w}(a)$ be the probability that a rational seller of type (v,w) chooses action $a \in P \cup \{e,c\}$ at 0^2. Hence, immediately after a seller's demand $p \in P_s$ and buyer's counterdemand $p' \in P_b$, the bargainers' reputations for commitment are:

$$\bar{z}_s^{p_s} = \frac{z_s\pi_s(p_s)}{z_s\pi_s(p_s) + (1-z_s)\mu_s(p_s)}, \quad \bar{z}_b^{p_s,p_b} = \frac{z_b\pi_b(p_b)}{z_b\pi_b(p_b) + (1-z_b)\sum_{(v,w)\in\Theta}\mu_b^{p_s,v,w}(p_b)}$$

and $\bar{z}_s(p_s) = 0$ if $p_s \notin P_s$ and $\bar{z}_b^{p_s}(p_b) = 0$ if $p_b \notin P_b$.[13] If $\mu_b^{p_s,v',w'}(p_b) > 0$ for some (v',w') then the probability that the buyer is of type (v,w) conditional on rationality is:

$$\bar{g}^{p_s,p_b}(v,w) = \frac{g(v,w)\mu_b^{p_s,v,w}(p_b)}{\sum_{(v',w')\in\Theta'} g(v',w')\mu_b^{p_s,v',w'}(p_b)}.$$

Conditional on reaching a continuation game at 0^3 with demands p_s, p_b, let the probability that player i concedes by time $t \in \{0^3, 0^4\} \cup (0,\infty]$ be $F_i^{p_s,p_b}(t)$, and let the probability that buyer exits by time t in that continuation game be $E_b^{p_s,p_b}(t)$. We can later back out the behavior of rational agents from these objects. At time t, the seller's updated

[13]This is without loss of generality even if $\mu_s(p_s) = 0$ or $\mu_s^{p_s}(p_b) = 0$; commitment types can't deviate.

reputation for commitment is then $\bar{z}_s^{p_s,p_b}(t) = \bar{z}^{p_s}/(1 - F_s^{p_s,p_b}(t))$ while buyer's updated reputation is $\bar{z}_b^{p_s,p_b}(t) = \bar{z}^{p_s,p_b}/(1 - E_b^{p_s,p_b}(t) - F_b^{p_s,p_b}(t))$. A rational seller's utility in the continuation game at 0^3 when she concedes at time t is then:

$$U_s^{p_s,p_b}(t) = \int^{\tau<t} p_s e^{-r\tau} dF_b^{p_s,p_b}(\tau) + (1 - F_b^{p_s,p_b}(t) - E_b^{p_s,p_b}(t))p_b e^{-rt} + \frac{1}{2}e^{-rt}\left((F_b^{p_s,p_b}(t) - F_b^{p_s,p_b}(t_-))(p_s + p_b) + (E_b^{p_s,p_b}(t) - E_b^{p_s,p_b}(t_-))p_b\right)$$

where $G(t_-) = \sup_{\tau<t} G(\tau)$ with $G(0^3_-) = 0$ for $G : \{0^3, 0^4\} \cup (0, \infty] \to [0, 1]$. The utility of a rational buyer with value v that concedes at time t is:

$$U_b^{p_s,p_b,v,c}(t) = \int^{\tau<t} (v - p_b)e^{-r\tau} dF_s^{p_s,p_b}(\tau) + (1 - F_s^{p_s,p_b}(t))(v - p_s)e^{-rt} + \frac{1}{2}e^{-rt}\left((F_s^{p_s,p_b}(t) - F_s^{p_s,p_b}(t_-))(2v - p_s - p_b)\right)$$

The utility of a rational buyer with type (v, w) that exits at time t is:

$$U_b^{p_s,p_b,v,w,e}(t) = \int^{\tau<t} (v - p_b)e^{-r\tau} dF_s^{p_s,p_b}(\tau) + (1 - F_s^{p_s,p_b}(t))we^{-rt} + \frac{1}{2}e^{-rt}\left((F_s^{p_s,p_b}(t) - F_s^{p_s,p_b}(t_-))(w + v - p_b)\right).$$

I will analyze weak perfect Bayesian equilibria of this game, where at each information set (0^1, 0^2 and 0^3) agents' strategies must be optimal given their beliefs, beliefs are consistent with Bayes' rule when possible (even off the equilibrium path), and an agent's actions do not affect her belief about her opponent. However, my main result, providing tight bounds on equilibrium outcomes also holds for any Nash equilibrium.

3 Equilibrium

This section characterizes equilibria of the game for arbitrary parameters. I follow a heuristic approach leaving many details for the appendix. I first characterize equilibria in the continuation game at 0^3, and then consider agents' initial demand choices.

In any equilibrium, rational sellers must imitate commitment demands. This is a standard result in the reputational bargaining literature. If instead a seller revealed rationality by demanding $p_s \notin P_b$ then by making the commitment demand $p_b = \underline{p} \in P_b$ a rational buyer could ensure the seller's immediate concession due a reputational Coase conjecture: If the seller didn't she must expect the buyer (he) will soon concede to her, and so if he doesn't, she will eventually become convinced that he never will and at that point will concede. If $T > 0$ is the last time she concedes, however, even a rational buyer won't concede just before T, and so the seller won't want to wait that long. Given that, the seller can always do weakly better by demanding $\max P_s > \max P_b$. I henceforth assume rational seller's always imitate commitment demands, $p_s \in P_s$.

It is also true that rational buyers must imitate commitment demands, but to show that I need to characterize the continuation equilibrium in the continuation game at 0^3 that would arise if they did not. I now proceed to characterize that equilibrium generally.

3.1 Equilibrium in the continuation game

I first describe equilibria in the continuation game at 0^3 assuming commitment demands $p_i \in P_i$ before highlighting what happens when the buyer makes a non-commitment demand ($p_b \notin P_b$). Since p_s and p_b are fixed, I drop them in superscripts on variables.

If there is just one rational buyer type who prefers to concede rather than exit, $v - p_s > w$, there is a unique equilibrium in the continuation game, which resembles that in Abreu and Gul (2000); since the buyer will never choose her outside option it becomes irrelevant. The equilibrium is characterized by three properties: 1) at most one agent concedes at time 0; 2) both agents reach a probability 1 reputation at the same time $T^* < \infty$; and 3) agents are indifferent between conceding at any time on $(0, T^*]$.

Property 3) implies that the seller and buyer must concede at the constant rates λ_s^v and λ_b respectively on $(0, T^*)$, where:

$$\lambda_s^v := \frac{r(v - p_s)}{p_s - p_b}, \qquad \lambda_b := \frac{rp_b}{p_s - p_b}.$$

The numerator of agent i's concession rate is her opponent's instantaneous cost of delay his concession, while denominator is the capital gain he receives when she concedes instead of him (so the rate equalizes his costs and benefits of waiting). Let $T_i = -ln(\bar{z}_i)/\lambda_i$ be the time it would take that agent i to reach a probability 1 reputation given that she concedes at rate λ_i on $(0, T^*)$ but not at time 0. Then we must have $T^* = \min\{T_s, T_b\}$ and time 0 concession satisfies $F_i(0^4) = 1 - \min\{\bar{z}_i \bar{z}_j^{-\lambda_i/\lambda_j}, 1\}$.

Given indifference to concession on $(0, T^*)$, the seller's continuation game payoff must be $U_s = F_b(0^4)p_s + (1 - F_b(0^4))p_b$ and the buyer's payoff must be $U_b = F_s(0^4)(v - p_b) + (1 - F_s(0^4))(v - p_s)$; these only exceed what an opponent offers if that opponent concedes at time 0 with positive probability. Increasing the relative generosity (utility gained from concession) of the seller's offer compared to the buyer's, $\lambda_s^v/\lambda_b = (v - p_s)/p_b$, causes the probability the buyer (seller) concedes at time 0 to increase (decrease) in order to ensure both agents still reach probability 1 reputation at the same time.

Let $\Theta^c = \{(v, w) \in \Theta : v - w > p_s\}$ be the set of rational buyer types for whom the seller's price is acceptable, $v - p_s > w$, and $\Theta^e = \{(v, w) \in \Theta : v - w < p_s\} = \Theta \setminus \Theta^c$ be the set of types for whom the seller's price is not acceptable.

Adding additional Θ^c buyer types into the continuation game doesn't greatly change the structure of the equilibrium highlighted above. Property 3) must be modified to account for the skimming property: on $(0, T^*)$ the seller and highest remaining buyer type are indifferent between one instant and the next. The skimming property says that high value agents always concede before low value agents, since they face greater (instantaneous) costs of delaying their concession, $r(v - p_s)$. Enumerate these Θ^c buyer values $v^1 < v^2 < ... < v^K$ and let t^k be the first time that all buyers with value v^k have conceded (so $T^* = t^1$ by the skimming property. Formally, let $v^1 = \min\{v \in V : v > \underline{w} + p_s\}$ and $v^{k+1} = \min\{v \in V : v > v^k\}$ until $v^K = \overline{v}$ for some $K < \infty$ and

$$t^k = \min\{t \geq 0^4 : F_b(t) \geq (1 - \bar{z}_b) \sum_{(v,w)\in\Theta^c : v \geq v^k} \overline{g}(v, w)\}.$$

Also let $t^{K+1} = 0$. The seller must then concede at rate $\lambda_s^{v^k}$ on (t^{k+1}, t^k) to make a buyer with the highest remaining value v^k, indifferent between immediately conceding and waiting an instant to do so, while the buyer continues to concede at rate λ_b on each interval. This equilibrium is still unique.

Notice the Coasean force at work in this equilibrium: high value buyers benefit from the presence of low value buyers. This is because the seller (he) concedes at a slower rate to low value buyers, which means he concedes with greater probability at time 0 to ensure both agents reach a probability 1 reputation at the same time T^* (compared to when all buyers have high value). More precisely, $F_s(0^4)$ is increasing in $\bar{g}(v^1, w)$.

Adding Θ^e buyer types, who never concede, into the continuation game, modifies the equilibrium structure further. Let $v^1 < v^2 < ... < v^K$ and $t^1 \geq t^2 \geq ... \geq t^{K+1}$ be defined as above, but now notice that the restriction of those definitions to Θ^c agents is substantive, e.g. there may be $(v, w) \in \Theta^e$ with $v < v^1$. The seller must still concede at the continuous rate $\lambda_s^{v^k}$ on (t^{k+1}, t^k) and the buyer at rate λ_b. Now, however, let $\underline{\lambda}^{v,w}$ be the concession rate which would make a buyer of type (v, w) indifferent between immediately exiting and waiting an instant to do so, that is:

$$\underline{\lambda}^{v,w} := \frac{rw}{v - p_b - w}.$$

Buyer $(v, w) \in \Theta^e$ will then choose to exit at: time $0 = t^{K+1}$ if $\underline{\lambda}^{v,w} > \lambda_s^{\bar{v}}$; at time t^k if $\underline{\lambda}^{v,w} \in (\lambda_s^{v^{k-1}}, \lambda_s^{v^k})$; and at $T^* = t^1$ if $\underline{\lambda}^{v,w} > \lambda_s^{v^1}$. When the buyer exits at $t^k < T^*$, she may also have to concede with positive probability to ensure the seller (he) is willing to concede just afterwards; if she didn't concede he would prefer to concede just before she exits rather than after. However, it $t^k > 0$ she can't concede too often or he would prefer to concede just after t^k than before. More precisely, suppose the buyer exits with (conditional) probability α at time t^k. If $t^k < T^*$ then the buyer must also concede with probability greater than $\alpha p_b/(p_s - p_b)$. If $t^k > 0$, she must not concede with probability greater than $\alpha p_b/(p_s - p_b)$. Clearly, therefore, if $t^k \in (0, T^*)$, she must exit with probability $\alpha p_b/(p_s - p_b)$ exactly.

The previous equilibrium property 1) need not hold when exit is possible, instead both agents may concede with positive probability at time 0. It may still be optimal for the seller to concede 0^3 when the buyer exit and concession at 0^4 with positive probability. Without loss of generality, however, the buyer never concedes at 0^3 and the seller never concedes at 0^4. Additionally, the equilibrium need not be unique. This is because total buyer concession is not determined at T^* when there is exit, nor is the identity of which types concede at $t^k \in (0, T^*)$; for instance, greater concession at T^* can reduce T^*.

If we assume the buyer make a non-commitment demand ($p_b \notin P_b$) while maintaining that the seller makes a commitment demand ($p_s \in P_s$), then it is impossible for both agents to reach a probability 1 reputation by the same time $T^* < \infty$. To accommodate this situation, define $T_b = \min(\{t \geq 0^4 : F_b(t) = (1 - \bar{z}_b)x\}$, $T_s = \min\{t \geq 0^4 : F_s(t) = 1 - \bar{z}_b\}$ and $T^* = \min\{T_s, T_b\}$ where $x = \sum_{(v,w)\in\Theta^c} \bar{g}(v, w)$ is the equilibrium probability that a rational buyer is some Θ^c type that is willing to concede, conditional on her being rational. The buyer must then have conceded or exited with probability 1 by time $T^* < \infty$ (to do so she must exit with positive probability at T^*), however, the seller may or may not have reached a probability 1 reputation by T^*. Prior to T^*, the other properties of equilibria (described above) must still hold. Those features will allow me to argue that in equilibrium the buyer only makes commitment demands ($p_b \in P_b$) in the next subsection.

Figure 1 displays an equilibrium of this sort. I summarize the above equilibrium characterization into the following Proposition.

Lemma 1. *Consider a continuation game at* 0^3 *after demands* $p_s \in P_s$ *and* $p_b \in P$ *and*

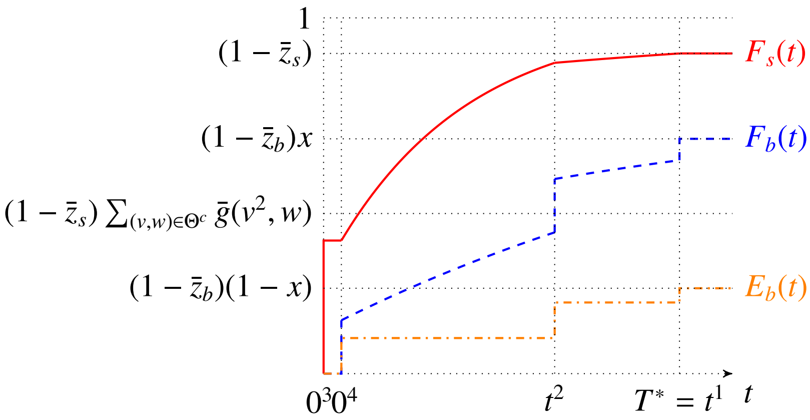

Figure 1. An example equilibrium

define. Without loss of generality, the buyer never concedes or exits at 0^3 *and the seller never concedes at* 0^4*. The skimming property holds before* $T^* < \infty$ *and:*

(1) the seller concedes at rate $\lambda_s^{v^k}$ *and the buyer at rate* λ_b *on* $(t^{k+1}, t^k) \cap (0, T^*)$.

(2) buyer $(v, w) \in \Theta^e$ *exits at time* $t^k \in \{t^1, ..., t^{K+1}\} \cap [0, T^*]$ *if* $\underline{\lambda}^{v,w} \in (\lambda_s^{v^{k-1}}, \lambda_s^{v^k})$ *where* $\lambda_s^{v^0} = 0$ *and* $\lambda_s^{v^{K+1}} = \infty$.

(3) if the buyer exits at time $t^k \in \{t^1, ..., t^{K+1}\} \cap [0, T^*]$ *with probability* α *then she must concede with probability greater than* $\alpha p_b/(p_s - p_b)$ *whenever* $t^k < T^*$ *and with probability less than* $\alpha p_b/(p_s - p_b)$ *if* $t^k \in (0, T^*]$.

(4) $F_b(T^*) + E_b(T^*) = 1 - \bar{z}_b$ *and if* $p_b \in P_b$ *then* $F_s(T^*) = 1 - \bar{z}_s$.

3.2 Equilibrium demand choices

I next consider the buyer's demand choice at 0^2 after the seller has announced a commitment demand. I show that Θ^e buyer types who find the seller's price unacceptable, always imitate the lowest commitment demand $\underline{p}$ (or immediately exit), while Θ^c buyer types, must imitate some commitment type and weakly (possibly strictly) prefer lower price demands. This is an important result for establishing the paper's main findings because it means Θ^e buyers demands are always extremely ungenerous to the seller when there is a rich set of commitment types, $\underline{p} \approx 0$, who consequently has little incentive to concede to them. The lemma below provides a formal statement:

Lemma 2. *Consider any equilibrium in the continuation game at* 0^2 *after a seller demand* $p_s \in P_s$*. Let* $p_b \in P_b$*, and* $p_b < p_b' \in P$ *then: all* Θ^c *type buyers weakly prefer demanding* p_b *to* p_b' *and without loss of generality* Θ^e *type buyers never demand* p_b'.

First consider the behavior of the Θ^c buyers who eventually concede; this dictates the behavior of the Θ^e buyers who exit. These Θ^c buyers mix between some subset of demands. As a group, if they demand p'_b with positive probability, they must also imitate all smaller commitment prices, $p_b < p'_b$ where $p_b \in P_b$.[14] If this wasn't true, a rational seller would believe buyers who demand p_b are either committed or will eventually exit, and so would immediately concede. However, in that case demanding $p'_b > p_b$ cannot be optimal for any buyer (who would prefer the lower price).

Since a value $\overline{v} = v^K$ buyer always concedes first (by the skimming property), she must be indifferent between demanding p'_b or p_b and then conceding at any time $t \in [0^4, t^{K,p_b}]$. Suppose then that a type v^k buyer is indifferent between the options: (i) demand p'_b and concede at t^{k,p'_b} and (ii) demand p_b and concede at t^{k,p_b}. The proof of the lemma then establishes that a value v^{k-1} buyer is either indifferent between (i) and (ii) or strictly prefers the lower price (ii). Moreover, if the buyer with value v^{k-1} is indifferent between these options then so are all buyers, and there is a greater time discounted probability that the seller has not conceded after the lower price p_b, that is: $e^{-rt^{k,p'_b}}(1 - F_s^{p'_b}(t^{k,p'_b})) \leq e^{-rt^{k,p_b}}(1 - F_s^{p_b}(t^{k,p_b}))$, strictly if $F_s^{p'_b}(t^{k,p'_b}) > 0$.

The key idea behind the claim that v^{k-1} must (weakly) prefer the lower price, option (ii) over (i), is that there is "more delay" after lower prices since the seller concedes more slowly ($\lambda_s^{v^k,p'_b} > \lambda_s^{v^k,p_b}$), but value v^{k-1} has a lower cost of delay than v^k who is indifferent between the options. For a more precise argument, consider the special case of $k = K$, and suppose (by way of contradiction) a value v^{K-1} buyer strictly preferred option (i) over (ii). All buyers with value $v < v^{K-1}$ must then likewise strictly prefer (i), because the benefit of (i) compared to (ii) is linear in the buyer's value. Since no buyer with value $v \leq v^{K-1}$ adopts (ii) we would have $T^{*,p_b} = t^{K,p_b}$ and since the seller concedes faster after p'_b than after p_b ($\lambda_s^{v^K,p'_b} > \lambda_s^{v^K,p_b}$) we would have $t^{K,p'_b} < t^{K,p_b} = T^{*,p_b}$. But then, consider the buyer's benefit of delaying her concession from time 0^4 to some time $t \in (0, t^{K,p'_b}]$ after demanding p'_b (case (a)) and after p_b (case (b)). Since the seller concedes faster after p'_b than after p_b ($\lambda_s^{v^K,p'_b} > \lambda_s^{v^K,p_b}$), agreements are more delayed in (b) than (a). Because delay is less costly for value v^{K-1} than for v^K, the benefit in (b) compared to (a) must be greater for v^K than for v^{K-1}. The benefit for value v^K is 0 in both (a) and (b), however, and so for v^{K-1} the benefit in (a) is strictly larger than in (b). Hence, v^{K-1} strictly prefers option (ii) over (i), a contradiction.

To illustrate why the time discounted probability that the seller has not conceded must be larger after the smaller price p_b than after p'_b when a value v^{k-1} buyer is indifferent

[14] In fact, something stronger is true: if some Θ^c buyer with value v sometimes demands p'_b, then some such buyer must also sometimes demand $p_b \in P_b$ with $p_b < p'_b$.

between (i) and (ii), consider the special case where $t^{k,p'_b} = t^{k,p_b} = 0^4$. For the value v^{k-1} buyer to be indifferent between (i) and (ii) the seller (she) must concede more often after the higher price p'_b. In fact, even if she conceded with the same strictly positive probability after each demand then the v^{k-1} buyer would strictly prefer the lower price option (ii). More precisely, we need $F_s^{p'_b}(0^3) = F_s^{p_b}(0^3)(p_s - p_b)/(p_s - p'_b) \geq F_s^{p_b}(0^3)$.

