News or Noise? The Missing Link*

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Abstract

The literature on belief-driven business cycles treats news and noise as distinct representations of agents' beliefs. We prove they are empirically the same. Our result lets us isolate the importance of purely belief-driven fluctuations. Using three prominent estimated models, we show that existing research understates the importance of pure beliefs. We also explain how differences in both economic environment and information structure affect the estimated importance of pure beliefs.

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1 Introduction

A large literature in macroeconomics has argued that changes in agents' beliefs about the future can be an important cause of economic fluctuations.[1] This idea, which dates at least to Pigou (1927), has been formalized in two ways. In the first way, which we call a "news representation," agents perfectly observe part of an exogenous fundamental in advance. As an analogy, this is like learning today that in next week's big game your favorite team will certainly win the first half. You don't know whether they will win the game, which is ultimately what you care about, because you are still unsure how the second half will turn out. In the second way, which we call a "noise representation," agents imperfectly observe an exogenous fundamental in advance. This is like your friend telling you that he thinks your team will win next week's game. He follows the sport more than you do, and is often right, but sometimes he gets it wrong.

Much of the literature emphasizes the differences between these two ways of representing agents' beliefs.[2] For example, in models with news, agents have full information and shocks are perfectly anticipated; in models with noise, agents have imperfect information and shocks are not perfectly anticipated. It has been suggested that models with noise shocks may be more theoretically flexible, and require weaker assumptions regarding the timing of information arrival. Others argue that models with news shocks may be easier to estimate using semi-structural empirical methods, which rely on fewer theoretical assumptions. Some studies include both news and noise shocks in the same model and attempt to determine which is more important.

In this paper, we argue that news and noise representations are more closely linked than the literature has recognized. Specifically, we prove that these two information structures are observationally equivalent. This means that even given an ideal data set with complete observations of exogenous fundamentals *and* agents' beliefs about those fundamentals, it would be impossible to tell them apart. It therefore follows that neither representation requires stronger modeling assumptions for theoretical work, or greater reliance on a model's structural details for empirical work.

Our main result is a representation theorem, which says that fundamentals and

[1] Throughout the paper, we use the words "beliefs," "expectations," and "forecasts" as synonyms.

[2] This emphasis is often implicit in discussions of news and noise. Relatively explicit examples include Sections 2 and 4.2.3 of Beaudry and Portier (2014), Sections 5 and 6 of Lorenzoni (2011), Sections II.B and II.C of Blanchard et al. (2013), and the introduction of Barsky and Sims (2012).

agents' beliefs about them always have both a news representation and a noise representation. This implies that associated with every noise representation is an observationally equivalent news representation and vice versa. We present a constructive proof of the theorem using Hilbert space methods. Because it is constructive, our proof also provides a method for explicitly deriving the mapping from one representation to another. We compute this mapping in closed form for several models of interest from the literature.

The main step in moving from noise to news amounts to finding the Wold representation of the noise model. This is because the shocks in the news representation are static rotations of the Wold innovations implied by the noise representation. Because the Wold innovations are contained in the space spanned by the history of variables that agents observe, the news representation is a way of writing models with noise "as if" agents have perfect information.[3] To move in the opposite direction, from news to noise, the idea is to reverse engineer the signal extraction problem that generates a given Wold representation. The challenge is to ensure that the noise shocks in that signal extraction problem are independent of fundamentals at all leads and lags, and that they capture all the non-fundamental variation in beliefs.

Beyond clarifying the link between news and noise, our representation theorem sheds new light on the importance of purely belief-driven fluctuations. Existing studies that either use models with only news shocks or some combination of news and noise shocks do not isolate the pure contribution of beliefs above and beyond fundamentals. The reason is that news shocks mix the fluctuations due purely to beliefs with the those due to fundamentals. News shocks can change beliefs on impact without any change in current fundamentals, but they are tied by construction to changes in future fundamentals. Beliefs change today, and on average fundamentals change tomorrow. But which is more important, the change in beliefs or the subsequent change in fundamentals?

To isolate the contribution of pure beliefs, it is necessary to disentangle the effects due only to expected changes in fundamentals from the consequences of their actual realizations.[4] One way to do this is to first find a noise representation of the news-

[3] A related result is Lemma 2 of Blanchard et al. (2013), which shows that their information structure has an observationally equivalent full information representation with correlated shocks.

[4] This point has been emphasized in the literature. For example, see the discussion in Section IV.A of Barsky, Basu, and Lee (2015), as well as the recent paper by Sims (2016).

shock model, and then consider the importance of noise shocks. Noise shocks isolate precisely those movements in beliefs that are independent of fundamentals at all horizons. Our representation theorem ensures that it is always possible to do this, and our constructive proof provides a procedure for doing so.

