ALGEBRA QUALIFYING EXAM SYLLABUS 2015

Recommended Textbooks

- *Abstract Algebra* by Dummit and Foote
- *Introduction to Commutative Algebra* by Atiyah and Macdonald
- *Algebra* by Lang.

Topics covered

I. Groups.
- Group actions, orbit-stabilizer theorem, Sylow theorems, semi-direct products, Jordan-Holder theorem.
- Examples: symmetric, alternating, dihedral groups, general and special linear groups.

II. Linear algebra.
- Modules over a PID, elementary divisor theorem.
- Invariant factors and similarity classes of matrices.
- Jordan and rational canonical forms, Cayley-Hamilton theorem.

III. Fields.
- Polynomial rings, Gauss lemma, Eisenstein criterion.
- Finite fields: construction, classification, structure of the units.
- Normal and separable extensions, Galois groups and the Galois correspondence.
- Computing Galois groups of low degree extensions, cyclotomic fields.
- Discriminants, symmetric polynomials, insolvability of the general quintic.
- Transcendence degree.

IV. Rings and commutative algebra.
- Noetherian and Artinian rings and modules.
- Discrete valuation rings, local rings, localization, Nakayama's lemma.
- Primary decomposition.
- Integral extensions. Going-up and going-down theorems.

V. Modules and homological algebra.
- Tensor product of modules and algebras.
- Exact sequences. Projective, injective, flat modules.
- Complexes. Projective and injective resolutions, Ext and Tor.
- Localization of modules.

VI. Algebraic Geometry.
- Zariski topology, Spec of a commutative ring, algebraic sets in affine space.
- Hilbert's Nullstellensatz, Noether Normalization, Krull dimension.

VII. Algebraic Number Theory.
- Algebraic integers, discriminants.
- Prime factorization in Dedekind rings.