I now turn to the claim that Θ^e buyers (who never concede), never demand the higher price p'_b (and consequently only ever demand $\underline{p} \in P_b$ or immediately exit). The key idea here is that the benefit to a buyer type $(v', w') \in \Theta^e$ of exiting instead of conceding at t^{k,p_b} after demand p_b is proportional to the discounted probability that the seller has not conceded $e^{-rt^{k,p_b}}(1 - F_s^{p_b}(t^{k,p_b}))(v' - (w' - p_s))$. However, when a value v^{k-1} buyer is indifferent between concession options (i) and (ii) so is value v' and the discounted probability that the seller has not conceded is larger after the smaller price p_b than after p'_b, $e^{-rt^{k,p_b}}(1 - F_s^{p_b}(t^{k,p_b})) \geq e^{-rt^{k,p'_b}}(1 - F_s^{p'_b}(t^{k,p'_b}))$, strictly if $F_s^{p'_b}(t^{k,p'_b}) > 0$. Hence either type (v', w') strictly prefers option (ii') demand p_b and exit at t^{k,p_b} over option (i') demand p_b and exit at t^{k,p'_b}, or $F_s^{p'_b}(t^{k,p'_b}) = 0$. In the latter case, if option (i') is ever an optimal strategy for type (v', w') we can assume she exits at time 0^2 instead, without loss of generality.

Given that only Θ^c buyers ever demand $p_b > \underline{p}$ they must only ever make commitment demands. If such a type made a non-commitment demand after the seller's commitment demand ($p_b \notin P_b$ after $p_s \in P_s$), standard arguments imply that she must immediately concede (the reputational Coase conjecture, outlined at the start of Section 3).

Finally, we can move back to the start of the game and the seller's demand choice at 0^1. I do not attempt a precise characterization here, but merely establish that an equilibrium exists, Proposition 1. The proof is in the *online* Appendix. It first identifies a particular continuation equilibrium structure in the continuation game at 0^3 that is continuous in agents' beliefs, $\bar{z}_i$ and $\bar{g}$. Given that, existence incorporating demand choice follows by a standard Kakutani fixed point argument.

Proposition 1. *An equilibrium exists.*

4 Vanishing commitment

This section presents the paper's main result: when the set of buyer values and commitment types are rich and commitment vanishes, bargaining outcomes are approximately equivalent to those when the seller can propose an ultimatum at a price below $p^* = \max\{\underline{w}, \underline{v}/2\}$. To get there, I again first focus on the continuation game at 0^3 (as agents' initial reputations vanish), before considering demand choices.

4.1 Vanishing commitment in the continuation game

First consider the simple case in which there is only a single rational buyer type who concedes $v - p_s > w$; a setting equivalent to Abreu and Gul (2000) (the outside option is irrelevant). If agents' initial reputations in the continuation game vanish at same rate ($\bar{z}_i^n \to 0$, $\bar{z}_i^n/\bar{z}_j^n \in [1/L, L]$ for some $L \geq 1$) then the agent who is less generous than her opponent must concede with probability approaching 1 at time 0; so the seller immediately concedes if $p_b > v - p_s$, and the buyer does if $p_b < v - p_s$. The reason for this result is that the generosity of agent i's offer is proportional to her opponent's cost of delay and is thus proportional to her concession rate, λ_i. Those concession rates determine the exponential growth rate of an agent's reputation during the continuation game, $(d\bar{z}_i(t)/dt)/\bar{z}_i(t) = \lambda_i$. When initial reputations are vanishingly small, reputations must grow a lot to reach probability 1 (it takes infinitely long in the limit). Absent time 0 concession, therefore, the faster growth rate of the more generous agent's reputation means she would reach a probability 1 reputation much faster than her opponent. To ensure both agents reach a probability 1 reputation at the same time, therefore, the less generous agent must concede with probability approaching 1 at time 0.

A straightforward corollary of the single buyer type case is a continuation game where the seller is *always* more generous than the buyer ($v^1 - p_s > p_b$) whose demand $p_b > \underline{p}$ implies that she never exits (by Lemma 2). In this case, if initial reputations vanish at the same rate the buyer must immediately concede with probability approaching 1. The more generous seller always concedes faster and so builds reputation faster. The buyer must, therefore, immediately concede in the limit so both agents reach a probability 1 reputation at the same time.

More interestingly, in a continuation game where the lowest value buyer who concedes is more generous than the seller ($v^1 - p_s < p_b$) the seller must concede immediately concede with probability approaching 1 if agents' initial reputations vanish at the same rate but the lowest value buyer's probability doesn't ($\lim_n \sum_{(v^1,w)\in\Theta^c} \bar{g}^n(v^1, w) > 0$). This prediction highlights the key Coasean force which drives the main results: any possibility of such a low value buyer makes the seller immediately back down in the limit.

The logic behind this result is as follows: The buyer's positive concession rate means that all high value buyers (who may be less generous than the seller, e.g. $\bar{v} - p_s > p_b$) must have the conceded in some bounded length of time. At that point, agents' reputations (for commitment) must still be vanishingly small, and so we are effectively

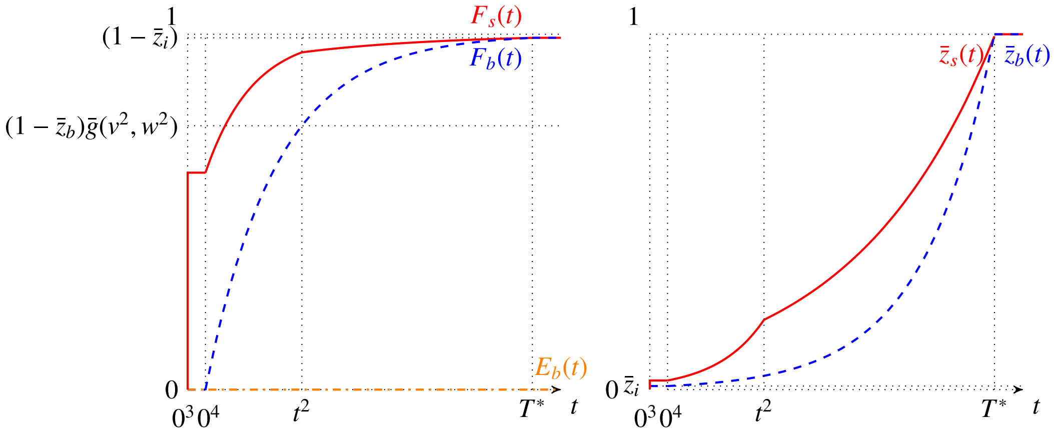

Figure 2. Continuation equilibrium with parameters: $v^2 = 6$, $v^1 = 4$, $\underline{w} < 1$, $p_s = 3$, $p_b = 2$, $\bar{g}(v^1, w) = 1/4$, $\bar{z}_i = 1/100$. Left: Concession. Right: Updated reputations.

back in the case of a single rational buyer type: the additional time it takes for agents to reach a probability 1 reputation is unbounded, and during that time the (more generous) low value buyer concedes quicker than the seller and so her reputation grows quicker. To ensure both agents reach a probability 1 reputation at the same time, therefore, the seller must concede with probability approaching 1 at time 0.

Figure 2 illustrates this logic in an example with two buyer values $v^2 = 6$, $v^1 = 4$ and outside option $\underline{w} < 1$. The announced price $p_s = 3$ and $p_b = 2$ imply that the seller is more generous than the high value buyer, and so concedes at a faster rate $\lambda_s^{v^2} = 3 > \lambda_b = 2$ on the interval $(0, t^2)$. However, she is less generous than the low value buyer, and so concedes slower thereafter $\lambda_s^{v^1} = 1 < \lambda_b = 2$. Initial reputations are small, $\bar{z}_i = 1/100$, and so even though the buyer is much more likely to have a high rather than low value, $\bar{g}(v^2, \underline{w})/\bar{g}(v^1, \underline{w}) = 3$, it takes much less time for all high value types to concede than all low value types, $t_2/T^* < 1/3$, because of the concave shape of the concession function F_b; if $\bar{z}_i^n \to 0$ then $t_2/T^* \to 0$. This means most reputation building occurs after t^2 when the buyer has a reputation building advantage (since $\lambda_s^{v^1} < \lambda_b$). To ensure that both agents reach a probability 1 reputation at the same time therefore the seller must concede with high probability at time 0, $F_s(0^3) = 0.61$.

What happens in the continuation game as initial reputations vanish, when there are some types that exit (so $p_b = \underline{p}$)? If there is some type which waits until T^* to exit ($\lim_n \bar{g}(v, w) > 0$ for some $(v, w) \in \Theta^e$ with $\underline{\lambda}^{v,w} < \lambda_s^{v^1}$) then the seller must again concede with probability approaching 1 at time 0. The buyer's positive concession rate means that the time it takes until only this exiting type remains is bounded. For the seller's reputation to reach probability 1 by that time, therefore, she must always immediately concede in the limit.

On the other hand, if $p_b = \underline{p}$ is sufficiently small and no buyers exit at T^* ($\underline{\lambda}^{v,w} > \lambda_s^{v^1}$ for $(v, w) \in \Theta^e$ where $\lim_n \sum_{(v^1,w)\in\Theta^c} \bar{g}^n(v^1, w) > 0$), then the buyer must immediately concede or exit with probability approaching 1. When $p_b = \underline{p}$ is very small (see Lemma 3 part (f) for a precise cutoff) not only is the buyer always less generous than the seller $p_b = \underline{p} \leq \underline{w} < v^1 - p_s$ but is so ungenerous that the seller would wait to receive concession from $(v^1, w) \in \Theta^c$ buyers at time t^2 even if all other buyer types exit at t^2. This means that agents' reputation must be vanishingly small at t^2 and since the seller concedes faster at that point, the buyer must concede and exit at time 0 with probability approaching 1 so that both agents can reach a probability 1 reputation at the same time.

Finally, unsurprisingly, if the seller's reputation vanishes but the buyer's doesn't ($\lim_n \bar{z}_b^n > 0$) then the seller must immediately concede with probability approaching 1 or the buyer would reach a probability 1 reputation before the seller. Similarly, if the buyer's reputation vanishes but the seller's doesn't, and $p_b > \underline{p}$ (so no buyers exit) then the buyer immediately concedes with probability approaching 1. Lemma 3 formally establishes these limit results for the continuation game.

Lemma 3. *Consider some fixed demands $p_s \in P_s$ and $p_b \in P_b$ and sequence of continuation games at 0^3 with updated equilibrium beliefs $(\bar{z}_i^n, \bar{g}^n)$.*
Suppose (for some subsequence) $\bar{z}_s^n \to 0$ and $\bar{z}_b^n \geq L\bar{z}_s^n$ for some constant $L > 0$. If:

(a) $\lim_n \bar{z}_b^n > 0$,

(b) or $\lim_n \bar{g}^n(v, w) > 0$ *for some* $(v, w) \in \Theta^c$ *with* $v - p_s < p_b$,

(c) or $p_b = \underline{p}$ *and* $\underline{\lambda}^{v,w} < \lambda_s^{v^1}$ *for some* $(v, w) \in \Theta^e$ *with* $\lim_n \bar{g}^n(v, w) > 0$,

then, $\lim_n F_s^n(0) = 1$.
Suppose instead $\bar{z}_b^n \to 0$ and $\bar{z}_s^n \geq L\bar{z}_b^n$ for some constant $L > 0$.

(d) If $p_b > \underline{p}$ *and* $\lim_n \bar{z}_b^n > 0$, *then* $\lim_n F_b^n(0^4) = 1$.

(e) If $p_b > \underline{p}$ *and* $v^1 - p_s > p_b$, *then* $\lim_n F_b^n(0^4) = 1$.

(f) If $p_b = \underline{p} < v^1 - p_s$ *and* $p_b(1 - \lim_n x^n) < (p_s - p_b) \lim_n \sum_{(v^1,w)\in\Theta^c} \bar{g}^n(v^1, w)$ *and* $\underline{\lambda}^{v,w} > \lambda_s^{v^1}$ *for all* $(v, w) \in \Theta^e$, *then* $\lim_n F_b^n(0^4) = 1 - \lim_n E_b^n(0^4) = \lim_n x^n$.

4.2 The main results

This subsection presents the paper's main results, which have two parts. First, Proposition 2 identifies tight bounds on the seller's payoff as commitment vanishes when the set of buyer's values and commitment types are rich: it is approximately the same as what she could get by making an ultimatum offer to the buyer at a price below $p^* = \max\{\underline{v}/2, \underline{w}\}$. The second part, Proposition 3 shows that $\max\{v - p^*, w\}$ is a lower bound on the payoffs of a buyer with type (v, w) in the above limit, and also provides regularity conditions that identify the ultimatum price the seller would charge, and so precisely predict limit equilibrium outcomes.

In order to formally present these results, I must first define what makes the sets of buyer values and commitment types rich. I say that the set of buyer values is $\varepsilon \geq 0$ rich if for any $d \in [\underline{v}, \overline{v}]$, there exists some $v \in V$ such that $|v - d| < \varepsilon$. This means that the difference between two consecutive buyer values must be less than 2ε. Given a rational buyer's type distribution, I say that the sets of agents' commitment types are $\varepsilon' > 0$ rich if for any $d' \in [0, \overline{v} - \underline{w}]$, there exists some $p_i \in P_i$ such that $|p_i - d'| < \varepsilon'$ for $i = 1, 2$. When I say that set of buyer values and commitment types are rich, I informally mean that they are respectively $\varepsilon > 0$ and $\varepsilon' > 0$ rich where $\varepsilon \approx \varepsilon' \approx 0$. However, in fact, my results first fix the ε richness of the buyer's values, and only then choose the ε' richness of commitment types. I also define $H(p) = \sum_{(v,w):v-w<p} g(v, w)$ for any $p \in [0, \infty)$, as the fraction of rational buyers that find the price p unacceptable, $v - p < w$; for instance if net values were approximately uniformly distributed on $[0, 1]$ then $H(p) \approx p$.

Proposition 2 then presents precise upper and lower bound on the seller's equilibrium payoff, V_s. For any $\varepsilon > 0$, consider any distribution of rational buyer types that is ε rich, and assume agents' prior probabilities of commitment vanishes at the same rate ($z_i^n \to 0$, $z_i^n/z_j^n \in [1/L, L]$ for some $L \geq 1$). Then for any $\delta > 0$, there exists $\varepsilon' > 0$ such that if the distribution of commitment types is ε' rich, the seller's limit payoff is at most δ greater than what she could from choosing an ultimatum at any price below $p^* + 2\varepsilon$ (the upper bound). Moreover, her limit payoff is least what she could from choosing an ultimatum at any price below p^* minus 2ε (the lower bound). Clearly, this tightly pins down seller's payoff for small ε.

Proposition 2. *For any $\delta > 0$, and any $\varepsilon > 0$ rich distribution of rational buyer types (g, Θ), there exists some $\varepsilon' > 0$ such that for any sequence of bargaining games $(z_i^n, \pi_i, P_i, g, \Theta, P)_{i \in s,b}$ with a ε' rich distribution of commitment types, $z_i^n \to 0$ and $z_s^n/z_b^n \in [1/L, L]$ for some $L \geq 1$, the seller's payoffs satisfy:*

$$\max_{p \in [0,p^*]} (1 - H(p))p - 2\varepsilon \leq \liminf_n V^n \leq \limsup_n V^n \leq \max_{p \in [0,p^*+2\varepsilon]} (1 - H(p))p + \delta.$$

To explain the logic for this result, I first highlight something special about $p^* = \max\{\underline{v}/2, \underline{w}\}$. It is the highest price p^* such that the seller can always guarantee her offer is more generous than the counterdemand of any buyer who eventually concedes.

To see that a seller proposing $p_s \leq p^*$ is always more generous than the counterdemand of any buyer who eventually concedes, notice that if $p_s \leq \underline{v}/2$ then $\underline{v}/2 \leq \underline{v} - p_s$ and so $p_b < p_s \leq \underline{v}/2 \leq \underline{v} - p_s$. Similarly, $p_s \leq \underline{w}$ implies $p_b < p_s \leq \underline{w} < v^{1,p_s} - p_s$.

This feature of p^* helps establish the lower bound on the seller's payoff. Suppose price $\hat{p}_s$ maximizes the seller's payoff when she can issue an ultimatum, but is restricted to prices below p^*. In a reputational bargaining game with a rich set of buyer values and commitment types, we can then find a commitment demand $p_s \in P_s$ that is very slightly smaller than $\hat{p}_s$, which ensures the buyer will either immediately concede or exit with probability approaching 1 as commitment vanishes, and so provide a limit profit of approximately $\hat{p}_s(1 - H(\hat{p}_s))$. Any counterdemand made with positive probability in the limit would make the buyer's updated reputation in the continuation game vanish at a weakly faster rate than the seller's. If that counterdemand is not minimal, $p_b > \underline{p}$, it is only made by Θ^{c,p_s} buyers who eventually concede, and because they are less generous than the seller (since $p_s \leq p^*$), we know they must then immediately concede with probability approaching 1 (Lemma 3, part (d)). If that counterdemand is minimal, however, then it is very ungenerous, $p_b = \underline{p} \approx 0$, since the set of commitment types is rich. The seller's limit demand p_s is chosen to ensure there is no Θ^{e,p_s} buyer type that would wait until T^* to exit, and is greater than $\hat{p}_s - 2\varepsilon$ when buyer values are ε rich.[15] And so, in this continuation game the buyer must immediately concede with probability approaching 1 (Lemma 3, part (f)).

On the other hand, when the seller proposes $p_s > p^*$ and the set of buyer values and commitment types is rich, there is a counterdemand for the lowest value buyer who concedes, v^{1,p_s} that buyer which is more generous than the seller's offer, where either $p_b \approx p^*$ or $p_b < p^*$. It is useful to distinguish between two cases depending on whether a buyer with value $\underline{v}$ ever concedes: (i) $p_s < \underline{v} - w$, and (ii) $p_s \in (\underline{v} - \underline{w}, \overline{v} - \underline{w})$. In case (i) we must have $p_s > p^* = \underline{v}/2$, and any counterdemand $p_b > \underline{v} - p_s$ for type $(\underline{v}, \underline{w}) \in \Theta^c$ is more generous than the seller's offer. With a rich set of commitment types there is such a (more generous) counterdemand with $p_b \approx \underline{v} - p_s < p^*$. In case (ii) if there is a rich set of buyer values, then the lowest value buyer that ever concedes v^{1,p_s} must be close to indifferent to taking the lowest outside option $v^{1,p_s} - p_s \approx \underline{w}$. Any counterdemand $p_b > v^{1,p_s} - p_s$ for type $(\underline{v}, \underline{w}) \in \Theta^c$ is more generous than the

[15] If $\hat{p}_s \leq \underline{v} - \underline{w}$ then no Θ^{e,p_s} will wait until T^* for any $p_s < \hat{p}_s$. If $\hat{p}_s > \underline{v} - \underline{w}$ then there is some $p_s \approx v^{1,p_s} - \underline{w}$ which ensures no such waiting. Since $\underline{\lambda}^{v,\underline{w},p_s,\underline{p}}$ is decreasing in v and $\underline{\lambda}^{v^{1,p_s},\underline{w},p_s,\underline{p}} \approx \lambda_s^{v^{1,p_s},p_s,\underline{p}}$ given $p_s \approx v^{1,p_s} - \underline{w}$ we have $\underline{\lambda}^{v,\underline{w},p_s,\underline{p}} > \lambda_s^{v,p_s,\underline{p}}$ for $v \leq v^{1,p_s}$. Higher outside option buyers have even less incentive to wait.

seller's offer. With a rich set of commitment types, there is such a (more generous) counterdemand $p_b \approx \underline{w} \leq p^* < p_s$.