The noise representation therefore allows us to decompose variation in endogenous variables into the part purely due to fundamentals and the part purely due to beliefs. It can also be used to further decompose the contribution of fundamentals into a part due to future fundamental shocks, and a part due to current and past fundamental shocks. The part due to future fundamental shocks represents the contribution of correctly anticipated fundamental changes. In order for future fundamental shocks to be an important driver of current actions, two things must be true. First, agents' actions must depend to a sufficient degree on their expectations of future fundamentals. Second, they must have access to accurate information about future fundamental shocks that is not already revealed by current or past fundamental realizations.

We use our result to compute the importance of pure beliefs implied by three different quantitative models of U.S. business cycles. The three models come from Schmitt-Grohé and Uribe (2012), Barsky and Sims (2012), and Blanchard et al. (2013). These models all appear to have very different information structures, which — combined with differences in the rest of the physical environment, estimation procedure, and data sample — has made it difficult to compare results across models. By allowing us to isolate the independent contribution of beliefs in each model, our representation theorem provides a way of coherently comparing them. We use the exact models and estimated parameters from the original papers. Because news and noise representations are observationally equivalent, the likelihood functions are the same under either representation.

In all three cases, the importance of pure beliefs has been understated. In the model of Schmitt-Grohé and Uribe (2012), there is no shock labeled "noise," but the implicit contribution of noise shocks is between 3% and 11% depending on the variable. In the model of Barsky and Sims (2012), noise shocks are responsible for 9% of the fluctuations in consumption, which is almost an order of magnitude larger than the original estimate of 1%. In the model of Blanchard et al. (2013), the contribution of noise to consumption is 57%, compared to the originally reported value of 44%.

While these models disagree sharply regarding the overall importance of noise shocks, we find that they all agree that future fundamental shocks play a very small role compared to current and past fundamental shocks. For example, in the model of Barsky and Sims (2012), future fundamentals are responsible for less than 0.5% of consumption fluctuations, while current and past fundamentals are responsible for over 80%. Future fundamentals matter the most in the model of Blanchard et al. (2013), but even in that model they are responsible for less than 7% of consumption fluctuations.

We conclude our paper by investigating the sources of disagreement across models regarding the overall importance of noise shocks. We show that the disagreement is due to differences both in the models' economic environments and information structures. For noise shocks to play a large role, agents' actions need to depend heavily on their forecasts of future fundamentals (economic environment), and their forecasts in turn need to depend heavily on noise-ridden signals (information structure). The model of Blanchard et al. (2013) has both of these features, which is why they find a large role for noise shocks. In their model, productivity is a random walk, so agents rely heavily on their noisy signal to forecast future productivity. Nominal price and wage rigidity and an accommodative monetary policy rule work together to make agents' consumption decisions highly forward-looking, and allow the model to generate empirically realistic patterns of co-movement in response to a noise shock.

To quantify the relative contribution of economic environment and information structure on the estimated importance of noise shocks, we re-estimate the models of Barsky and Sims (2012) and Blanchard et al. (2013) using the same data, exogenous shocks, and estimation procedure (maximum likelihood) across both models. Consistent with the authors' original estimates, we find that noise shocks play a small role in the model of Barsky and Sims (2012) and a much larger role in the model of Blanchard et al. (2013). This suggests that differences in data, shocks, and estimation procedure are not the primary reasons these models deliver different estimates of the importance of noise shocks.

We then swap information structures and re-estimate both models. Substituting the information structure of Blanchard et al. (2013) into the economic environment of Barsky and Sims (2012) does almost nothing to change the estimated importance of noise shocks. On the other hand, substituting the information structure of Barsky and Sims (2012) into the economic environment of Blanchard et al. (2013) results in an estimated importance of noise shocks that is about halfway between the original estimates. This suggests that, while both economic environment and information structure play an important role in generating a large role for noise shocks, differences in environment turn out to be quantitatively more important in explaining the disagreement between these two models.

The literature on both news and noise shocks is large. In the noise literature, Lorenzoni (2009), Angeletos and La'O (2013), and Benhabib et al. (2015) have explored models in which dispersed information across agents can generate fluctuations in beliefs that are independent of aggregate fundamentals; we restrict our analysis to cases with a single, representative information set. In the news literature, Cochrane (1994), Beaudry and Portier (2006), and Beaudry and Lucke (2010) all provide VAR-based evidence pointing to an important role for news, and some empirical DSGE studies not cited above, including Forni et al. (2017) and Christiano et al. (2014), have estimated large roles for such shocks. Walker and Leeper (2011) and Leeper et al. (2013) explore how the specification of news processes alters the effects of news shocks on the dynamics of endogenous variables. Other related papers include Jaimovich and Rebelo (2009), Beaudry et al. (2011), Lorenzoni (2011), Barsky and Sims (2011), Born et al. (2013), Kurmann and Otrok (2013), and Jinnai (2014).