This second feature of p^* establishes the upper bound on the seller's payoff. Suppose the seller charged some $p_s > p^*$ with positive limit probability as commitment vanished, then her updated reputation in the continuation game must vanish (at a weakly faster rate than the buyer's). As highlighted above, given a rich set of buyer values and commitment types there is a commitment counterdemand $p_b \in P_b$ which would make the lowest value buyer who concedes more generous than seller, $p_b > v^{1,p_s} - p_s$, and furthermore either $p_b < p^* = \underline{v}/2$ (in case (i)) or $p_b \approx \underline{w} \leq p^*$ (in case (ii)). If the lowest value buyer, $v^{1,p_s} - p_s$, makes that demand p_b with positive limit probability, therefore, the seller would immediately concede with probability approaching 1 due to the Coasean force in the model (Lemma 3 part b), and in equilibrium she must do so.[16]

The argument above implies a lower bound on the payoff $V_b^{v,w}$ of buyer type (v, w) of approximately $\max\{v - p^*, w\}$ as commitment vanishes (given a rich set of buyer values and commitment types); this is claim (a) in Proposition 3. To get a tighter bound on the buyer's payoff and the outcome of the game, requires imposing regularity conditions on the seller's payoff function in the ultimatum game. These are similar in spirit to requiring that $\arg\max_{p \leq p^*} p(1 - H(p))$ is unique, but must be slightly stronger to account for the richness of the sets of buyer values and commitment types. Part (b) says that if the seller strictly prefers an ultimatum price close to p^* to any lower price then $\max\{v - p^*, w\}$ is also an upper bound on the buyer's limit payoff. Part (c) says that if an ultimatum at a price $\hat{p}_s \leq p^*$ gives higher seller profits than any other price below or just above p^*, then as commitment vanishes in the reputational game, the seller will charge a price close to $\hat{p}_s$, and the buyer will either immediately concede or exit.

The definition below helps to formally describe conditions on the seller's ultimatum game payoff function. Given a distribution of buyer types and price $p \in [0, \infty)$ define

$$\check{p}(p) = \min\{p, \max\{v - \underline{w} \leq p : v \in V\}\}.$$

where $\max \emptyset = \infty$. Clearly, $\check{p}(p) = p$ if $p \leq \underline{v} - \underline{w}$, and otherwise $\check{p}(p) \leq p$ is the largest net value associated with a type $(v, \underline{w})$ that prefers her outside option to accepting p. When the set of buyer values is $\varepsilon > 0$ rich, we have $\check{p}(p) > p - 2\varepsilon$. Proposition 3's proof is in the Online Appendix.

[16]If v^{1,p_s} didn't demand p_b, then some higher value v' buyer must instead. If v^{1,p_s} demands $p'_b < p_b$ she would have to immediately concede in the limit, while if she demanded $p'_b > p_b$ the seller would immediately concede. In either case, however, she would benefit by deviating to p_b to receive the fast seller concession to the higher value buyers over a unbounded interval (in addition to any time 0 concession).

Proposition 3. *Consider any $\varepsilon > 0$ rich distribution of rational buyer types (g, Θ).*

(a) There exists some $\varepsilon' > 0$ such that $\liminf_n V_b^{v,w,n} \geq \max\{w, v - p^* - 2\varepsilon\}$...

(b) If $\check{p}(p^*)(1 - H(\check{p}(p^*))) > p(1 - H(p))$ for all $p < \check{p}(p^*)$ and $\check{p}(p^*)(1 - H(\check{p}(p^*))) > (p^* + 2\varepsilon)(1 - H(p))$ for all $p > p^* + y$ for some $y \geq 2\varepsilon$,[17] then there exists $\varepsilon' > 0$ such that $\limsup_n V_b^{v,w,n} \leq \max\{w + y, v - p^* + 2\varepsilon\}$....

(c) If $\check{p}(\hat{p}_s)(1 - H(\check{p}(\hat{p}_s))) > p(1 - H(p)$ for some $\hat{p}_s \leq p^*$ and $p \in [0, \check{p}(\hat{p}_s)) \cup (\hat{p}_s, p^* + 2\varepsilon]$, then there exists $\varepsilon' > 0$ such that $\lim_n \sum_{p_s \in [\hat{p}_s - 2\varepsilon, \hat{p}_s)} \mu_s(p_s) = 1$, and without loss of generality $\lim_n \mu_b^{p_s,v,w}(\{c, e\}) = 1$ when $\lim_n \mu_s(p_s) > 0$, so $\max\{w, v - \hat{p}_s\} \leq \liminf_n V_b^{v,w,n} \leq \limsup_n V_b^{v,w,n} \leq \max\{w, v - \hat{p}_s - 2\varepsilon\}$...

...in any sequence of bargaining games $(z_i^n, \pi_i, P_i, g, \Theta, P)_{i \in s,b}$ with a ε' rich distribution of commitment types, $z_i^n \to 0$ and $z_s/z_b \in [1/L, L]$ for some $L \geq 1$.[18]

5 Extensions and discussion

This section discusses some of the implications of my model, and presents extensions of it. In particular, I show how: results extend to an alternating offers bargaining protocol; seller payoffs can increase in the buyer's outside option or sunk costs the buyer must pay to initiate bargaining (allowing endogenous participation); results depend on the richness of the sets of agents' types.

5.1 Alternating offers

Below I outline an alternating offers bargaining protocol, which is a minimal modification of Board and Pycia (2014)'s protocol, where outcomes must converge to those of the continuation game as offers become frequent.

In period 1 the seller can propose a price $p_s \in [0, \infty)$. The buyer observes this and can then accept, reject, or exit (taking her outside option). If still bargaining in period $n \geq 2$, the buyer can propose a price $p_b \in [0, \infty)$. The seller observes this and can accept, or make a counterdemand $p_s \in [0, \infty)$. If the seller makes a counterdemand the buyer observes this and can accept, reject, or exit. If one of the players accepts, or exits,

[17] This last condition must be satisfied for large enough y. It ensures the seller wouldn't want to announce a high price $p_s > p^* + y$ if that always elicited a counterdemand $p_b \leq p^* + 2\varepsilon$.

[18] The statement after the ellipsis should follow each item (a), (b) and (c).

the game ends. If the price p is agreed in period n, a rational seller gets $\delta^{n-1}p$ and a rational buyer $\delta^{n-1}(v-p)$ where $\delta = e^{-r\Delta}$ for some period length $\Delta > 0$. If the buyer exits in period n, rational payoffs are 0 and $\delta^{n-1}w$ respectively. The description of types is unchange except now assume $\underline{p} = \min P_b < \min_{(v,w)\in\Theta} v - w$.

In this model, in any equilibrium the buyer never reveals rationality before the game ends. This is an immediate consequence of Lemma 1 from Board and Pycia (2014). The reason is that if a rational buyer did reveal rationality, a rational seller will never propose or accept a price strictly below that of the lowest net value type she still considers feasible. Given that, the lowest net value type's continuation payoff will be (weakly) less than her outside option w. Of course, if such a type faces a committed seller, her continuation payoff is below $\max\{v - p_s, w\}$. In either case, however, the buyer could have obtained the payoff $\max\{v - p_s, w\}$ in the previous period without it being discounted, hence, she would never wait.

On the other hand, for any $\varepsilon > 0$, there exists $\overline{\Delta} > 0$ such that if $\Delta < \overline{\Delta}$ and the seller has revealed rationality but the buyer has not at history h_t, then the buyer's continuation payoff is at least $\max\{v - p_b, w\} - \varepsilon$ and the seller's is at most $p_b + \varepsilon$. This follows almost immediately from Abreu and Gul (2000)'s Lemma 1, a reputational Coase conjecture argument (a discrete time analogue of the argument at the start of Section 3). As offers become frequent, $\Delta \to 0$, therefore, the game approaches the concession/exit game. Abreu and Gul (2000)'s Proposition 4, provides a complete proof of such a convergence result in a fixed surplus setting, e.g. one rational buyer type with $v > w = 0$.

With more general bargaining protocols, there will typically always still always be an equilibria which converges to the continuation game I analyze. Assume the seller always believes she faces a buyer compatible with her commitment demand $p_s < \overline{v} - \underline{w}$ if the buyer reveals rationality, then such a buyer must concede almost immediately when offers are frequent. However, it might be possible to construct other equilibria where the buyer sometimes reveals rationality, since the possibility that she is a rational type that would never accept the seller's commitment demand provides its own form of commitment.

5.2 Effects of outside options and sunk bargaining costs

In this subsection, I highlight how the buyer's private information about values and outside options can have quite different effects. In particular, I show that the seller's payoff can increase in the buyer's outside option. A higher buyer outside options cause her to opt out of bargaining (exit) allowing the seller to effectively commit to a higher price. Such endogenous participation also means the seller may benefit from a sunk cost that the buyer must pay to initiate bargaining. Interestingly, when sunk costs are non-negligible, bargaining outcomes may always appear fully Coasean (in particular efficient) with respect to those who initiate bargaining.

Both a buyer's value and her outside option determine her net value, $v - w$, the potential gains from trade with the seller. However, the model predicts that these two components can have very different effects even when holding the distribution of net values fixed. To see this, consider three simple examples, in all of which the buyer's net values are approximately uniformly distributed on $[0, 1]$. Example i): the buyer's value is known to be $v = 1$ but her outside option is approximately uniform on $[0, 1]$, so $p^* = \underline{v}/2 = 1/2$. Example ii): the buyer's outside option is know to be $w \approx 0$ but her value is approximately uniform on $[0, 1]$, so $p^* = \underline{w} \approx 0$. Example iii): the buyer's outside option is known to be $w = 1/2$ and her value is approximately uniform on $[0.5, 1.5]$, so $p^* = \underline{w} \approx 1/2$. In all three examples the seller will charge $p_s \approx p^*$, however, the value of p^* varies. In i) and iii) $p^* = 1/2$ and so the seller's payoff is approximately $1/4$, whereas in ii) $p^* \approx 0$ and so the seller's payoff is likewise close to 0. The key difference is that in i) and iii), the seller price $p_s \approx 1/2$ is generous to all buyer types who find it acceptable, $v - p_s \geq 1/2$ and so they cannot make a more generous counterdemand (we must have $p_b < v - p_s$). In ii), however, a price $p_s \approx 1/2$ is very ungenerous to some buyer types who find it acceptable $v^{1,p_s} - p_s \approx w \approx 0$, and hence such types can make a low counterdemand $p_b \approx w \approx 0$ which is more generous than the seller's, $p_b > v^{1,p_s} - p_s$.

Extending example ii) shows how the seller's payoff can increase in the buyer's outside option. Continue to assume buyer's values are approximately uniform on $[0, 1]$, but now the outside option is $w \in (0, 0.25]$. Net values are then approximately uniform on $[-w, 1 - w]$ and the seller will charge $p_s \approx p^* = w$ for a payoff of $(1 - 2w)w$, which is increasing in w.[19] This suggest the seller may benefit from competitors, who provide more attractive outside options for buyers, and allow her to partially escape the Coase conjecture. Although a key message of Board and Pycia (2014) was similarly that the seller could benefit from positive buyer outside options, in their model the seller's payoff is always decreasing in the buyer's outside option (so long as it is positive).

In the above example, I allowed the buyer's outside option to exceed her value $w > v$, however, in other examples the seller can benefit from higher outside options even when always maintaining $w < v$.[20] Formally, in the baseline model, I assumed $v > w$ for all

[19]If $w \in (0.25, 0.33]$ the seller again charges $p_s \approx p^* = w$ for a payoff of $(1-2w)w$, and if $w \in (0.33, 1]$ she charges $p_s \approx (1 - w)/2$ for a payoff of $(1 - w)^2/4$.

[20]Suppose $v \sim U[1, 5]$ and $w \in [0.5, 1]$, so net values are approximately uniformly on $[1 - w, 5 - w]$ and the seller will charge $p_s \approx p^* = w$ for a payoff of $w(5 - 2w)/4$, which is increasing in w.

$(v, w) \in \Theta$. This simplified the exposition but had no effect on the results because rational buyers with $v - w < \underline{p}$ would always exit at time 0^2 (see Lemma 4). Below, I highlight additional ways in which the types who choose to initiate bargaining can be endogenously determined.

First suppose that buyers can take their outside option very slightly before the start of bargaining, something I call "negligible" delay. If there is a rich set of commitment types, all positive net value rational buyer types (i.e. our previously defined Θ) will wait for this negligible delay because they receive payoffs strictly above their outside option when a committed seller type demands $\min P_s < \min\{v - w > 0 : (v, w) \in \Theta\}$. This outcome is in contrast to Board and Pycia (2014), where even negligible delay would cause the whole market to unravel.

What happens if the delay required before bargaining is non-negligible? The benefit to all buyer types from bargaining outlined above (occasional low prices $\min P_s$ from committed sellers) becomes vanishingly small as commitment vanishes, and may no longer justify delay in taking an outside option. Suppose, therefore, that the buyer can either immediately take her outside option w, or wait to bargain with any bargaining payoffs then discounted by $\delta < 1$ (i.e. she has to wait $-ln(\delta)/r$ before getting to bargain). In this case, with a rich set of buyer values and commitment types outcomes are approximately equivalent to those when the seller charges an ultimatum price of exactly $p^* = \underline{v}/2 > \underline{w}$ where now $\underline{v} = \min\{v \geq 2\underline{w}/\delta : (v, w) \in \Theta\}$ is the lowest value buyer who initiates bargaining.[21] Only rational buyers with $\delta(v - p^*) > w$ initiate bargaining. With respect to such buyers, outcomes may appear Coasean: there is immediate agreement at the price $\underline{v}/2$, the same price as would be agreed if the buyer was known to have type $(\underline{v}, \underline{w})$. It may also appear that the buyers' "small" outside options $w < \underline{v}/2 = p^* \leq v - p^*$ are irrelevant to bargaining (as in Binmore et al. (1989) under complete information). However, both appearances are misleading. There be substantial inefficiency due to types with $\delta v > w$ choosing not to bargain, and outside options do play a role in determining prices by determining $\underline{v}$.

The seller's equilibrium payoff can be increasing in the buyer's cost of delay $(1 - \delta)$, because delay deters low value buyers from bargaining (it acts as a screening device),

[21]To see this, assume the seller charges an arbitrary deterministic limit price $\check{p}$, let $\Theta^{\check{p}} = \{(v, w) \in \Theta : w \leq \delta(v - \check{p})\}$ and $\underline{v}^{\check{p}} = \min\{v : (v, \underline{w}) \in \Theta^{\check{p}}\}$. Outcomes must then be approximately those where the seller chooses any ultimatum $p_s \leq p^{*,\check{p}} = \max\{\underline{v}^{\check{p}}/2, \underline{w}\}$ where for consistency we need $p_s = \check{p}$. Suppose that $p_s < p^*$ and $p_s < \underline{u}^{\check{p}} = \min\{v - w : (v, w) \in \Theta^{\check{p}}\}$ then the seller could profitably increase her demand. Hence, if $p_s < p^{*,\check{p}}$ we need $p_s = \underline{u}^{\check{p}}$, but we can never have $p_s = \underline{u}^{\check{p}}$ because then $w > \delta(v - p_s)$ for some $(v, w) \in \Theta^{\check{p}}$, a contradiction. And so we must have $p_s = p^{*,\check{p}} < \underline{u}^{\check{p}}$. If $p_s = p^{*,\check{p}} = \underline{w} \geq \underline{v}^{\check{p}}/2$ then $\underline{v}^{\check{p}} - p_s \leq \underline{v}^{\check{p}}/2 \leq \underline{w} < \underline{w}/\delta$, a contradiction to $p_s < \underline{u}^{\check{p}}$. Hence, we must have $\check{p} = p^{*,\check{p}} = \underline{v}^{\check{p}}/2 > \underline{w}$. And so, $\underline{u}^{\check{p}} > \check{p} = \underline{v}/2$ where $\underline{v} = \underline{v}^{\check{p}} = \min\{v \geq 2\underline{w}/\delta : (v, w) \in \Theta\}$.

allowing the seller to charge a higher price.[22] This can help explain why some sellers appear to make their goods intentionally hard to purchase, beyond just restricting supply (e.g. Birkin bags[23]). Interestingly, even as $\delta \to 1$, outcomes need not converge to those of the negligible delay model.[24] This is because waiting is a form of sunk investment cost for the buyer and there is a hold up problem: the seller cannot commit to the low prices that would encourage greater investment. Of course, when delay is lengthy, $\delta < 2\underline{w}/\overline{v}$, the hold up problem will inefficiently deter any buyer from initiating bargaining

Analogous predictions hold when the buyer must pay an additive sunk cost $c > 0$ to initiate bargaining.[25] Assuming negligible delay, the lowest value buyer which bargains is then $\underline{v} = \min\{v \geq 2(c + \underline{w}) : (v, w) \in \Theta\}$, and there is again immediate agreement at a price of $p^* = \underline{v}/2$.

5.3 Rich type space requirements

In this subsection I illustrate the need for the sets of buyer values and commitment types to be rich for my results to be meaningful.

I first highlight that if the set of buyer values is not sufficiently rich, prices may be much higher than the Coase conjecture would lead us to expect. My main result shows that as commitment vanishes rational seller's will never demand more than $p^* + 2\varepsilon$ as commitment vanishes when the set of buyer values is ε-rich (and there is a rich set of commitment types). For the set of values to be ε rich we need that for any $d \in [\underline{v}, \overline{v}]$, there exists some $v \in V$ such that $|v - d| < \varepsilon$. If the set of values is sparse, however, ε may need to be large.

For instance, consider the case of binary values, $v \in \{\underline{v}, \overline{v}\}$. The set of values is only ε rich if $2\varepsilon > \overline{v} - \underline{v}$. If we additionally assume $\min\{\overline{v}/2, 2\underline{w}\} > \underline{v}$ the seller can in fact charge $p_s = \overline{v}/2 \in (p^*, p^* + 2\varepsilon)$. This equals the highest price she could charge if the buyer was known to have value $\overline{v}$, even when $\overline{v}$ is very large. As commitment

[22] For example, suppose rational buyer's values are approximately uniform on $[0, 1]$ and their outside option is $w \in (0, \delta/2)$. For $\delta < 1$, we have $\underline{v} = 2w/\delta$ and the seller charges w/δ for expected profits of $(1 - 2w/\delta)w/\delta$. The seller's payoff is decreasing in δ when $\delta \geq 1/(2 - 4w)$, so that for $w < 0.25$ the seller benefits from some delay, $\delta < 1$.

[23] For example see *https://baghunter.com/blogs/insights/how-to-get-birkin-bag-from-hermes* on the obstacles to acquiring such bags.

[24] For the example in footnote 22, as $\delta \to 1$ the seller's payoff approaches $(1 - 2w)w$. When $w > 0.33$, this is strictly less than her payoff in the negligible delay model of $(1 - w)^2/4$. If $w < 0.33$ then her payoff in the negligible delay model is also $(1 - 2w)w$. The order of limits matters here: first $z_i^n \to 0$ then $\delta \to 0$.

[25] These costs may take the form of fees for lawyers and advisors in negotiations. Initial contracts specifying breakup fees before entering advanced negotiations (typical in mergers) can play a similar role (while also affecting disagreement payoffs).

vanishes, any high value buyer immediately concedes to that demand, while any low value buyer immediately exits. This is because that demand is more generous than any counterdemand $p_b \in (\underline{p}, p_s)$ of the high value seller, $\overline{v} - p_s > p_b$, so the seller concedes and builds reputation faster.[26] Introducing a third buyer value v' that is slightly higher than $\overline{v}/2 + \underline{w}$ rules out the price $p_s = \overline{v}/2$, because the value v' buyer could counterdemand slightly more than $\underline{w}$ which is more generous than the seller, $v' - p_s < p_b$, so she would builds reputation more quickly, causing the seller to immediately concede.

A rich set of buyer values is only needed because outside options are positive, $\underline{w} > 0$. If $\underline{w} = 0$ my results easily extend to show the buyer would choose her best ultimatum price $p_s \leq p^* = \underline{v}/2$ regardless of the richness of buyer values. In that case, if the seller made a demand $p_s > \underline{v}$ with positive limit probability, then buyers with type $(\underline{v}, 0)$ would demand $\underline{p}$ and then wait until at least T^* to exit, which would mean the seller must immediately concede with probability approaching 1, by the same logic as Lemma 3, part (c). If $p_s \in (\underline{v}/2, \underline{v})$, however, then $(\underline{v}, 0)$ can counterdemand $p_b \approx \underline{v} - p_s < \underline{v}/2$, to which the seller would again have to immediately concede by Lemma 3, part (b).

As stated previously, the assumption that for any value $v \in V$ there is a positive probability of the lowest outside option, $g(v, \underline{w}) > 0$, is also implicitly about the richness of buyers' types. Without something similar to this assumption, the main results can break down. In particular, suppose that instead $w = h(v) \geq v/2$ where $h(v)/v$ is strictly decreasing, then outcomes are approximately equal to those where the seller can choose any ultimatum given a rich set of commitment types. In this case any seller price p_s is more generous than the counterdemand of any Θ^{c,p_s} buyer who eventually concedes, and so buyers immediately concede or exit as commitment vanishes; $v - p_s > h(v)$ implies $p_s < (v - h(v)) \leq v/2$ and so for any counterdemand $p_b < p_s < v - p_s$.[27]

The main result also depends on a rich set of commitment types, and in particular buyer commitment types that make ungenerous offers $\underline{p} \approx 0$. All types of rational buyer could benefit if they were constrained to make more generous offers, $\underline{p} >> 0$. Consider an example where buyer has value 5, or 6 each with probability 0.24, value 13 with prob-

[26]Moreover, after the ungenerous counterdemand $\underline{p} \approx 0$, a low value buyer never waits as the seller concedes at rate $\lambda_s^{\overline{v},\underline{p},p_s} = r(\overline{v} - p_s)/(p_s - \underline{p}) \approx r$ since $\underline{\lambda}^{\underline{v},\underline{w}} = r\underline{w}/(\underline{v} - \underline{p} - \underline{w}) < r$.

[27]If $\hat{p}_s \in \arg\max_p(1-H(p))p$ then $\hat{p}_s = v - h(v)$ for some $v \in V$, and there exists $p_s \in (\hat{p}_s - 2\varepsilon', \hat{p}_s) \cap P_s$ given a $\varepsilon' > 0$ rich set of commitment types. If $p_s < \underline{v} - h(\underline{v})$, then by Lemma 3 all must buyers immediately concede in the limit. If $p_s > \underline{v} - g(\underline{v})$ then $\lambda_s^{v^{1,p_s},p_s,\underline{p}} = (v^{1,p_s} - p_s)/(p_s - \underline{p}) \leq (h(v^{1,p_s}) + 2\varepsilon')/(v^{1,p_s} - h(v^{1,p_s}) - 3\varepsilon')$ whereas $\underline{\lambda}^{v,g(v),p_s,\underline{p}} = h(v)/(v - h(v) - \underline{p}) \leq h(v)/(v - h(v))$ for $v < v^{1,p_s}$. We have $h(v^{1,p_s})/(v^{1,p_s} - h(v^{1,p_s})) < h(v)/(v - h(v))$ given that $h(v)/v$ is decreasing in $v < v^{1,p_s}$, and so for all $\varepsilon' > 0$ small enough $\lambda_s^{v^{1,p_s},p_s,\underline{p}} < \underline{\lambda}^{v,g(v),p_s,\underline{p}}$. By Lemma 3, therefore, the seller can guarantee a payoff of $\arg\max_p(1 - H(p))p - 2\varepsilon'$.

ability 0.48, and values $\{7, 8, ..., 12\}$ each with probability 1/150, and a known outside option of $w = 3 = p^*$. With a rich set of commitment types we must have an upper bound on seller prices is 4. However, the seller will actually choose $p_s \approx p^* = 3$, for a limit payoff of 2.28 (buyer $\underline{v} = 5$ immediately exits). On the other hand, if the buyer can only imitate commitment prices greater than $\underline{p} = 1.5$, there are multiple equilibrium limits, which include one where the seller always proposes a price just below 2, which is accepted by all buyers; if the seller charges higher prices, a value $\underline{v} = 5$ buyer demands $\underline{p} = 1.5$ and waits until T^* to exit.[28]

5.4 No buyer offers

Many settings relevant to the Coase conjecture, may seem to offer buyers very limited opportunities to make counteroffers. Of course, buyer counteroffers may be vanishingly unlikely in equilibrium in my model, even when the potential for such counteroffers significantly determines the seller's (ultimatum) price; only if the seller charged higher prices would the buyer make counteroffers. However, it is also of interest to understand what would happen if the buyer could not make counteroffers.

If we continue to assume that there are a rich set of buyer commitment types (who don't accept less than a target price), then as commitment vanishes the seller can effectively choose any ultimatum price, consistent with Board and Pycia (2014); this shows the clear benefits to the buyer of making generous counteroffers, consistent with findings from the previous subsection. In a discrete time game with no buyer offers and frequent seller offers, Abreu and Gul (2000)'s reputational Coase conjecture implies that if the seller reveals rationality she must almost immediately propose a price acceptable to the lowest commitment demand of $\underline{p} \approx 0 < \min_{(v,w)\in\Theta} v - w$. Hence, outcomes must effectively converge to a continuous time concession game where the seller can choose any commitment price from a rich set but the buyer can only choose $\underline{p}$ (the seller screens

[28] There are also equilibrium limits similar to those when $\underline{p} \approx 0$, where the seller effectively makes an ultimatum offer just below 3. To see how the low price equilibrium holds together, first notice that a rational seller certainly prefers to charge just below 2 instead of $p_s \in (2, 2.63)$ as those prices risk much higher disagreement with little benefit. After demands $p_s \in (2.63, 3)$, which aren't never made (only) in the limit, the buyer immediately concedes with probability approximately $(2 - \underline{p})/(p_s - \underline{p}) < 0.44$ to make the seller indifferent between this demand and demanding just below 2. With the residual limit probability the buyer demands $\underline{p}$. Buyer and seller then concede continuously at rates $\lambda_s^{p_s,\underline{p},\bar{v}}$ and $\lambda_b^{p_s,\underline{p}}$ until time T^*, at which the buyer exits with probability 0.24 (in the limit, when she has value $\underline{v} = 5$) and concedes with probability $0.24\underline{p}/(p_s - \underline{p})$. This concession is much larger when $\underline{p} >> 0$, which helps the buyer build her reputation. It is also important for this construction (and can be easily verified) that the buyer with value $\underline{v}$ prefers to wait to exit at time T^* given the high initial rate of concession $\lambda_s^{p_s,\underline{p},\bar{v}} > \underline{\lambda}_b^{p_s,\underline{p},\underline{v},w}$.

through commitment types that accept larger prices arbitrarily quickly). Given that, for any $d \in [\underline{v}, \overline{v}] - \underline{w}$, the seller can choose a price arbitrarily close to d which is more generous than $\underline{p} < v^{1,p_s} - p_s$ for buyers who find it acceptable. And so, as the probability of commitment vanishes, the buyer must either immediately accept this price or exit.

Unlike my results when the buyer can make offers, the above conclusion depends on buyer outside options being strictly positive. If instead $\underline{w} = 0$ the seller would propose a price below $\underline{v}$ as commitment vanished, in order to ensure no buyer exited at T^* or waited forever. That alternative prediction is broadly consistent with Inderst (2005), who assumes the buyer is always rational and cannot make offers (and so accepts $\underline{v}$), while the seller might be a commitment type.

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A Appendix: Proofs of results

I first prove the supporting lemma below, which helps establish Proposition 1.

Lemma 4. *In any equilibrium in the continuation game at* 0^3 *after demands* p_s *and* p_b

(a) It is without loss of generality to assume $\bar{g}(v, w) = 0$ *if* $v - w < p_b$ *(henceforth, this is assumed throughout).*

(b) If $x = 0$*, as when* $\Theta^c = \emptyset$*, then without loss of generality* $F_s(0^3) = 1 - \bar{z}_s$*,* $E_b(0^3) = 0$*,* $E_b(0^4) = 1 - \bar{z}_b$*.*

(c) If $p_s \in P_s$ *then* $T^* < \infty$*.*

(d) *If* $p_s \notin P_s$*, but* $p_b \in P_b$*, then* $F_s(0^4) = 1$ *(and so clearly* $T^* < \infty$*).*

(e) *If* $p_s \in P_s$*,* $p_b \notin P_b$ *and* $x = 1$ *then* $F_b(0^4) = 1$.

(f) *If* $p_s \in P_s$*, then* $F_b(T^*) = (1 - \bar{z}_b)x$ *and* $E_b(T^*) = (1 - \bar{z}_b)(1 - x)$*. Similarly, if* $p_b \in P_b$ *then* $F_s(T^*) = (1 - \bar{z}_s)$*.*

(g) *If* F_s *jumps at* $t \geq 0^3$ *then* F_b *and* E_b *are constant on* $[t - \varepsilon, t]$ *for some* $\varepsilon > 0$*.*

(h) F_s *is continuous at* $t > 0^4$*.*

(i) *If* F_s *is continuous at* t *then so is* $U_b^{c,v,w}$ *and* $U_b^{e,v,w}$*. Likewise if* F_b *and* E_b *are continuous at* t *then so is* U_s*.*

(j) F_s *and* F_b *are strictly increasing on* $(0, T^*]$*.*

(k) *The skimming property holds: if a buyer with value* v *concedes at* t *then a buyer with value* $v' > v$ *will not concede after* $\max\{t, 0^4\}$*.*

Proof. For (a), suppose that $\mu_b^{p_s,v,w}(p_b) > 0$ for some $v - w < p_b$, then such an agent would certainly always exit before 0^4 as her best payoff in the continuation game is less than w, and if the buyer ever conceded to her with positive probability she would have a strictly profitably deviation of conceding at 0^2 instead.

For (b), notice that since buyer can never concede in equilibrium, without loss of generality, $F_b(t) = 0$. Suppose that $F_s(t) < 1 - \bar{z}_s$ for $t > 0$ then deviating to concede at 0^3 is always a profitable deviation. This deviation would also be profitable for the seller if she conceded at 0^4 and $E_b(0^4) > 0$, while if $E_b(0^4) = 0$ then it is still weakly better for the seller to concede at 0^3 than 0^4, hence in all cases we may assume $F_s(0^3) = 1 - \bar{z}_s$. If $F_s(0^3) > 0$, then clearly $E_b(0^3) = 0$ (as exit at 0^4 would be a profitable deviation for the buyer given (a)). If $\bar{z}_s < 1$, then we must, however, have $E_b(0^4) = 1 - \bar{z}_b$ given $w > 0$.

The argument for (c) is standard (e.g. see Abreu and Gul (2000)). If $p_s \in P_s$, then if a rational buyer does not concede or exit, she must believe the seller will concede shortly afterwards, and so her belief that the seller is committed increases if there is no concession. Repeating this argument, the buyer must eventually become convinced of the seller's commitment by some time $T^* < \infty$ and will then concede or exit.

The reasoning for (d) is similar: given $p_b \in P_b$ if the seller does not immediately concede, she must eventually become convinced of the buyer's commitment by some $T^* < \infty$ and will then concede. Given that, however, no rational buyer will concede to her on $[T^* - \varepsilon, T^*]$ for sufficiently small $\varepsilon > 0$ (strictly preferring to wait for the seller's concession), implying that she must have conceded by $T^* - \varepsilon$, a contradiction. The argument for (e) is analogous.

For the first part of (f), notice that at time T^*, the buyer knows that the seller is committed to her demand and so will immediately either concede or quit. For the second part, the seller likewise knows that the buyer will never concede after T^*, and so will immediately concede herself.

For (g), we can assume that $v - p_b > w$ for all buyers by (a). Given $p_b < p_s$ and the seller's positive concession at t, the buyer would strictly prefer to concede, or respectively exit, an instant after time t than on $[t - \varepsilon, t]$ for $\varepsilon > 0$ small. Given (g), if F_s jumped at $t > 0^4$ then F_b is constant on $[t - \varepsilon, t]$, and hence the seller would prefer to concede strictly before t, a contradiction which implies (h). Part (i) is immediate from the definitions.

Suppose that (j) did not hold, so that $F_i(t) = F_i(t')$ for some $0 < t < t' \leq T^*$ and i. Let $t_i^* = \sup\{\tau : F_i(\tau) = F_i(t)\}$. Clearly, agent j will not concede at $\tau \in (t, t_i^*)$ as conceding slightly beforehand would strictly improve her payoff, and hence $t_s^* = t_b^*$. As F_s and hence $U_b^{c,v,w}$ is continuous after t, conceding at or slightly after t_b^* delivers a strictly lower buyer payoff than conceding at $\tau \in (t, t_i^*)$. Hence, t_b^* cannot be the supremum, a contradiction.

For (k), given that conceding at t is optimal for type (v, w) we can assume $t \geq 0^4$ and that F_s is continuous at t (if the seller conceded with positive probability at 0^3 or 0^4 then the buyer wouldn't), and at t' by (g) and (h). So let $D(v) = U_b^{v,w,c}(t) - U_b^{v,w,c}(t')$ for $t' > t$:

$$D(v) = -\int^{\tau\in(t,t')} (v - p_b)e^{-r\tau} dF_s(\tau) + (v - p_s)\left((1 - F_s(t))e^{-rt} - (1 - F_s(t'))e^{-rt'}\right) \geq 0$$

Notice that

$$\begin{aligned} dD(v)/dv &= -\int^{\tau\in(t,t')} e^{-r\tau} dF_s(\tau) + (1 - F_s(t))e^{-rt} - (1 - F_s(t'))e^{-rt'} \\ &\geq \left(1 - \frac{(v - p_s)}{(v - p_b)}\right)\left((1 - F_s(t))e^{-rt} - (1 - F_s(t'))e^{-rt'}\right) > 0 \end{aligned}$$

where the first inequality uses $D(v) \geq 0$, and the second uses $(1 - F_s(t))e^{-rt} > (1 - F_s(t'))e^{-rt'}$ and $p_s > p_b$. Hence, $D(v') > 0$. □

Proof of Proposition 1. Part (f) of Lemma 4 establishes the part (4) of the Proposition. By part (a) we assume $\bar{g}(v, w) = 0$ if $v - w < p_b$. Part (b) means we can focus on continuation games where $\Theta^c \neq \emptyset$, so v^1 is well defined.

For such games, I next establish parts (1) and (2) of the Proposition. By Lemma 4 part (j), F_i is strictly increasing on $(0, T^*)$. This implies that if $T^* > 0$, the set of times O_i^c at which it is optimal for some type of agent i to concede, must be dense in $(0, T^*) \cap (t^{k+1}, t^k)$. By the skimming property only types $(v^k, w) \in \Theta^c$ concede on (t^{k+1}, t^k). By Lemma 4 parts (h) and (i), we also have that F_s and so $U_b^{c,v,w}$ are continuous at $t > 0$. Combined with the density of O_b^c in (t^{k+1}, t^k) therefore, we must have that $U_b^{c,v^k,w}$ is differentiable on that interval, with a derivative equal to zero, $dU_b^{c,v^k,w}(t)/dt = 0$. This immediately implies that the seller must concede at rate

$\lambda_s^{v^k}$ on that interval.

A buyer of type $(v, w) \in \Theta^e$, with $\underline{\lambda}^{v,w} > \lambda_s^{v^k}$ prefers to exit earlier on (t^{k+1}, t^k) than later, as the inequality implies $dU_b^{e,v^k,w}(t)/dt < 0$ on that interval. Moreover, given the skimming property and the continuity of $U_b^{e,v^k,w}$ at $t > 0$ (parts (h) and (i) of Lemma 4), such a buyer would prefer to concede at some point in (t^{k+1}, t^k) rather than at any later time (as any buyer concession after t^k is even slower). Likewise, if $\underline{\lambda}^{v,w} < \lambda_s^{v^k}$ for $(v, w) \in \Theta^e$, then such a buyer prefers to concede later on (t^{k+1}, t^k) than earlier as $dU_b^{e,v^k,w}(t)/dt > 0$. Given the skimming property, therefore, she will not concede before t^k (as any buyer concession before t^{k+1} is even faster).

I next claim that F_b is continuous on (t^{k+1}, t^k). If F_b jumped at some $t \in (t^{k+1}, t^k)$, then F_s would necessarily be constant on $[t-\varepsilon, t]$, for some small $\varepsilon \in (0, t - t^{k+1}]$, because we have established that the buyer will not exit on (t^{k+1}, t^k), while the seller prefers that the buyer concedes to her, rather than that she concedes. This, however, would contradict the required seller concession rate of $\lambda_s^{v^k}$ on that interval.

Given the continuity of F_b and E_b on (t^{k+1}, t^k), U^s is also continuous, by Lemma 4 part (i). Combined with the fact that O_s^c is dense in (t^{k+1}, t^k), we must then have $dU_s(t)/dt = 0$ and so the buyer must concede at rate λ_b.

I next argue that (without loss) the buyer never concedes or exits at 0^3 and the seller never concedes at 0^4. Suppose instead that a seller conceded with positive probability at time 0^4, then certainly a rational buyer cannot concede or exit at 0^3 or 0^4 (or the buyer would strictly prefer to concede or exit an instant after 0^4). Hence, outcomes are not affected by switching such seller concessions to time 0^3. Likewise, if the buyer conceded or exited at 0^3, then certainly the seller cannot concede at 0^3 or 0^4, or the buyer's decision would not be optimal. Hence, outcomes are not affected by moving any buyer concession or exit to time 0^4.

Next consider part (3) or the proposition, and suppose the seller concedes with probability strictly greater than $\alpha p_b/(p_s - p_b)$ at t^k. If $t^k \in (0, T^*]$, since F_b has at most finitely many jumps at times $t^K, ..., t^1$, there exists some $\varepsilon > 0$ such that the seller would prefer to concede an instant after t^k than on $(t^k - \varepsilon, t^k]$. However, this would contradict that F_s is strictly increasing on $(0, T^*)$, Lemma 4 part (j). Hence, such large concession requires $t^k = 0^4$ (recall, we can assume no buyer concession or exit at 0^3). Clearly, in this case seller will not concede at 0^3 (or 0^4).

Now suppose the seller concedes with probability strictly less than $\alpha p_b/(p_s - p_b)$ at time t^k. If $t^k < T^*$, then the seller would prefer to concede at t^k compared to conceding on $(t, t + \varepsilon]$ for sufficiently small $\varepsilon > 0$. However, this would contradict that F_s is strictly increasing on $(0, T^*)$, Lemma 4 part (j). Hence, such small concession requires $t^k = T^*$. If $t^k = 0^4$, a rational seller would prefer to concede at 0^3 rather than at 0^4 or $(0, \varepsilon)$, and hence without loss, any rational buyer must have always conceded by $0^4 = T^*$.

□

Proof of Lemma 2. First notice that if $F_s^{p_b}(0^4) = 0$ for all $p_b \in P$ then any exit and concession by a rational buyer that occurs by 0^4 can instead be moved to 0^2 without affecting outcomes or incentives so that $E_b^{p_b}(0^4) = F_b^{p_b}(0^4) = 0$. Henceforth, assume this.

Let $\tilde{v}^{p_b} = \max\{v : \sum_{(v,w)\in\Theta^c} \mu_s^{p_s}(p_b) > 0\}$. By the skimming property (Lemma 4, part (k)) the payoff for $(\tilde{v}^{p_b}, w) \in \Theta^c$ from demanding $p_b \in P$ is $F_s^{p_b}(0^4)(\tilde{v}^{p_b} - p_b) + (1 - F_s^{p_b}(0^4))(\tilde{v}^{p_b} - p_s)$, while her payoff from demanding p'_b is at least $F_s^{p'_b}(0^4)(\tilde{v}^{p_b} - p_b) + (1 - F_s^{p'_b}(0^4))(\tilde{v}^{p_b} - p_s)$. If $F_s^{p'_b}(0^4)(p_s - p'_b) > F_s^{p_b}(0^4)(p_s - p_b)$ then type $\tilde{v}^{p_b}$ will not imitate type p_b (a contradiction). On the other hand, if $F_s^{p'_b}(0^4)(p_s - p'_b) < F_s^{p_b}(0^4)(p_s - p_b)$ then type $\tilde{v}^{p'_b}$ will not imitate type p'_b. Hence, if p_b and p'_b are both imitated with positive probability by some buyer in Θ^c then $F_s^{p'_b}(0^4)(p_s - p'_b) = F_s^{p_b}(0^4)(p_s - p_b)$; if $p_b < p'_b$ therefore $F_s^{p'_b}(0^4) \geq F_s^{p_b}(0^4)$. If $p'_b > p_b \in P_b$, and p'_b is demanded with positive probability by some rational buyer, then some Θ^c buyer must demand p_b with positive probability: if not, $(1-\bar{z}_b^{p_b})x^{p_b} = 0$, so a rational seller will immediately concede and $F_s^{p'_b}(0^4)(p_s - p'_b) < F_s^{p_b}(0^4)(p_s - p_b)$, a contradiction.