2 Observational Equivalence

News and noise representations are two different ways of describing economic fundamentals and agents' beliefs about them. "Fundamentals" are stochastic processes capturing exogenous changes in technology, preferences, endowments, or government policy. Throughout this section, fundamentals are summarized by a single scalar process $\{x_t\}$. Agents' decisions depend on expected future realizations of x_t, so both representations specify what agents can observe at each date and how they use their observations to form beliefs about the future.

The main result of the paper, which is presented in this section, is a representation theorem linking news and noise representations. The first subsection presents the basic result in a simple example with news or noise regarding fundamentals only one period in the future while the second subsection presents the more general result.

2.1 Simple Example

In the simplest of news representations, x_t is equal to the sum of two shocks, $a_{0,t}$ and $a_{1,t-1}$, which are independent and identically distributed (i.i.d.) over time, and which are independent of one another:

$$x_t = a_{0,t} + a_{1,t-1}, \qquad \begin{bmatrix} a_{0,t} \\ a_{1,t} \end{bmatrix} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, \begin{bmatrix} \sigma_{a,0}^2 & 0 \\ 0 & \sigma_{a,1}^2 \end{bmatrix}\right). \tag{1}$$

At each date t, agents observe the whole history of the two shocks up through that date, $\{a_{0,\tau}, a_{1,\tau}\}$ for all integers $\tau \leq t$. Their beliefs regarding fundamentals are rational; the probabilities they assign to future outcomes are exactly those implied by system (1). The shock $a_{1,t}$ is a news or anticipated shock because agents see it at date t but it doesn't affect the fundamental until date $t+1$. The shock $a_{0,t}$ is a surprise or unanticipated shock.

Now consider instead a noise representation. The fundamental variable x_t is i.i.d. over time, and there is a noisy signal of the fundamental one period into the future:

$$s_t = x_{t+1} + v_t, \qquad \begin{bmatrix} x_t \\ v_t \end{bmatrix} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}\right). \tag{2}$$

At each date t, agents observe the whole history of fundamentals and signals up through that date, $\{x_\tau, s_\tau\}$ for all integers $\tau \leq t$. Even though agents only have imperfect information about x_{t+1}, their beliefs are nevertheless still rational. The shock v_t is a noise or error shock because it affects beliefs but is totally independent of fundamentals.

Our point is that these two representations are observationally equivalent. But before making that point, it is important to be clear about what types of things we are considering to be "observable." To be concrete, imagine an econometrician who is able to observe the entire past, present, and future history of the fundamental process $\{x_t\}$, along with the entire past, present, and future history of agents' subjective beliefs regarding $\{x_t\}$. More concisely, we will say that the econometrician observes "fundamentals and beliefs." All of our results are stated from the perspective of such an econometrician, and are to be understood with respect to those observables.

An important feature of our concept of equivalence is that we treat beliefs, as well as fundamentals, as observable. We take this approach for three reasons. First, it is a stronger condition; observational equivalence with respect to a larger set of observables implies observational equivalence with respect to any smaller set of those observables. Second, beliefs are observable in economics, in principle. Beliefs may be measured directly, using surveys, or indirectly, using the mapping between beliefs and actions implied by an economic model. That actions reflect beliefs is, after all, a basic motivation for the literature on belief-driven fluctuations. Third, in a broad class of linear rational expectations models with unique equilibria, endogenous processes are purely a function of current and past fundamentals and beliefs about future fundamentals. So observational equivalence of fundamentals and beliefs implies observational equivalence of the entire economy.

We would also like to emphasize that the observability of beliefs distinguishes our concept of observational equivalence from that often encountered in time series analysis. To use a familiar example (cf. Hamilton, 1994, pp. 64-67), it is well-known that

$$y_t = \epsilon_t - \theta\epsilon_{t-1} \quad \epsilon_t \overset{\text{iid}}{\sim} \mathcal{N}(0,\sigma^2) \quad \text{and} \quad y_t = \tilde{\epsilon}_t - \tilde{\theta}\tilde{\epsilon}_{t-1} \quad \tilde{\epsilon}_t \overset{\text{iid}}{\sim} \mathcal{N}(0,\tilde{\sigma}^2) \tag{3}$$

are two observationally equivalent representations of the stationary MA(1) process $\{y_t\}$ when $\tilde{\theta} = 1/\theta$ and $\sigma^2 = \theta^2\tilde{\sigma}^2$. However, this applies only when $\{y_t\}$ is the sole observable. If (rational) expectations of future values of $\{y_t\}$ are also observable, then the two representations in (3) are no longer the same. To see why, note that the variance of the one-step-ahead rational forecast $\hat{y}_t \equiv E_t[y_{t+1}]$ is equal to $\theta^2\sigma^2$ under the first representation, but σ^2 under the second. Therefore, an econometrician observing $\{\hat{y}_t\}$ *and* $\{y_t\}$ (or independent functions of these objects) could discriminate between these two representations.