Let $\check{v}^{p_b}$ be the maximum value buyer such that some $(\check{v}^{p_b}, w) \in \Theta^c$ demands p_b with positive probability, but has not always conceded by time 0^4. Suppose that $p'_b > p_b \in P_b$ is demanded with positive probability by rational buyers but $\check{v}^{p'_b}$ is not well defined because all those buyers concede or exit by 0^4, $F_b^{p'_b}(0^4) + E_b^{p'_b}(0^4) = 1 - \bar{z}_b > 0$. A rational seller must therefore concede at 0^3 after p'_b with strictly positive probability (or we could move the buyer's concession and exit to 0^2), and so $F_s^{p'_b}(0^4)(p_s - p'_b) = F_s^{p_b}(0^4)(p_s - p_b) > 0$. The payoff of type $(v, w) \in \Theta^e$ who demands p'_b and exits at 0^4 is then $w + F_s^{p'_b}(0^4)(v - p'_b - w)$. Her payoff to demanding p_b and exiting at 0^4 is then strictly larger $w + F_s^{p_b}(0^4)(v - p_b - w) = w + F_s^{p'_b}(0^4)(v - p_b - w)(p_s - p'_b)/(p_s - p_b)$ since $(v - p_b - w)/(p_s - p_b)$ is decreasing in $p_b < p'_b$ when $v - p_s > w$. This implies $x^{p'_b} = 1$, and so since the buyer concedes at 0^4 after p'_b with positive probability ($F_b^{p'_b}(0^4) = 1 - \bar{z}_b$) the seller strictly prefers to concede an instant after 0^4 than at 0^3, a contradiction. We know that $F_s^{p'_b}(0^4)(p_s - p'_b) = F_s^{p_b}(0^4)(p_s - p_b)$ and so must have $F_s^{p_b}(0^4) < 1 - \bar{z}_b = F_s^{p_b}(T^{*,p_b})$ for $p_b \in P_b$ with $p_b < p'_b$, and hence $\check{v}^{p_b}$ is also well defined. The argument above shows more generally that (without loss of generality) a buyer with value $(v, w) \in \Theta^e$ will never demand $p'_b > p_b \in P_b$ and then exit at time 0.

I next claim that $\check{v}^{p_b} = \check{v}^{p'_b}$. Suppose instead $\check{v}^{p_b} < \check{v}^{p'_b}$. The payoff to $(\check{v}^{p_b}, w) \in \Theta^c$ from demanding p_b is $F_s^{p_b}(0^4)(p_s - p_b) + (\check{v}^{p_b} - p_s)$, which is also her payoff from demanding p'_b and then conceding an instant after 0^4 (we established $F_s^{p'_b}(0^4)(p_s - p'_b) = F_s^{p_b}(0^4)(p_s - p_b)$ above). However, the payoff to type $(\check{v}^{p_b}, w)$ from demanding p'_b and waiting to concede after the positive interval on which she receives concession at rate $\lambda_s^{\check{v}^{p'_b}, p'_b} > \lambda_s^{\check{v}^{p_b}, p'_b}$, (by Lemma 1) is strictly larger, a contradiction. Similarly, if $\check{v}^{p_b} > \check{v}^{p'_b}$ then $\check{v}^{p'_b}$ will profitably deviate by demanding p_b. Hence, $\check{v}^{p'_b} = \check{v}^{p_b}$.

Recall that an agent with value v^m with $m \geq 1$ is indifferent between conceding at any point in the interval $[t^{m+1,p_b}, t^{m,p_b}]$ after demanding p_b. Now assume: (i) for any v, a rational buyer with that value is indifferent between demanding p_b before conceding at t^{m+1,p_b} or demanding

$p_b' > p_b$ before conceding at $t^{m+1,p_b'}$; and (ii) $t^{m+1,p_b'} \geq t^{m+1,p_b}$ and $F_s^{p_b'}(t^{m+1,p_b'}) \geq F_s^{p_b}(t^{m+1,p_b})$ (both strictly if $t^{m+1,p_b'} > 0$). Clearly (ii) implies $e^{-rt^{m+1,p_b}}(1 - F_s^{p_b}(t^{m+1,p_b})) \geq e^{-rt^{m+1,p_b'}}(1 - F_s^{p_b'}(t^{m+1,p_b'}))$. Let the difference in payoffs for a buyer with value v between demanding $p_b \in P_b$ before conceding at t^{m,p_b} or demanding $p_b' > p_b$ before conceding at time t^{m,p_b} be $D^m(v) = U^{c,p_b,v}(t^{m,p_b}) - U^{c,p_b',v}(t^{m,p_b'})$. Given (i) $D^{m+1}(v) =$, we have $D^m(v) = D^m(v) - D^{m+1}(v)$, and so:

$$D^m(v) = \int^{t^{m+1,p_b}<\tau<t^{m,p_b}} e^{-r\tau}(v - p_b)dF_s^{p_b}(\tau) - (e^{-rt^{m+1,p_b}}(1 - F_s^{p_b}(t^{m+1,p_b})) - e^{-rt^{m,p_b}}(1 - F_s^{p_b}(t^{m,p_b})))(v - p_s)$$
$$- \int^{t^{m+1,p_b'}<\tau<t^{m,p_b'}} e^{-r\tau}(v - p_b')dF_s^{p_b'}(\tau) + (e^{-rt^{m+1,p_b'}}(1 - F_s^{p_b'}(t^{m+1,p_b'})) - e^{-rt^{m,p_b'}}(1 - F_s^{p_b}(t^{m,p_b'})))(v - p_s)$$

which implies,

$$\frac{dD^m(v)}{dv} = \int^{t^{m+1,p_b}<\tau<t^{m,p_b}} e^{-r\tau}dF_s^{p_b}(\tau) - (e^{-rt^{m+1,p_b}}(1 - F_s^{p_b}(t^{m+1,p_b})) - e^{-rt^{m,p_b}}(1 - F_s^{p_b}(t^{m,p_b})))$$
$$- \int^{t^{m+1,p_b'}<\tau<t^{m,p_b'}} e^{-r\tau}dF_s^{p_b'}(\tau) + (e^{-rt^{m+1,p_b'}}(1 - F_s^{p_b'}(t^{m+1,p_b'})) - e^{-rt^{m,p_b'}}(1 - F_s^{p_b}(t^{m,p_b'})))$$
$$= -\frac{p_s - p_b}{v^m - p_b}e^{-rt^{m+1,p_b}}(1 - F_s^{p_b}(t^{m+1,p_b}))(1 - e^{-r(t^{m,p_b}-t^{m+1,p_b})}\frac{1 - F_s^{p_b}(t^{m,p_b})}{1 - F_s^{p_b}(t^{m+1,p_b})})$$
$$+ \frac{p_s - p_b'}{v^m - p_b'}e^{-rt^{m+1,p_b'}}(1 - F_s^{p_b'}(t^{m+1,p_b'}))(1 - e^{-r(t^{m,p_b'}-t^{m+1,p_b'})}\frac{1 - F_s^{p_b'}(t^{m,p_b'})}{1 - F_s^{p_b'}(t^{m+1,p_b'})}))$$

where the second line imposes that type v^m is indifferent between conceding at t^{m+1,p_b} or t^{m,p_b} for any demand p_b (as required by Lemma 1), that is:

$$\int^{t^{m+1,p_b}<\tau<t^{m,p_b}} e^{-r\tau}dF_s^{p_b}(\tau) = (e^{-rt^{m+1,p_b}}(1 - F_s^{p_b}(t^{m+1,p_b})) - e^{-rt^{m,p_b}}(1 - F_s^{p_b}(t^{m,p_b}))\frac{v^m - p_s}{v^m - p_b}$$

and also $(v^m - p_s)/(v^m - p_b) - 1 = -(p_s - p_b)/(v^m - p_b)$.

Other things equal it is clear that $dD^m(v)/dv$ is strictly decreasing in t^{m,p_b} and strictly increasing in $t^{m,p_b'}$ and equals 0 when both $t^{m,p_b'} = t^{m+1,p_b'}$ and $t^{m,p_b} = t^{m+1,p_b}$.

Given some equilibrium $t^{m,p_b'}$, $t^{m+1,p_b'}$ and t^{m+1,p_b} we must have $T^{*,p_b} \geq t^{m+1,p_b} + t^{m,p_b'} - t^{m+1,p_b'}$. Suppose not, then let $t^{m,p_b'} - t^{m+1,p_b'} = q > T^{*,p_b} - t^{m+1,p_b}$. Since $\lambda_s^{v,p_b} \leq \lambda_s^{v^m,p_b} < \lambda_s^{v^m,p_b'}$ for all $v \leq v^m$ (since $p_b < p_b'$) we have $(1 - F_s^{p_b}(T^{*,p_b}))/(1 - F_s^{p_b}(t^{m+1,p_b})) > e^{-\lambda_s^{v^m,p_b}q} \geq e^{-\lambda_s^{v^m,p_b'}q} = (1 - F_s^{p_b'}(t^{m,p_b'}))/(1 - F_s^{p_b'}(t^{m+1,p_b'}))$, and so given $F_s^{p_b'}(t^{m+1,p_b'}) \geq F_s^{p_b}(t^{m+1,p_b})$ by (ii) we would then have $(1 - F_s^{p_b}(T^{*,p_b})) > 1 - F_s^{p_b}(t^{m,p_b'})) \geq \bar{z}_s$, a contradiction since $p_b \in P_b$. Suppose next that $q = t^{m,p_b'} - t^{m+1,p_b'} = t^{m,p_b} - t^{m+1,p_b}$ and in this case let $\hat{D}^v(q)$ be $dD^m(v)/dv$ defined as a function of q. We then have:

$$\frac{d\hat{D}^v(q)}{dq} = -re^{-rt^{m+1,p_b}}(1 - F_s^{p_b}(t^{m+1,p_b}))e^{-(r+\lambda_s^{v^m,p_b})q} + re^{-rt^{m+1,p_b'}}(1 - F_s^{p_b'}(t^{m+1,p_b'}))e^{-(r+\lambda_s^{v^m,p_b'})q}$$

where I use the identity $r + \lambda_s^{v^m,p_b} = r(v^m - p_b)/(p_s - p_b)$. Given that $e^{(r+\lambda_s^{v^m,p_b'})q}d\hat{D}^v(q)/dq$

is strictly decreasing in q (since $\lambda_s^{v^m,p_b'} > \lambda_s^{v^m,p_b}$) and $e^{-rt^{m+1,p_b}}(1-F_s^{p_b}(t^{m+1,p_b})) \geq e^{-rt^{m+1,p_b'}}(1-F_s^{p_b'}(t^{m+1,p_b'}))$ (by (ii)) we have $d\hat{D}^v(0)/dq \leq 0$, and $d\hat{D}^v(q)/dq < 0$ for all $q > 0$. Since $\hat{D}^v(0) = 0$ we must have $\hat{D}^v(q) < 0$ for all $q > 0$. Since $dD^m(v)/dv$ is strictly decreasing in t^{m,p_b}, if $t^{m,p_b} - t^{m+1,p_b} \geq t^{m,p_b'} - t^{m+1,p_b'}$ then $dD^m(v)/dv \leq 0$ with $dD^m(v)/dv < 0$ when $t^{m,p_b'} > t^{m+1,p_b'}$. On the flip side, if $dD^m(v)/dv \geq 0$ then we must certainly have $t^{m,p_b} - t^{m+1,p_b} \leq t^{m,p_b'} - t^{m+1,p_b'}$.

I next claim that we always have $dD^m(v)/dv \leq 0$. Suppose not, so that $dD^m(v)/dv > 0$. Since $D^m(v^m) = 0$ we must have $D(v) < 0$ for all $v < v^m$. Hence, all such buyers would strictly prefer to demand p_b' and concede at $t^{m,p_b'}$ than demand p_b and concede at t^{m,p_b}. This would then imply that $T^{*,p_b} = t^{m,p_b}$. However, we observed above that $T^{*,p_b} \geq t^{m+1,p_b} + t^{m,p_b'} - t^{m+1,p_b'}$ and so $t^{m,p_b} - t^{m+1,p_b} \geq t^{m,p_b'} - t^{m+1,p_b'}$, but this implies $d\hat{D}^v(q)/dq \leq 0$, a contradiction.

We have established that one of the following hold: (a) $dD^m(v)/dv < 0$ and no agent with $v < v^m$ imitates p_b' only to concede, or (b) $dD^m(v)/dv = 0$. In case (b) we have $t^{m,p_b} - t^{m+1,p_b} \leq t^{m,p_b'} - t^{m+1,p_b'}$ and so given (ii), $t^{m,p_b'} \geq t^{m,p_b}$ and $F_s^{p_b'}(t^{m,p_b'}) \geq F_s^{p_b}(t^{m,p_b})$, strictly if $t^{m,p_b'} > 0$. Furthermore, in either case (a) or (b), given $D^{m+1}(v) = 0$ for all v by (i), there is some $\hat{t}^{m,p_b} \in [t^{m+1,p_b}, t^{m,p_b}]$ such that all buyer types are indifferent between demanding p_b' before conceding at $t^{m,p_b'}$ or demanding p_b before conceding at $\hat{t}^{m,p_b}$ where $\hat{t}^{m,p_b} \leq t^{m,p_b'}$ and $F_s^{p_b'}(t^{m,p_b'}) \geq F_s^{p_b}(t^{m,p_b})$, both strictly if $t^{m,p_b'} > 0$ (where $\hat{t}^{m,p_b} = t^{m,p_b}$ if $dD^m(v)/dv = 0$). Since m is arbitrary, induction establishes that all $(v, w) \in \Theta^c$ weakly prefer $p_b \in P_b$ over $p_b' > p_b$.

Next, consider the incentives of an agent $(v, w) \in \Theta^e$. I claim that such a buyer would never demand p_b'. We already saw that (without loss of generality) such a buyer would never demand p_b' only to exit at 0^4. Suppose then that it was optimal for such an agent to demand p_b' before exiting at $t^{m,p_b'} > 0$. We can assume $t^{m,p_b'} > t^{m+1,p_b'}$, as otherwise it is optimal to exit at $t^{m+1,p_b'}$, and hence $D^{m+1}(v) = 0$. Given $t^{m,p_b'} > t^{m+1,p_b'}$ we established that all rational buyers must be indifferent between demanding p_b before conceding at time $\hat{t}^{m,p_b}$ and demanding p_b' before conceding at time $t^{m,p_b'} > 0$. However, in that case such a buyer must strictly prefer to demand p_b before exiting at $\hat{t}^{m,p_b}$ to demanding p_b' before exiting at $t^{m,p_b'}$. To see this, let $\hat{D}^m(v, w) = U^{e,v,w,p_b}(\hat{t}^{m,p_b}) - U^{e,v,w,p_b'}(t^{m,p_b'})$ be the increase in payoffs from this deviation:

$$\hat{D}^m(v, w) = (e^{-r\hat{t}^{m,p_b}}(1 - F_s^{p_b}(\hat{t}^{m,p_b})) - e^{-rt^{m,p_b'}}(1 - F_s^{p_b}(t^{m,p_b'})))(w - v + p_s) > 0$$

where the first equality follows from $U^{e,v,w,p_b}(t) = U^{c,v,p_b}(t) + e^{-rt}(1 - F_s^{p_b}(t))(w - v + p_s)$ and $U^{c,v,p_b}(\hat{t}^{m,p_b}) = U^{c,v,p_b'}(t^{m,p_b'})$, and the inequality from $e^{-r\hat{t}^{m,p_b}}(1 - F_s^{p_b}(\hat{t}^{m,p_b})) > e^{-rt^{m,p_b'}}(1 - F_s^{p_b}(t^{m,p_b'}))$ and $w > v - p_s$. Hence, demanding p_b' is never optimal for $(v, w) \in \Theta^e$.

□

Proof of Lemma 3. I first establish parts (a), (b), and (c). Suppose by way of contradiction that $\lim_n F^n(0^4) < 1$. Since $\bar{z}_s^n = 1 - F_s^n(T^*) \geq (1 - F_s^n(0^4))e^{-\lambda_s^{\bar{v}} T^{*,n}}$ by Lemma 1, we must have $T^{*,n} \to \infty$.

For (a), define $t^* = -ln(\lim_n \bar{z}_b^n - \varepsilon)/\lambda_b < \infty$ for some $\varepsilon \in (0, \lim_n \bar{z}_b^n)$. For all large enough n we must have $T^{*,n} \leq t^*$ since $\lim_n \bar{z}_b^n - \varepsilon < \bar{z}_b^n = 1 - E_b^n(T^{*,n}) - F_b^n(T^{*,n}) \leq e^{-\lambda_b T^{*,n}}$ by Lemma 1, a contradiction. Given this, for claims (b), and (c) assume $\lim_n \bar{z}_s^n = 0$

For (b), notice that $1 - E_b^n(t) - F_b^n(t) \leq e^{-\lambda_b t}$ by Lemma 1. Hence by the skimming property (Lemma 4 part (k)), for any $\varepsilon \in (0, \lim_n \bar{g}^n(v, w))$ for large n, at time $t^* = -ln(\lim_n \bar{g}^n(v', w') - \varepsilon)/\lambda_b < \infty$ all remaining rational buyers with $(v, w) \in \Theta^c$ must have $v \leq v' < p_b + p_s$ and hence $\lambda_b > \lambda_s^v$. But since $\bar{z}_b^n = 1 - E_b^n(T^{*,n}) - F_b^n(T^{*,n}) \leq e^{-\lambda_b T^{*,n}}$ and $\bar{z}_s^n = 1 - F_s^n(T^{*,n}) \geq (1 - F_s^n(0^4))e^{-\lambda_s^{\bar{v}} t^* - \lambda_s^v (T^{*,n} - t^*)}$ (by Lemma 1) we have

$$1 - F_s^n(0^4) \leq \frac{\bar{z}_s^n}{\bar{z}_b^n} e^{(\lambda_s^v - \lambda_b)(T^{*,n} - t^*) + (\lambda_s^{\bar{v}} - \lambda_b)t^*} \leq L e^{(\lambda_s^{v'} - \lambda_b)(T^{*,n} - t^*) + (\lambda_s^{\bar{v}} - \lambda_b)t^*}$$

where the right hand side converges to 0 as $T^{*,n} \to \infty$ since $\lambda_s^{v'} - \lambda_b < 0$. This clearly contradicts $\lim_n F_s^n(0) \neq 1$.

For (c), notice that type $(v', w') \in \Theta^e$ always demands $p_b > \underline{p}$ (given Lemma 2 and $\bar{z}_s^n < 1$) so that $\lim_n \bar{g}^n(v', w') \geq g(v', w') > 0$, and will not exit until after any type $(v^1, w) \in \Theta^c$, by Lemma 1. For any $\varepsilon \in (0, \lim_n \bar{g}^n(v', w'))$ let $t^* = -ln(\lim_n \bar{g}^n(v, w) - \varepsilon)/\lambda_b < \infty$. Since $1 - E_b^n(t) - F_b^n(t) \leq e^{-\lambda_b t}$, for large n, by time t^* all $(v^1, w) \in \Theta^c$ must have conceded, and so $t^* \geq T^{*,n}$, which contradicts $T^{*,n} \to \infty$.

I now turn to the proof of parts (d) and (e) and (f). The logic for (d) and (e) is almost identical to that for (a) and (b). Given $p_b > \underline{p}$ we must have $x^n = 1$ (given Lemma 2) so that $\bar{z}_b^n = 1 - F_b^n(T^{*,n}) = (1 - F_b^n(0^4))e^{-\lambda_b T^{*,n}}$ by Lemma 1. Hence, if $\lim_n F_b^n(0^4) < 1$ then we must have $T^{*,n} \to \infty$.

For (d) notice that $\bar{z}_s^n = 1 - F_s^n(T^{*,n}) \leq e^{-\lambda_s^{v^1} T^{*,n}}$ by Lemma 1, which implies $\lim_n T^{*,n}$ is bounded above by $-ln(\lim_n \bar{z}_s)/\lambda_s^{v^1}$, a contradiction.

For (f), by assumption $v^1 - p_s > p_b$ and so $\lambda_s^{v^1} > \lambda_b$. We need $\bar{z}_s^n = 1 - F_s^n(T^{*,n}) \leq e^{-\lambda_s^{v^1} T^{*,n}}$ and so

$$(1 - F_b^n(0^4)) \leq \frac{\bar{z}_b}{\bar{z}_s} e^{(\lambda_b - \lambda_s^{v^1})T^{*,n}} \leq L e^{(\lambda_b - \lambda_s^{v^1})T^{*,n}}$$

where the right-hand side clearly converges to 0 as $T^{*,n} \to \infty$, implying $\lim_n F_b^n(0^4) = 1$, a contradiction.

For (e) let $\bar{t}^n = \min\{t \geq 0^4 : F_b^n(t) \geq \sum_{(v,w) \in \Theta^c : v > v^1} \bar{g}^n(v^1, w)\}$. By time $\bar{t}^n$ only rational buyers with type $(v^1, w) \in \Theta^c$ remain. We clearly have $F_b^n(\bar{t}_-^n) = \sup_{s < \bar{t}^n} F_b^n(s) \leq \sum_{(v,w) \in \Theta^c : v > v^1} \bar{g}^n(v, w)$. First consider some subsequence for which $\bar{t}^n > 0$ for all n. By Lemma 1 the probability of concession at $\bar{t}^n > 0$ must satisfy $(F_b^n(\bar{t}^n) - F_b^n(\bar{t}_-^n))(p_s - p_b)/p_b \leq E_b^n(\bar{t}^n) - E_b^n(\bar{t}_-^n)$ where the right hand side is certainly less than $1 - x^n$ and so for small enough $\varepsilon > 0$, for all sufficiently large n

$$F_b^n(\bar{t}^n) \leq (1 - x^n)p_b/(p_s - p_b) + \sum_{(v,w) \in \Theta^c : v > v^1} \bar{g}^n(v, w) < \lim_n \sum_{(v,w) \in \Theta^c} \bar{g}^n(v, w) - \varepsilon.$$

And so, we have $1 - F_b^n(\bar{t}^n) - E_b^n(\bar{t}^n) \geq \varepsilon$ for all sufficiently large n.

Similarly, suppose along some subsequence we always have $\bar{t}^n = 0^4$, then $E_b^n(0^4) = 1 - x^n$. For this subsequence, if $\lim_n F_b^n(0^4) < \lim_n x^n$ we must then again have $1 - F_b^n(\bar{t}^n) - E_b^n(\bar{t}^n) \geq \varepsilon$ for some $\varepsilon > 0$. For any subsequence with $\bar{t}^n = 0^4$ or $\bar{t}^n > 0^4$, therefore, we must have $1 - F_b^n(\bar{t}^n) - E_b^n(\bar{t}^n) \geq \varepsilon$ for some $\varepsilon > 0$ for all large n. In that case, we must have $\bar{z}_b^n = 1 - F_b^n(T^{*,n}) - E_b^n(T^{*,n}) = (1 - F_b^n(\bar{t}^n) - E_b^n(\bar{t}^n))e^{-\lambda_b(T^{*,n} - \bar{t}^n)}$ and so clearly $(T^{*,n} - \bar{t}^n) \to \infty$. Combined with $\bar{z}_s = 1 - F_s^n(T^{*,n}) \leq e^{-\lambda_s^{v^1} T^{*,n}}$ we get

$$(1 - F_b^n(\bar{t}^n) - E_b^n(\bar{t}^n)) \leq \frac{\bar{z}_b}{\bar{z}_s} e^{(\lambda_b - \lambda_s^{v^1})(T^{*,n} - t^n) - \lambda_s^{v^1} t^n} \leq L e^{(\lambda_b - \lambda_s^{v^1})(T^{*,n} - t^n)}$$

where the right hand side must converge to 0 given that $\lambda_b < \lambda_s^{v^1}$ and $(T^{*,n} - t^n) \to \infty$. This contradicts $(1 - F_b^n(\bar{t}^n) - E_b^n(\bar{t}^n)) \geq \varepsilon > 0$ for large n. □

Proof of Propositions 2. I first present some preliminary observations. Notice that by choosing $\varepsilon' > 0$ sufficiently small, a ε' rich commitment type space must have $\underline{p} \leq \varepsilon' < \min\{v - w : (v, w) \in \Theta\}$. Moreover, let $\tilde{p}_s = \max\{p_s \in P_s : p_s \leq \min\{p^*, v - w : (v, w) \in \Theta\}\}$, then for small enough $\varepsilon' > 0$, we have $\tilde{p}_s > \underline{p}$, and the seller will always demand $p_s \in P_s$ such that $p_s \geq \tilde{p}_s$. To see this, notice that demanding $\tilde{p}_s$ guarantees that $x^{\tilde{p}_s} = 1$ and since $\tilde{p}_s \leq p^*$, any counterdemand $p_b \in P$ will imply $\lambda_s^{\underline{v}, p_b, \tilde{p}_s} > \lambda_b^{p_b, \tilde{p}_s}$ given $p_b < \tilde{p}_s \leq \min\{p^*, \underline{v} - \underline{w}\}$. After any counterdemand $p_b \in P$ the buyer makes with positive limit probability (for some subsequence) we must have $\bar{z}_b^{p_b, \tilde{p}_s} / \bar{z}_s^{\tilde{p}_s} \leq L'$ for some constant L' and for all n sufficiently large. Hence, by Lemma 3, the buyer must concede with probability approaching 1 in the limit. This would guarantee the seller a payoff of at least $\tilde{p}_s$ in the limit and so she certainly won't demand less. She will also never demand $p_s \notin P_s$ as then the highest limit payoff she could expect would be $\underline{p}$. Nor will a rational seller ever demand $p_s > \bar{v} - \underline{w}$ as she would then need to immediately concede against any counterdemand (Lemma 4, part (a)), again giving her a limit payoff of $\underline{p}$.

For $p_s > \underline{v} - \underline{w}$ we can define $v^{0, p_s} = \max\{v \in V : v < v^{1, p_s}\}$. Suppose that the seller demands p_s with positive limit probability such that $\underline{\lambda}^{v^{0, p_s}, \underline{w}, p_s, \underline{p}} < \lambda_s^{v^{1, p_s}, p_s, \underline{p}}$. Lemma 2, implies that $(v^{0, p_s}, \underline{w}) \in \Theta^{e, p_s}$ counterdemands $\underline{p}$. Hence, by Lemma 3, we know that the seller must then concede with probability approaching 1 in the limit, providing a limit payoff of $\underline{p}$ (as all buyers will then demand $\underline{p}$). This is a contradiction, as we already established the seller can guarantee a payoff of $\tilde{p}_s > \underline{p}$. Hence, the seller can never make such a demand with positive limit probability and we can restrict attention to seller demands, $p_s \in P_s^* = \{p_s \in P_s : p_s < -\underline{w}$ or $\underline{\lambda}^{v^{0, p_s}, \underline{w}, p_s, \underline{p}} > \lambda_s^{v^{1, p_s}, p_s, \underline{p}}\}$. Also notice that for sufficiently small $\varepsilon' > 0$ we must also have that $(p_s - \underline{p})g(v, w) > \underline{p}(1 - g(v, w))$ for all $p_s \geq \tilde{p}_s$ and $(v, w) \in \Theta$.

I next establish the upper bound on the buyer's payoff in Proposition 2. Recall that if the seller demands $p_s \in P_s^*$, with $p_s \geq \tilde{p}_s$, then by Lemma 2 no buyer with $(v, w) \in \Theta^{e, p_s}$ will counterdemand $p_s > \underline{p}$, where $\underline{p} \leq \varepsilon' \leq \delta$ in a small $\varepsilon' > 0$ rich set of commitment types. Hence, the best case for the seller who demands price p_s is that all types $(v, w) \in \Theta^{c,p_s}$ accept her demand and all types $(v, w) \in \Theta^{e,p_s}$ demand $\underline{p}$, giving her a payoff of at most $(1 - H(p_s))p_s + \delta$. For the seller to obtain a limit payoff larger than $\max_{p\in[0,p^*+2\varepsilon]}(1 - H(p))p + \delta$, therefore, she must demand $p_s > p^* + 2\varepsilon$ with positive limit probability; assume this.

Define $\hat{p}_b^{p_s} = \min\{p_b \in P_b : p_b > v^{1,p_s} - p_s\}$. I claim that $\hat{p}_b^{p_s}$ is well defined and $\hat{p}_b^{p_s} < p^* + 2\varepsilon$ given a small $\varepsilon' > 0$ rich set of commitment types. There are two cases to consider, (a) $p_s < \underline{v} - \underline{w}$ and (b) $p_s > \underline{v} - \underline{w}$. First consider case (a) where $v^{1,p_s} = \underline{v}$. Since $\underline{v} - \underline{w} > p_s > p^* + 2\varepsilon$ we must have $\underline{w} < \underline{v}/2 < p_s - 2\varepsilon$, and so $\underline{v} - p_s < p_s - 4\varepsilon$. When $\varepsilon' \leq \varepsilon/2$, there exists $p_b \in [\underline{v} - p_s, \underline{v} - p_s + \varepsilon] \cap P_b$ in any ε' rich commitment type space, and so $\hat{p}_b^{p_s}$ is not only well defined but $\hat{p}_b^{p_s} \leq \underline{v} - p_s + \varepsilon \leq \underline{v}/2 - \varepsilon < p^*$.

Next consider case (b). Let $\hat{\varepsilon} = \max_{d\in[\underline{v},\overline{v}]} \min_{v\in V} |d - v|$. Given that a rational buyer's type space is ε rich, we must have that $\hat{\varepsilon} < \varepsilon$. Since $v^{1,p_s} \neq \underline{v}$ we have v^{0,p_s} well-defined. Moreover, since $v^{1,p_s} - 2\hat{\varepsilon} \leq v^{0,p_s} < p_s + \underline{w}$, we must have $v^{1,p_s} - p_s < \underline{w} + 2\hat{\varepsilon}$. Given $\varepsilon' \leq \varepsilon - \hat{\varepsilon}$, there must be some $p_b \in [\underline{w} + 2\hat{\varepsilon}, \underline{w} + 2\varepsilon) \cap P_b$ in a ε' rich commitment type space, and hence $\hat{p}_b^{p_s}$ is well-defined with $\hat{p}_b^{p_s} < \underline{w} + 2\varepsilon < p_s$. Also notice that $\hat{p}_b^{p_s} > v^{1,p_s} - p_s > \underline{w} > \underline{p}$.

Without loss of generality, I will assume that types $(v^{1,p_s}, w) \in \Theta^c$ never concede with positive probability at time 0^2. They will certainly never do so if the buyer concedes at time 0^3 or 0^4 for some counterdemand, but if $F_s^{p_s,p_b}(0^4) = 0$ for all $p_b < p_s$ then conceding at 0^4 is no different from conceding at 0^2.

Notice that if following some demand $p_b \in P_b$, with $p_b \geq \hat{p}_b^{p_s} > \underline{p}$, we have $\lim_n \bar{g}^n(v^{1,p_s}, w) > 0$ for some type $(v^{1,p_s}, w) \in \Theta^{c,p_s}$, then by Lemma 3, then the seller must immediately concede with probability 1 in the limit. A similar conclusion holds if $\lim_n \bar{z}_b^{p_s,p_b,n} > 0$. Clearly, if $\lim_n F_s^{p_s,\hat{p}_b^{p_s},n}(0^4) = 1$ then no buyer would imitate $p_b > \hat{p}_b^{p_s}$ in the limit and so the seller's payoff would be less than $\hat{p}_b^{p_s}(1 - H(\hat{p}_b^{p_s})) + \delta$, establishing the desired seller payoff bound.

Suppose instead, therefore, that $\lim_n F_s^{p_s,\hat{p}_b^{p_s},n}(0^4) < 1$, implying $\lim_n \bar{z}_b^{p_s,\hat{p}_b^{p_s},n} = 0$ and $\lim_n \bar{g}^n(v^{1,p_s}, w) = 0$ for all $(v^{1,p_s}, w) \in \Theta^{c,p_s}$. In turn, this implies v^{2,p_s} is well defined and $t^{2,p_s,\hat{p}_b^{p_s},n} \to \infty$. Without loss of generality, all types $(v^{1,p_s}, w) \in \Theta^{c,p_s}$ make some counterdemand and cannot demand $\hat{p}_b^{p_s}$ with positive limit probability. Suppose such types instead demand $p_b' > \hat{p}_b^{p_s}$ with positive limit probability, then since $v^{1,p_s} - p_s < p_b'$, they will receive a limit payoff of $v^{1,p_s} - p_b'$ (by Lemma 3). We must then have $\lim_n F_s^{p_s,\hat{p}_b^{p_s},n}(0^4) = (p_s - p_b')/(p_s - \hat{p}_b^{p_s}) < 1$ to ensure rational buyers demand $\hat{p}_b^{p_s}$ (see the argument in the proof of Lemma 2). Since $\hat{p}_b^{p_s} > \underline{p}$, and Θ^{e,p_s} types never demand $p_b > \underline{p}$, we have $F_b^{p_s,\hat{p}_b^{p_s},n}(0^4) = 0$. And so,

$$z_b^{p_s,\hat{p}_b^{p_s}} + (1 - z_b^{p_s,\hat{p}_b^{p_s}}) \sum_{(v^{1,p_s},w)\in\Theta^{c,p_s}} \bar{g}^{p_s,\hat{p}_b^{p_s}}(v^{1,p_s}, w) = 1 - F_b^{p_s,\hat{p}_b^{p_s}}(t^{2,p_s,\hat{p}_b^{p_s},n}) = e^{-\lambda_b t^{2,p_s,\hat{p}_b^{p_s},n}}$$

converges to zero, implying $t^{2,p_s,\hat{p}_b^{p_s},n} \to \infty$. Hence, by demanding $\hat{p}_b^{p_s}$ type $(v^{1,p_s}, w) \in \Theta^{c,p_s}$

secures a payoff of at least

$$(v^{1,p_s} - \hat{p}_b^{p_s})\Big(F_s^{p_s,\hat{p}_b^{p_s},n}(0^4) + \int^{0^4<t<t^{2,p_s,\hat{p}_b^{p_s}}} e^{-rt} dF_s^{p_s,\hat{p}_b^{p_s},n}(t)\Big) \tag{1}$$

$$\geq (v^{1,p_s} - \hat{p}_b^{p_s})\Big(F_s^{p_s,\hat{p}_b^{p_s},n}(0^4) + \frac{v^{2,p_s} - p_s}{v^{2,p_s} - \hat{p}_b^{p_s}}(1 - F_s^{p_s,\hat{p}_b^{p_s},n}(0^4) - e^{-rt^{2,p_s,\hat{p}_b^{p_s}}}(1 - F_s^{p_s,\hat{p}_b^{p_s},n}(t^{2,p_s,\hat{p}_b^{p_s},n})))\Big)$$

where the second inequality follows from the fact that v^{2,p_s} would find it optimal to concede at $t^{2,p_s,\hat{p}_b^{p_s}}$ conditional on demanding $\hat{p}_b^{p_s}$, that is:

$$\int^{0^4<t<t^{2,p_s,\hat{p}_b^{p_s}}} e^{-rt} dF_s^{p_s,\hat{p}_b^{p_s},n}(t) \geq \frac{v^{2,p_s} - p_s}{v^{2,p_s} - \hat{p}_b^{p_s}}(1 - F_s^{p_s,\hat{p}_b^{p_s},n}(0^4) - e^{-rt^{2,p_s,\hat{p}_b^{p_s}}}(1 - F_s^{p_s,\hat{p}_b^{p_s},n}(t^{2,p_s,\hat{p}_b^{p_s},n})))$$

but as $t^{2,p_s,\hat{p}_b^{p_s},n} \to \infty$, the right hand side of (1) converges to

$$(v^{1,p_s} - \hat{p}_b^{p_s})\Big(\lim_n F_s^{p_s,\hat{p}_b^{p_s},n}(0^4) + \frac{v^{2,p_s} - p_s}{v^{2,p_s} - \hat{p}_b^{p_s}}(1 - \lim_n F_s^{p_s,\hat{p}_b^{p_s},n}(0^4))\Big). \tag{2}$$

This equals $(v^{1,p_s} - \hat{p}_b^{p_s})(v^{2,p_s} - p_b')/(v^{2,p_s} - \hat{p}_b^{p_s})$ given $\lim_n F_s^{p_s,\hat{p}_b^{p_s}}(0^4) = (p_s - p_b')/(p_s - \hat{p}_b^{p_s})$. That exceeds $v^{1,p_s} - p_b'$ given that $(v^{2,p_s} - p_b')/(v^{2,p_s} - \hat{p}_b^{p_s})$ is strictly increasing in v^{2,p_s}. This, however, contradicts the optimality of a value v^{1,p_s} buyer demanding p_b'

Next suppose that type $(v^{1,p_s}, \underline{w}) \in \Theta^{c,p_s}$ imitates $p_b' \in (\underline{p}, \hat{p}_b^{p_s}) \cap P_b$ with positive limit probability, then since $p_b' < v^{1,p_s} - p_s$ the buyer must concede with probability approaching 1 (by Lemma 3), to give $(v^{1,p_s}, w) \in \Theta^{c,p_s}$ a limit payoff of $v^{1,p_s} - p_s$. However, since $t^{2,p_s,\hat{p}_b^{p_s},n} \to \infty$ such a buyer could secure a limit payoff of at least 2 by imitating $\hat{p}_b^{p_s}$ which exceeds $v^{1,p_s} - p_s$ even when $\lim_n F_s^{p_s,\hat{p}_b^{p_s},n}(0^4) = 0$ since $(v^{2,p_s} - p_s)/(v^{2,p_s} - \hat{p}_b^{p_s})$ is strictly increasing in v^{2,p_s}. Again, this is a contradiction.

The final possibility is that $\lim_n \mu_b^{p_s,v^{1,p_s},w,n}(\underline{p}) = 1$ for all $(v^{1,p_s}, w) \in \Theta^{c,p_s}$. Given $p_s \in P_s^*$ we have $\underline{\lambda}^{v',w',p_s,\underline{p}} < \lambda_s^{v^{1,p_s},p_s,\underline{p}}$ for all $(v', w') \in \Theta^{c,p_s}$. Hence, since $(p_s - \underline{p})g(v^{1,p_s}, \underline{w}) > \underline{p}(1 - g(v^{1,p_s}, \underline{w}))$, the buyer must either concede or exit immediately with probability approaching 1 in the limit by Lemma 3, giving $(v^{1,p_s}, w) \in \Theta^{c,p_s}$ a payoff of $v^{1,p_s} - p_s$, which is again strictly less than the payoff of (2) she could have obtained by demanding $\hat{p}_b^{p_s}$. This contradiction ensures $F_s^{p_s,\hat{p}_b^{p_s},n}(0^4)$ approaches 1 in the limit, so no buyer proposes a higher price and the seller's payoff is at most $(1 - H(p_s))\hat{p}_b^{p_s} + \delta \leq \max_{p\in[0,p^*+2\varepsilon]}(1 - H(p))p + \delta$ since $\hat{p}_b^{p_s} < p^* + 2\varepsilon < p_s$. This also shows (whether or not the seller demands $p_s > p^* + 2\varepsilon$ with positive limit probability), that the buyer enjoys a limit payoff of at least $\max\{v - (p^* + 2\varepsilon), w\}$, establishing the lower bound on the buyer's payoffs in Proposition 3, part (a).

I now turn to the lower bound on seller payoffs in Proposition 2. Let $\hat{p} \in \arg\max_{p\in[0,p^*]}(1-H(p))$ and recall that $\check{p}(p) = \min\{p, \max\{v - \underline{w} \leq p : v \in V\}\}$ and $\hat{\varepsilon} = \max_{d\in[\underline{v},\overline{v}]} \min_{v\in V} |d - v| < \varepsilon$, so that $\check{p}(p) \in [p - 2\hat{\varepsilon}, p]$. Let $\overline{p}_s = \max\{p_s \in P_s : p_s < \check{p}(\hat{p}_s)\}$, where $\overline{p}_s \geq \tilde{p}_s < \min\{v - w : (v, w) \in \Theta\}$ and $\overline{p}_s \in [\hat{p}_s - 2\varepsilon, \check{p}(\hat{p}_s)) \cap P_s$ in any $\varepsilon' \leq \varepsilon - \hat{\varepsilon}$ rich commitment type space.

I next claim $\overline{p}_s \in P_s^*$ for a small $\varepsilon' > 0$ rich set of commitment types. If $\overline{v} = \underline{v}$ then since $\hat{p}_s \leq \overline{v} - \underline{w}$ we have $\check{p}(\hat{p}_s) = p \leq \underline{v} - \underline{w}$, otherwise let $\check{\varepsilon} = \min\{v - v' : v \neq v' \in V\} \in (0, \overline{v})$ and assume that $2\varepsilon' \leq \underline{w}\check{\varepsilon}/(\overline{v} - \check{\varepsilon})$. Clearly, if $\check{p}(\hat{p}_s) \leq \underline{v} - \underline{w}$ then $v^{1,\overline{p}_s} = \underline{v}$. For $v \geq \underline{v}$, $\lambda_s^{v,\overline{p}_s,\underline{p}} < \lambda_s^{v,\overline{p}_s,\underline{p}}$. For $v - w < p_s$, $\lambda_s^{v,\overline{p}_s,\underline{p}} < \underline{\lambda}^{v,w,p_s,\underline{p}}$ (since the latter is increasing in w), and so $\overline{p}_s \in P_s^*$.

On the other hand, suppose that $\check{p}(\hat{p}_s) > \underline{v} - \underline{w}$ and so $v^{1,\overline{p}_s} - \underline{w} = \check{p}(\hat{p}_s) < \overline{p}_s + 2\varepsilon'$ and $v^{0,\overline{p}_s} \leq v^{1,\overline{p}_s} - \check{\varepsilon}$. In this case we have,

$$\begin{aligned}
(v^{1,\overline{p}_s} - \overline{p}_s)(v^{0,\overline{p}_s} - \underline{w}) - \underline{w}\overline{p}_s < \quad & (\underline{w} + 2\varepsilon')(v^{0,\overline{p}_s} - \underline{w}) - \underline{w}(v^{1,\overline{p}_s} - \underline{w} - 2\varepsilon') \\
\leq (\underline{w} + 2\varepsilon')(v^{1,\overline{p}_s} - \check{\varepsilon} - \underline{w}) - \underline{w}(v^{1,\overline{p}_s} - \underline{w} - 2\varepsilon') \leq \quad & (\underline{w} + 2\varepsilon')(\overline{v} - \check{\varepsilon} - \underline{w}) - \underline{w}(\overline{v} - \underline{w} - 2\varepsilon') \leq 0
\end{aligned}$$

where the first inequality follows from $\overline{p}_s > v^{1,\overline{p}_s} - \underline{w} - 2\varepsilon'$, the second from $v^{0,\overline{p}_s} \leq v^{1,\overline{p}_s} - \check{\varepsilon}$, the third from $v^{1,\overline{p}_s} \leq \overline{v}$ and the fourth from $2\varepsilon' \leq \underline{w}\check{\varepsilon}/(\overline{v} - \check{\varepsilon})$. Furthermore notice that

$$(v^{1,\overline{p}_s} - \overline{p}_s)(v^{0,\overline{p}_s} - p_b - \underline{w}) - \underline{w}(\overline{p}_s - p_b) \tag{3}$$

is decreasing in p_b given $v^{1,\overline{p}_s} - \overline{p}_s > \underline{w}$ and so must be negative for any $p_b \in (0, \overline{p}_s)$. I claim this implies $\lambda_s^{\overline{p}_s,\underline{p},v^{1,\overline{p}_s}} < \underline{\lambda}^{\overline{p}_s,\underline{p},v,w}$ for all $(v, w) \in \Theta^{e,\overline{p}_s}$, so that $\overline{p}_s \in P_s^*$. Clearly, the inequality holds for $(v^{0,\overline{p}_s}, \underline{w})$ by the negativity of (3). Since (3) is increasing in $v^{0,\overline{p}_s}$, it must likewise hold for any $(v, \underline{w})$ with $v < v^{0,\overline{p}_s}$. Since (3) is decreasing in $\underline{w}$, the claim must also hold for all $(v, w) \in \Theta$ with $v \leq v^{0,\overline{p}_s}$ and $w \geq \underline{w}$. If $v \geq v^{1,\overline{p}_s}$ and $v - w < p_s$, we again have $\lambda_s^{v^{1,\overline{p}_s},\overline{p}_s,\underline{p}} \leq \lambda_s^{v,\overline{p}_s,\underline{p}} < \underline{\lambda}^{v,w,p_s,\underline{p}}$ (since the latter is increasing in w). And so, $\overline{p}_s \in P_s^*$.

Hence, suppose the seller demands $\overline{p}_s$, then since $\overline{p}_s \leq p^*$ for any counterdemand $p_b < \overline{p}_s$ we must have $v - \overline{p}_s > p_b$ for all $(v, w) \in \Theta^{c,\overline{p}_s}$. To see this, notice that if $\underline{v}/2 \geq \overline{p}_s$ then $v - \overline{p}_s \geq \underline{v}/2 \geq \overline{p}_s > p_b$ whereas if $\underline{w} \geq \overline{p}_s$ then $v - \overline{p}_s > w \geq \underline{w} \geq \overline{p}_s > p_b$ for $(v, w) \in \Theta^{c,\overline{p}_s}$. Hence, if a buyer with $(v, w) \in \Theta^{c,\overline{p}_s}$ demands $p_b > \underline{p}$ with positive probability in the limit, she must subsequently immediately concede with probability 1 in the limit by Lemma 3 to give her payoffs of $(v - \overline{p}_s)$ for all large n (since $\Theta^{e,\overline{p}_s}$ buyers only demand $\underline{p}$).

As argued previously, it is without loss of generality to assume that type $(v^{1,\overline{p}_s}, \underline{w})$ always makes some counterdemand $p_b \in P_b$, and so without loss to she counterdemands $\underline{p}$ with probability approaching 1 in this limit (or she will get exactly $(v^{1,p_s} - \overline{p}_s)$). However, by Lemma 3, therefore, the buyer concedes with probability approaching $\lim_n x^{\overline{p}_s,\underline{p}}$ and exits with probability approaching $1 - \lim_n x^{\overline{p}_s,\underline{p}}$ at time 0[4] since $(\overline{p}_s - \underline{p})g(v^{1,\overline{p}_s}, \underline{w}) > \underline{p}(1 - g(v^{1,\overline{p}_s}, \underline{w}))$. And so, the demand $\overline{p}_s$ secures the seller a limit payoff of at least $(1 - H(\overline{p}_s))\overline{p}_s \geq \max_{p \in [0,p^*]}(1 - H(p))p - 2\varepsilon$ where the inequality follows from $\overline{p}_s \in [\hat{p}_s - 2\varepsilon, \hat{p}_s]$. □

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Proof of Proposition 1

In order to prove this result I first define what I call a straightforward "straightforward" equilibrium in the continuation continuation game at 0^3 given $p_s \in P_s$ and $p_b \in P_b$ and beliefs $(\bar{z}_s, \bar{z}_b, \bar{g})$ where $\bar{z}_i > 0$.

I first define some preliminary objects that will help to describe such an equilibrium. For $y \in [0, (1-\bar{z})x]$ let

$$\bar{k}(y) = \max\{k \leq K+1 : \sum_{(v^m,w)\in\Theta^c:m\geq k} \bar{g}(v,w)(1-\bar{z}_b) \geq y\},$$

Clearly, $\bar{k}(0) = K+1$, and $\bar{k}((1-\bar{z})x) = 1$ if $\bar{g}(v^2, w) > 0$ for $(v^2, w) \in \Theta^c$. This is decreasing and upper semi continuous in y. Also define $\underline{k}(y) = \bar{k}(y)$ if $y < (1-\bar{z})x$ and $\underline{k}((1-\bar{z})x) = 0$. Loosely, if fraction y of buyers have conceded by time t then $t \in (t^{\bar{k}(y)+1}, t^{\bar{k}(y)}]$.

For $k \in \{1, ..., K\}$ let

$$\bar{G}^e(k) = \sum_{(v,w)\in\Theta^e:\underline{\lambda}^{v,w}>\lambda_s^{v^k}} \bar{g}(v,w)$$

while $\bar{G}^e(K+1) = 0$ and $\bar{G}^e(0) = 1-x$. Notice that $\bar{G}^e(\bar{k}(y))$ is increasing and lower semi continuous in y.

Next define

$$\pi(y,\hat{y}) = (p_s - p_b)(\hat{y} - y) - p_b(1-\bar{z}_b)(G^e(\underline{k}(\hat{y})) - G^e(\bar{k}(y))).$$

Loosely, this the difference between the present value payoff of p_b a seller gets by conceding an instant before time t, and the payoff she would receive conceding an instant after t, if at time t a fraction $(\hat{y} - y)$ of buyers concede and $(1-\bar{z})(G^e(\underline{k}(\hat{y})) - G^e(\bar{k}(y)))$ exit. And then let:

$$\tilde{y}(\hat{y}) = \min\{y \geq 0 : \pi(y,\hat{y}) \leq 0\}.$$

Loosely, $\hat{y} - \tilde{y}(\hat{y})$ is the maximum probability of concession at time t such that the buyer prefers to concede an instant before t compared to an instant after where $\hat{y}$ is the total fraction of buyers who have conceded before time t.

It is useful to outline equilibrium strategies starting at time $T^* = t^1$, which I relabel as "time" $\tau^1 = 0$, and more generally will define equilibrium objects in terms of $\tau = T^* - t \in [0,\infty)$. Define $\hat{F}_s^1(\tau_1) = (1-\bar{z}_s)$, $\hat{F}_b^1(\tau_1) = (1-\bar{z}_b)x$, $\hat{E}_b^1(\tau_1) = (1-\bar{z}_b)G^e(1)$, and then by induction for $k \in \{1, ..., K\}$ and $\tau \geq \tau^k$, let $1-\hat{F}_s^k(\tau) = (1-\hat{F}_s^k(\tau^k))e^{\lambda_s^{v^k}(\tau-\tau_1)}$, $\hat{E}_b^k(\tau) = \hat{E}_b^k(\tau^k)$, $1-\hat{E}_b^k(\tau^k)-\hat{F}_b^k(\tau) = (1-\hat{E}_b^k(\tau^k)-\hat{F}_s^k(\tau^k))e^{\lambda_b(\tau-\tau^k)}$. Effectively, $\hat{F}_s^k$ (respectively $\hat{F}_b^k$) correspond to the concession probability of the seller (buyer) assuming she concedes at rate $\lambda_s^{v^k}$ (λ_b) on $(t, t^k) = (T^*-\tau, T^*-\tau^k)$

if $F_i(t^{k-}) = \hat{F}_i^k(\tau^k)$ and $E_b(t^{k-}) = \hat{E}_b^k(\tau^k)$. Then for $k \leq K$ (where recall that $v^K = \overline{v}$) define

$$\tau^{k+1} = \min\{\tau \geq \tau^k : \tilde{y}(\hat{F}_b^k(\tau)) < \hat{F}_b^k(\tau) \text{ or } \overline{k}(\hat{F}_b^k(\tau)) > k\}$$

with $\hat{F}_s^{k+1}(\tau^{k+1}) = \hat{F}_s^k(\tau^{k+1})$, $\hat{F}_b^{k+1}(\tau^{k+1}) = \tilde{y}(\hat{F}_b^k(\tau))$ and $\hat{E}_b^{k+1}(\tau^{k+1}) = (1 - \bar{z}_b)G^e(k+1)$. Notice that we can have $\tau^{k+1} = \tau^k$. In fact, define $\ell^k = \max\{\ell : \tau^\ell \leq \tau^k\} \geq k$ so that $\tau^{\ell^k} = \tau^k$.

Next define $\hat{F}_s(0) = (1 - \bar{z}_s)$, $\hat{F}_b(0) = (1 - \bar{z}_b)x$, $\hat{E}_b(0) = (1 - \bar{z}_b)(1 - x)$, and if $\tau \in (\tau^k, \tau^{k+1}]$ then $\hat{F}_s(\tau) = \hat{F}_s^k(\tau)$, $\hat{F}_b(\tau) = \hat{F}_b^k(\tau)$, $\hat{E}_b(\tau) = \hat{E}_b^k(\tau) = (1 - \bar{z}_b)G^e(k)$. Let $\hat{F}_s(\tau) = \hat{F}_s^K(\tau)$ for $\tau \geq \tau^{K+1}$ and then define $\tau_s = \min\{\tau : \hat{F}_s(\tau) \geq 0\}$, $\tau_b = \tau^{K+1}$ and $\tau^* = T^* = \min\{\tau_b, \tau_s\}$. Finally, let $F_s(0^3) = \hat{F}_s(\tau^*)$, $E_b(0^3) = F_b(0^3) = 0$, then for $t \in [0^4, T^*]$ let $F_s(t) = \hat{F}_s(\tau^* - t)$, $F_b(t) = \hat{F}_b(\tau^* - t)$ and $E_b(t) = \hat{E}_b(\tau^* - t)$.

By construction, for $k \in \{1, ..., K\}$, we have $t^k = \tau^* - \tau^k$ if $\tau^k < \tau^*$ and $t^k = 0^4$ otherwise. Rational agent concession and exit strategies can clearly be backed out from these functions by skimming property and Lemma 1; all such equilibria are payoff equivalent. Up to that equivalence, the equilibrium is unique by construction. Also by construction, no agent has a profitable deviation (so such strategies form an equilibrium). In particular, concession on (t^{k+1}, t^k) is at rates λ_b and $\lambda_s^{v^k}$ respectively to make a rational seller or buyer $(v^k, w) \in \Theta^c$ indifferent between conceding on that interval. If $\tau^* = 0$ then $F_s(0^3) = (1 - \bar{z}_s)$. Otherwise, buyer concession at $t^k \geq 0^4$ is calibrated to always leave a rational seller indifferent between conceding an instant before or after t^k (given the probability of exit at t^k). As the next lemma shows, such an equilibrium is continuous in agents' beliefs.

Lemma 5. *Consider the continuation game at 0^3 after demands $p_s \in P_s$ and $p_b \in P_b$ with fixed Θ. A unique "straightforward" continuation equilibria exists, for which agents' continuation payoffs are continuous at the beliefs $(\bar{z}_s, \bar{z}_b, \bar{g})$ where $\bar{z}_i \geq z_i\pi_i(p_i) > 0$.*

Proof. To prove the result it is first necessary to establish the following inductive *Claim:* Consider an arbitrary sequence of distributions $(\bar{z}_s^n, \bar{z}_b^n, \bar{g}^n) \to (\bar{z}_s^n, \bar{z}_b^n, \bar{g}^n)$. If $\lim_n \tau^{k,n} = \tau^k$ as well as $\lim_n \hat{F}_b^{k,n}(\tau^{k,n}) = F_b^k(\tau^k)$ and $\lim_n \hat{E}_b^{k,n}(\tau^{k,n}) = E_b^k(\tau^k)$, then $\lim_n \tau^{\ell,n} = \tau^\ell$ for all $\ell \in \{k+1, ..., \ell^{k+1}\}$ and $\lim_n \hat{F}_b^{\ell^{k+1},n}(\tau^{\ell^{k+1},n}) = F_b^{\ell^{k+1}}(\tau^{\ell^{k+1}})$ and $\lim_n \hat{E}_b^{\ell^{k+1},n}(\tau^{\ell^{k+1},n}) = E_b^{\ell^{k+1}}(\tau^{\ell^{k+1}})$.

Subclaim 1. *For any $\tau > \tau^k$ we must have $\tau > \tau^{k,n}$ for large n,* $\lim_n \hat{F}_b^{k,n}(\tau) = F_b^k(\tau)$ *and* $\lim_n \overline{k}^n(\hat{F}_b^{k,n}(\tau)) \leq \overline{k}(\hat{F}_b^k(\tau))$ *taking subsequences if necessary so that limits is defined.* To see this, notice that $1 - \hat{F}_b^{k,n}(\tau) = (1 - \hat{F}_b^{k,n}(\tau^{k,n}))e^{\lambda_s^{v^k}(\tau - \tau^{k,n})} \to 1 - \hat{F}_b^k(\tau)$ then $\lim_n \overline{k}^n(\hat{F}_b^{k,n}(\tau)) \leq \overline{k}(\hat{F}_b^k(\tau))$ follows from the upper semi continuity of $\overline{k}$. More precisely, if $\sum_{(v^m,w)\in\Theta^c:m\geq k'} \bar{g}^n(v,w)(1 - \bar{z}_b^n) \geq \hat{F}_b^{k,n}(\tau)$ for all n, then the inequality also holds in the limit.

Subclaim 2. *If $\overline{k}(\hat{F}_b^k(\tau)) = k' > k$ for $\tau \geq \tau^k$ then* $\lim_n \overline{k}^n(\hat{F}_b^{k,n}(\tau + \varepsilon)) \geq k'$ *for any $\varepsilon > 0$, and so if $\overline{k}(\hat{F}_b^k(\tau^{k+1})) > k$ then* $\lim_n \tau^{k+1,n} \leq \tau^{k+1}$. This follows from $\sum_{(v^m,w)\in\Theta^c:m\geq k'} \bar{g}^n(v,w)(1 - \bar{z}_b^n) > \hat{F}_b^k(\tau + \varepsilon/2) \geq \hat{F}_b^{k,n}(\tau + \varepsilon)$ for all large n.

Subclaim 3. *We must have* $\lim_n \tau^{k+1,n} \geq \tau^{k+1}$. Suppose not, so that $\lim_n \tau^{k+1,n} < \tau^{k+1}$. Since

$\lim_n \overline{k}^n(\hat{F}_b^{k,n}(\tau)) \leq \overline{k}(\hat{F}_b^k(\tau)) = k$ for $\tau < \tau^{k+1}$, we must have $y^n = \tilde{y}^n(\hat{F}_b^{k,n}(\tau^{k+1,n})) < \hat{F}_b^{k,n}(\tau^{k+1,n})$ and

$$\pi^n(y^n, \hat{F}_b^{k,n}(\tau^{k+1,n})) = (p_s - p_b)(\hat{F}_b^{k,n}(\tau^{k+1,n}) - y^n) - p_b(1-\overline{z})(G^{e,n}(k) - G^{e,n}(\overline{k}^n(y^n))) \leq 0$$

where the inequalities are preserved in the limit, $\pi(\lim_n y^n, \hat{F}_b^k(\lim_n \tau^{k+1,n})) \leq 0$. We must have $\lim_n y^n \leq \lim_n \hat{F}_b^{k,n}(\tau^{k+1}) = \hat{F}_b^k(\tau^{k+1})$ otherwise $y^n > \hat{F}_b^k(\tau)$ for some $\tau < \tau^{k+1}$ and all large n so that $\bar{k}^n(y^n) \leq \bar{k}^n(\hat{F}_b^k(\tau)) = k$, so $\pi^n(y^n, \hat{F}_b^{k,n}(\tau^{k+1,n})) = (p_s - p_b)(\hat{F}_b^{k,n}(\tau^{k+1,n}) - y^n) > 0$, a contradiction. This in turn implies $\lim_n y^n < \hat{F}_b^k(\lim_n \tau^{k+1,n})$ so that $\pi(\lim_n y^n, \hat{F}_b^k(\lim_n \tau^{k+1,n})) \leq 0$ contradicts the definition of $\tau^{k+1} > \lim_n \tau^{k+1,n}$, establishing the subclaim.

Subclaim 4. *We must have* $\lim_n \tau^{k+1,n} = \tau^{k+1}$. Suppose not so that $\lim_n \tau^{k+1,n} > \tau^{k+1} + \varepsilon$ for some $\varepsilon > 0$ and $\overline{k}^n(\hat{F}_b^{k,n}(\tau^{k+1}+\varepsilon) = k$ for large n. For small enough $\varepsilon' > 0$, we must have $\pi(y, \hat{F}_b^k(\tau^{k+1}))$ is continuous and strictly decreasing in y on some interval $y \in [-\varepsilon', 0] + \tilde{y}(\hat{F}_b^k(\tau^{k+1}))$ and $\overline{k}(y)$ is constant. Then define $y^\delta = \min\{\tilde{y}(\hat{F}_b^k(\tau^{k+1})) - \delta, 0\}$, for small enough $\delta > 0$, we must have

$$\begin{aligned}&\lim_n (p_s - p_b)(\hat{F}_b^{k,n}(\tau^{k+1} + \varepsilon) - y^\delta) - p_b(1-\overline{z})(G^{e,n}(k) - G^{e,n}(\overline{k}^n(y^\delta)))\\ =&(p_s - p_b)(\hat{F}_b^k(\tau^{k+1} + \varepsilon) - y^\delta) - p_b(1-\overline{z})(G^e(k) - G^e(\overline{k}(y^\delta))) < 0.\end{aligned}$$

And so, for all sufficiently large n we must have $\pi^n(y^\delta, \hat{F}_b^{k,n}(\tau^{k+1} + \varepsilon) < 0$, which contradicts $\lim_n \tau^{k+1,n} > \tau^{k+1} + \varepsilon$, establishing the subclaim.

Subclaim 5. *We must have* $\tau^{\ell^{k+1}} = \tau^{k+1} = \lim_n \tau^{k+1,n} = \lim_n \tau^{\ell^{k+1},n}$ *and* $F_b^{\ell^{k+1},n}(\tau^{\ell^{k+1},n}) \leq F_b^{\ell^{k+1}}(\tau^{\ell^{k+1}})$. This adapts the arguments for subclaim 4. If the first part of subclaim 5 didn't hold, then $\lim_n \tau^{\ell^{k+1},n} > \tau^{k+1}$, and so $\lim_n \tau^{l,n} = \tau^{k+1}$ for $l \in \{k+1, ..., k'\}$ but $\lim_n \tau^{k'+1,n} > \tau^{k+1} + \varepsilon$ for some $\varepsilon > 0$ and $k' \in \{k+1, ..., \ell^k - 1\}$. Define $\check{y}^{l,n} = \hat{F}_b^{n,l}(\tau^{l,n})$, $\hat{y}^{l,n} = \hat{F}_b^{n,l}(\tau^{l+1,n})$, $\alpha^n(1) = k$ and $\alpha^n(j+1) = \overline{k}^n(\tilde{y}^n(\hat{y}^{j,n}))$. Again taking a subsequence if necessary, $\alpha^n(j)$ is constant in n for large n, and then let $k' = \alpha^n(j')$. Given $\pi^n(\check{y}^{\alpha^n(j+1),n}, \hat{y}^{\alpha^n(j),n}) = 0$, we have

$$\sum_{j=1}^{j'-1} \pi^n(\check{y}^{\alpha^n(j+1),n}, \hat{y}^{\alpha^n(j),n}) = (p_s - p_b)(\hat{y}^{k,n} - \check{y}^{k',n} + \sum_{j=2}^{j'-1}(\hat{y}^{\alpha^n(j),n} - \check{y}^{\alpha^n(j),n}) - p_b(1-\bar{z}_b)(G^{e,n}(k) - G^{e,n}(k')) = 0$$

Similarly, letting, $\check{y}^l = \hat{F}_b^{n,l}(\tau^{l,n})$, $\hat{y}^l = \hat{F}_b^n(\tau^{l+1})$ we know $\pi^n(\check{y}^{\ell^{k+1}}, \hat{y}^k) = 0$. Let $y^\delta = \min\{\check{y}^{\ell^{k+1}} -$

$\delta, 0\}$ be defined as before, then for small $\delta > 0$, we then get

$$\pi^n(y^\delta, \hat{F}_b^{n,k'}(\tau^{k+1}+\varepsilon)) = \pi^n(y^\delta, \hat{F}_b^n(\tau^{k+1}+\varepsilon)) + \sum_{j=1}^{j'-1} \pi^n(\check{y}^{\alpha^n(j+1),n}, \hat{y}^{\alpha^n(j),n}) - \pi^n(\check{y}^{\ell^{k+1}}, \hat{y}^k)$$

$$=(p_s - p_b)((\check{y}^{\ell^{k+1}} - y^\delta) + (\hat{y}^{k,n} - \hat{y}^k) + (\hat{F}_b^{n,k'}(\tau^{k+1}+\varepsilon) - \check{y}^{k',n})$$

$$+ \sum_{j=2}^{j'-1}(\hat{y}^{\alpha^n(j),n} - \hat{y}^{\alpha^n(j),n})) - p_b(1-\bar{z}_b)((G^{e,n}(k) - G^e(k)) + (G^e(\ell^{k+1}) - G^{e,n}(\ell^{k+1})))$$

$$\rightarrow (p_s - p_b)((\check{y}^{\ell^{k+1}} - y^\delta) + (\lim_n \hat{F}_b^{n,k'}(\tau^{k+1}+\varepsilon) - \check{y}^{k',n}) < 0$$

where the limit follows from $\lim_n \tau^{l,n} = \tau^{k+1}$ for $l \in \{k+1, ..., k'\}$ and the inequality from $\lim_n \hat{F}_b^{n,k'}(\tau^{k+1}+\varepsilon) - \check{y}^{k',n} < 0$ and with $\delta > 0$ chosen sufficiently small. However, of course, this implies a contradiction to $\lim_n \tau^{k'+1,n} > \tau^{k+1} + \varepsilon$.

Finally, suppose that $\lim_n \check{y}^{\ell^{k+1},n} > \check{y}^{\ell^{k+1}}$ then for $\delta = (\check{y}^{\ell^{k+1}} - \lim_n \check{y}^{\ell^{k+1},n})/2 < 0$, we have $\overline{k}^n(y^\delta) = \ell^{k+1}$ for large n and so

$$\pi^n(y^\delta, \hat{y}^{k,n}) = \pi^n(y^\delta, \hat{y}^{k,n}) - \pi^n(\check{y}^{\ell^{k+1}}, \hat{y}^k)$$

$$=(p_s - p_b)((\check{y}^{\ell^{k+1}} - y^\delta) + (\hat{y}^{k,n} - \hat{y}^k) - p_b(1-\bar{z}_b)((G^{e,n}(k) - G^e(k)) + (G^e(\ell^{k+1}) - G^{e,n}(\ell^{k+1})))$$

which converges to $(p_s - p_b)\delta < 0$, contradicting the definition of $\check{y}^{\ell^{k+1},n} = \tilde{y}(\hat{y}^{k,n}) > y^\delta$ for large n.

Subclaim 6. *We have* $\lim_n F_b^{\ell^{k+1},n}(\tau^{\ell^{k+1},n}) = F_b^{\ell^{k+1}}(\tau^{\ell^{k+1}})$.

Let $\alpha^n(j') = \ell^{k+1}$ then

$$0 = \sum_{j=1}^{j'-1} \pi^n(\check{y}^{\alpha^n(j+1),n}, \hat{y}^{\alpha^n(j),n})$$

$$= (p_s - p_b)(\hat{y}^{k,n} - \check{y}^{\ell^{k+1},n} + \sum_{j=2}^{j'-1}(\hat{y}^{\alpha^n(j),n} - \hat{y}^{\alpha^n(j),n}) - p_b(1-\bar{z}_b)(G^{e,n}(k) - G^{e,n}(\ell^{k+1}))$$

$$\rightarrow (p_s - p_b)(\hat{y}^k - \lim_n \check{y}^{\ell^{k+1},n}) - p_b(1-\bar{z}_b)(G^e(k) - G^e(\ell^{k+1})) = \pi(\lim_n \check{y}^{\ell^{k+1},n}, \hat{y}^k)$$

where the limit follows from $\tau^{k+1} = \lim_n \tau^{k+1,n} = \lim_n \tau^{\ell^{k+1},n}$. Hence, $\lim_n \check{y}^{\ell^{k+1},n} \geq \check{y}^{\ell^{k+1}}$, by the definition of $\tilde{y}$, establishing the subclaim, and completing the proof of the Claim.

Given the Claim, it is clear that $\tau^{k,n} \rightarrow \tau^k$, $\tau_b^n \rightarrow \tau_b$, $\tau_s^n \rightarrow \tau_s$, as well as $F_s^n(0^3) \rightarrow F_s(0^3)$. The payoff of a rational buyer with value v who concedes at t^k is

$$U_b^{v,c}(t^k) = (v - p_b)F_s(0^3) + (v - p_b)\int^{t\in(0,t^k)} e^{-rt} dF_s(t) + (v - p_s)e^{-rt^k}(1 - F_s(t^k))$$

Given that $F_s^n \rightarrow_w F_s$ where F_s is continuous at t^k, it is clear that $U_b^{v,n}(t^{k,n}) \rightarrow U_b^v(t^k)$. Similarly, the payoff of a rational buyer who exits at time t^k is $U_b^{v,w,e}(t^k) = U_b^{v,c}(t^k) + (w - v + p_s)e^{-rt^k}(1 - F_s(t^k))$ so that $U_b^{v,w,e,n}(t^{k,n}) \to U_b^{v,w,e}(t^k)$.

We now turn to the rational seller, who's payoff can be expressed as $V_s = \max\{p_b, U_s(T_+^*)\}$ where $U_s(T_+^*) = \int^{s \leq T^*} p_s e^{-rs} dF_b(s) + e^{-rT^*}(1 - \bar{z}_s)p_b$ is the payoff from conceding an instant after T^*. Given that $\lim_n F_b^n(T^{*,n}) = F_b(T^*) = 1 - \bar{z}_b$, $\lim_n T^{*,n} = T^*$ and $F_i^n \to_w F_i$ it is immediate that $U_s^n(T_+^{*,n}) \to U_s(T_+^*)$ and so $V_s^n \to V_s$. This completes the proof.

□

We are now ready to complete the proof of Proposition 1. Given the parameters of a bargaining game $(z_i, \pi_i, g, \Theta)_{i=s,b}$, let $\Delta_s = \Delta(P_s)$ be the set of seller demand choice distributions at 0^1. Let $\Delta_s^{p_s} \subset \Delta(P_b \cup \{e\})$ be the set of rational buyer demand choice distributions at 0^2 after seller demand p_s such that $\mu_b^{v,w,p_s}(e) = \mathbb{1}_{v-w>\underline{p}}$ and $\mu_b^{v,w,p_s}(p_b) = 0$ for $p_b \geq p_s$. Then $\Delta_b = \prod_{p_s \in P_s} \Delta_b^{p_s}$. Let $U_b^{v,w,p_b,p_s}(\mu_s, \mu_b)$ be the expected payoff of rational buyer (v, w) at 0^3 given demands $p_i \in P_i$, the demand choice distributions, $\mu_s \in \Delta_s(P_s)$ and $\mu_b \in \Delta_b$ combined with straightforward equilibrium continuation play. Also let $U_s^{p_s}(\mu_s, \mu_b)$ be the expected payoff of the seller at 0^2 given the demand $p_s \in P_s$, the demand choice distributions $\mu_s \in \Delta_s(P_s)$ and $\mu_b^{p_s} \in \Delta_b$ with straightforward equilibrium continuation play at 0^3. We then define:

$$\begin{aligned} B(\mu_s, \mu_b) = \{(\hat{\mu}_s, \hat{\mu}_b) \in \Delta_s \times \Delta_b : \hat{\mu}_s(p_s) > 0 \Rightarrow U_s^{p_s}(\mu_s, \mu_b) \geq U_s^{p_s'}(\mu_s, \mu_b), \forall p_s' \in P_s, \\ \hat{\mu}_b^{p_s}(p_b) > 0 \Rightarrow U_b^{v,w,p_b,p_s}(\mu_b, \mu_s) \geq U_b^{v,w,p_b'}(\mu_s, \mu_b), \forall p_b' \in P_b\}. \end{aligned}$$

It is clear that this self-correspondence is non-empty and convex-valued and has a closed graph given that $U_b^{v,w,p_b,p_s}(\mu_s, \mu_b)$ and $U_s^{p_s}(\mu_s, \mu_b)$ are continuous in (μ_b, μ_s) by Lemma 5. Hence, by Kakutani, it admits a (non-empty) fixed-point. This fixed point describes equilibrium demand choices and implies beliefs for $p_i \in P_i$. After the demand $p_b \notin P_b$, the seller always believes the rational buyer has a type $(\bar{v}, \underline{w})$. The buyer then immediately concedes if $p_s \leq \bar{v} - \underline{w}$ and the seller immediately concedes otherwise. □

Proof of Proposition 3

This proof directly builds on from the proof of Proposition 2, and references definitions and arguments first stated there. In fact, that proof already established part (a) of Proposition 3. I will first, therefore, address part (c), where (by assumption) for some $\hat{p}_s \leq p^*$, $\check{p}(\hat{p}_s)(1 - H(\check{p}(\hat{p}_s))) > p(1 - H(p))$ for $p \in [0, \check{p}(\hat{p}_s)) \cup (\hat{p}_s, p^* + 2\varepsilon]$. As in Proposition 2, let $\overline{p}_s = \max\{p_s \in P_s : p_s < \check{p}(\hat{p}_s)\}$ where $\overline{p}_s \in (\check{p}(\hat{p}_s) - 2\varepsilon', p^*]$ with a $\varepsilon' > 0$ rich set of commitment types. As argued in the proof of Proposition 2, for $\varepsilon' > 0$ small enough $\overline{p}_s \in P_s^*$. Moreover, for any $p_s \in P_s^*$ with $p_s \in [\tilde{p}_s, p^*]$, all buyers immediately concede or exit with probability approaching 1 in the limit so the seller's limit payoff is exactly $(1 - H(p_s))p_s$. Hence, the seller's payoff from demanding

$\overline{p}_s$ is at least $(\check{p}(\hat{p}_s))(1 - H(\check{p}(\hat{p}_s))) - 2\varepsilon'$.

If $\overline{p}_s > \min\{v - w : (v, w) \in \Theta\}$ then let $p^\dagger = \max\{p < \check{p}(\hat{p}_s) : H(p) < H(\check{p}(\hat{p}_s))\}$; notice that H is constant on the non-degenerate interval $(p^\dagger, \check{p}(\hat{p}_s))$ and $(1 - H(p_s))p_s$ is increasing in p_s on this interval so that $(1 - H(p_s))p_s < (1 - H(\overline{p}_s))\overline{p}_s$ for $p_s \in (p^\dagger, \overline{p}_s)$. Let $2\varepsilon' < \check{p}(\hat{p}_s)(1 - H(\check{p}(\hat{p}_s))) - \max_{p \le p^\dagger} p(1 - H(p))$, where the right hand side is strictly positive by assumption. Given this, the seller's limit payoff from demanding $p_s < \overline{p}_s$ is, therefore, less than from demanding $\overline{p}_s$ and so she won't make such a demand positive limit probability.

On the other hand, suppose that the seller demands $p_s > p^* + 2\varepsilon$, then I showed in the proof of Proposition 2 that for small $\varepsilon' > 0$ the buyer will counterdemand $p_b \le p^* + 2\varepsilon$, which the seller will immediately concede to with strictly positive probability, giving her a payoff less than $(p^* + 2\varepsilon)(1 - H(p^* + 2\varepsilon)) + \varepsilon'$. And so, the seller's best possible limit payoff from demanding $p_s > \hat{p}_s$, is always less than $\max_{p \in (\hat{p}_s, p^*+2\varepsilon]} p(1 - H(p)) + \varepsilon'$. This payoff is strictly then less than her payoff from proposing $\overline{p}_s$ whenever $3\varepsilon' < \check{p}(\hat{p}_s)(1 - H(\check{p}(\hat{p}_s))) - \max_{p \in (\hat{p}_s, p^*+2\varepsilon]} p(1 - H(p))$, where the right hand side is strictly positive by assumption. Hence, the seller will never demand $p_s > \hat{p}_s$ with positive limit probability limit. Hence, the seller only demands $p_s \in [\hat{p}_s - 2\varepsilon, \hat{p}_s] \cap P_s^*$ with positive limit probability, and since $\hat{p}_s \le p^*$, buyers' either immediately concede or exit in the limit. The bound on the buyer's payoff is then immediate.

Finally, I turn to the upper bound on buyer payoffs in Proposition 3, part (b). Let $\overline{p}_s$ be defined as above given $\hat{p}_s = p^*$, then the argument for part (c) shows the seller will never charge $p_s < \overline{p}_s \in [p^* - 2\varepsilon, p^*] \in P_s^*$ given a small $\varepsilon' > 0$ rich set of commitment types. Demanding $\overline{p}_s$ gives the seller a payoff of at least $(1 - H(\overline{p}_s))\overline{p}_s > (1 - H(\check{p}(\hat{p})))\check{p}(\hat{p}) - 2\varepsilon'$. The buyer's payoff after any seller demand $p_s \in P_s^* \in [\overline{p}_s, p^*]$ is $\max\{v - p_s, w\} \le \max\{v - p^* - 2\varepsilon, w\}$. If the seller charges $p_s > p^*$ with positive limit probability but doesn't (for large n) immediately concede to some counterdemand p_b, then the buyer's limit payoff is bounded above by $\max\{v - p^*, w\}$.

Suppose, therefore, that the seller charges $p_s > p^*$ with positive limit probability and immediately concedes to some buyer counterdemand p_b. Given $p_s \in P_s^*$, we must therefore have $v^{1,p_s} - p_s < \hat{p}_b^{p_s} = \min\{p_b \in P_b : p_b > v^{1,p_s} - p_s\} < p^* + 2\varepsilon$. The seller must then concede with limit probability $\lim_n F_s^{p_s,p_b,n}(0^4) = (p_s - \hat{p}_b^{p_s})/(p_s - p_b)$ to commitment demands $p_b \le \hat{p}_b^{p_s}$. The buyer $(v^{1,p_s}, w) \in \Theta^{c,p_s}$ must demand $\hat{p}_b^{p_s}$ for all small $\varepsilon' > 0$ (as $v^{2,p_s} - p_s > \hat{p}_b^{p_s}$)) and so $\lim_n \bar{g}^{p_s,p_b,n}(v', w') = 0$ for $p_b < \hat{p}_b^{p_s}$ with $(v', w') \in \Theta^{c,p_s}$ and $v' > v^{1,p_s}$ (or (v^{1,p_s}, w) would deviate to get the payoff (1) instead). Hence, any buyer $(v, w)\Theta^{c,p_s}$ is indifferent to demanding $\hat{p}_b^{p_s}$ and so receives a payoff $v - \hat{p}_b^{p_s}$.

As shown in the proof of Proposition 2, the seller's payoff in this case is at most $(1 - H(p_s))\hat{p}_b^{p_s} + \varepsilon'$. If $\hat{p}_b^{p_s} \le p^* - 2\varepsilon$ then the seller's limit payoff would be less than from demanding $\overline{p}_s$ for $3\varepsilon' < (1 - H(\check{p}(\hat{p})))\check{p}(\hat{p}) - (1 - H(p_s))(p^* - 2\varepsilon)$, where the right hand side is positive by assumption. And so, we must have $\hat{p}_b^{p_s} > p^* - 2\varepsilon$, which establishes the payoff of any $(v, w)\Theta^{c,p_s}$ buyer is at most $v - p^* + 2\varepsilon$.

First suppose that $p_s \in (p^*, \underline{v} - \underline{w})$ so that $p^* = \underline{v}/2$. If $p_s \le \underline{v}/2 + \varepsilon$ then $p_s - 2\varepsilon \le \underline{v} - p_s < \hat{p}_b^{p_s}$ and so $p_s - \hat{p}_b^{p_s} < 2\varepsilon$. If $p_s > \underline{v}/2 + \varepsilon$ then with a ε' rich set of commitment types we have $\hat{p}_b^{p_s} < \underline{v} - p_s + 2\varepsilon' < \underline{v}/2 - \varepsilon + 2\varepsilon'$ and so the seller's payoff is at most $(1 - H(p^*))(p^* - \varepsilon + 2\varepsilon') + \varepsilon'$ which is less than her payoff from charging $p_s' \in [p^* - 2\varepsilon', p^*] \in P_s^*$ of at least $(p^* - 2\varepsilon')(1 - H(p^*))$, when $\varepsilon' < \varepsilon(1 - H(p^*))/(5 - 4H(p^*))$, a contradiction (if $H(p^*) = 1$ the seller would certainly never charge $p_s > p^*$). Hence, in this case we must have $p_s - \hat{p}_b^{p_s} < 2\varepsilon$.

Now suppose that $p_s > \underline{v} - \underline{w}$. If $\underline{v}/2 = p^* \ge \underline{w}$ then if $\hat{p}_b^{p_s} < p^* - K\varepsilon'$ where $K > 2 + 1/(1 - H(p^*))$ then the seller's payoff with a ε' rich set of commitment types of at most $(1 - H(p^*))\hat{p}_b^{p_s} + \varepsilon'$ is less than her payoff from demanding $p_s \in [p^* - 2\varepsilon', p^*]$ which is at least $(1 - H(p^*))(p^* - 2\varepsilon') + \varepsilon'$. If $\underline{w} = p^* > \underline{v}/2$ then given $p_s > p^*$ we must have $\hat{p}_b^{p_s} > v^{1,p_s} - p_s > \underline{w}$. And so, we must have $\hat{p}_b^{p_s} > p^* - K\varepsilon'$. Next define:

$$\underline{y}^{\varepsilon'} = \max\{\hat{y} \ge 0 : (1 - H(p^* + \hat{y})(p^* + 2\varepsilon) + \varepsilon' \ge (1 - H(\check{p}(p^*)))(\check{p}(p^*) - 2\varepsilon')\},$$

where $\underline{y}^{\varepsilon'} = 0$ if the maximizing set is empty. Given $\varepsilon' > 0$ small enough, we must have $\underline{y}^{\varepsilon'} < y$ given $(1 - H(p^* + y))(p^* + 2\varepsilon) < (1 - H(\check{p}(p^*)))\check{p}(p^*)$ by assumption. In a $\varepsilon' > 0$ rich set of commitment types, therefore, the seller will never charge $p_s > p^* + \underline{y}^{\varepsilon'}$ and so whenever $K\varepsilon' < y - \underline{y}^{\varepsilon'}$ we have $p_s - \hat{p}_b^{p_s} < y$.

Given $p_s \in P^*$ and $\lim_n \bar{g}^{p_s, \underline{p}, n}(v', w') = 0$ for $(v', w') \in \Theta^{c, p_s}$ with $v' > v^{1, p_s}$, the limit payoff to type $(v, w) \in \Theta^{e, p_s}$ can be written $w + (v - \underline{p} - w) \lim_n F_s^{p_s, p_b, n}(0^4)$. This is increasing in v, where clearly $v - w \le p_s$. And so that payoff is certainly less than $w + y$ given that $(p_s - \underline{p}) \lim_n F_s^{p_s, p_b, n}(0^4) = (p_s - \hat{p}_b^{p_s}) \le y$. □