The following proposition states the equivalence result for the simple example of this subsection, and provides the parametric mapping from one representation to the other. Its proof is in the Appendix.

Proposition 1. *The news representation* (1) *is observationally equivalent to the noise representation* (2) *if and only if:*

$$\sigma_x^2 = \sigma_{a,0}^2 + \sigma_{a,1}^2 \quad \textit{and} \quad \frac{\sigma_v^2}{\sigma_x^2} = \frac{\sigma_{a,0}^2}{\sigma_{a,1}^2}.$$

The intuition behind the result comes from the fact that the noise representation implies an observationally equivalent innovations representation (cf. Anderson and

Moore, 1979, ch. 9) of the form:

$$\begin{aligned} x_t &= \hat{x}_{t-1} + w_{0,t} \\ \hat{x}_t &= \kappa w_{1,t} \end{aligned} \qquad \begin{bmatrix} w_{0,t} \\ w_{1,t} \end{bmatrix} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, \begin{bmatrix} \kappa\sigma_v^2 & 0 \\ 0 & \sigma_x^2 + \sigma_v^2 \end{bmatrix}\right), \tag{4}$$

where $\kappa = \sigma_x^2/(\sigma_x^2 + \sigma_v^2)$ is a Kalman gain parameter controlling how much agents trust the noisy signal, and $w_t \equiv (w_{0,t}, w_{1,t})'$ is the vector of Wold innovations. But system (4) is the same as the news representation in system (1) when $a_{0,t} = w_{0,t}$ and $a_{1,t} = \kappa w_{1,t}$. The news shocks are linear combinations of the Wold innovations.

A direct implication of Proposition (1) is that the news representation is identified if and only if the noise representation is identified. By observational equivalence, both representations have the same likelihood function. Because the relations in Proposition (1) define a bijection, it is always possible to go from one set of parameters to the other and vice versa. This suggests that the distinction often made between news and noise representations in the literature on semi-structural empirical methods may be misleading.

Proposition (1) also reveals that noise shocks are closely related to a popular thought experiment in the news-shock literature, which some researchers have used to isolate the effects of a change in beliefs that does not correspond to any change in fundamentals (e.g. Christiano et al. (2010) Section 4.2, Schmitt-Grohé and Uribe (2012) Section 4.2, Barsky, Basu, and Lee (2015) Section IV.A, or Sims (2016) Section 3.3). This experiment involves computing the impulse responses of endogenous variables to a current news shock followed by an offsetting future surprise shock.

In this simple example, it is easy to see that the noise shocks generate exactly the sort of offsetting news shocks envisioned by this thought experiment. Using the Kalman filter, the surprise and news shocks can be expressed as functions of fundamental and noise shocks:

$$a_{1,t} = \kappa x_{t+1} + \kappa v_t \quad \text{and} \quad a_{0,t} = (1-\kappa)x_t - \kappa v_{t-1}.$$

Therefore, a positive noise shock at date t generates a positive news shock at date t and an exactly offsetting surprise shock at date $t+1$.

This example shows that it may be possible to mimic noise shocks using particular linear combinations of news shocks. Nevertheless, there are a number of advantages to working directly with noise shocks. First, we can think about how often these situations arise, since we have an explicit probability distribution for the noise shocks: for example, how big is a "one standard deviation impulse" of a news reversal? Second, we can ask how important these types of news reversals are in the data overall; that is, we can do a proper variance decomposition. Third, in models with news shocks that are not i.i.d., it is not as straightforward to determine the configuration of news shocks that correspond to a noise shock. Therefore, it is desirable to have a more general characterization of the link between news and noise shocks. We turn to this more general characterization next.

2.2 Representation Theorem

This subsection generalizes the previous example to allow for news and noise at multiple future horizons, and potentially more complex time-series dynamics. To fix notation, we use $\mathcal{L}^2$ to denote the space of (equivalence classes of) complex random variables with finite second moments, which is a Hilbert space when equipped with the inner product $(a, b) = E[a\bar{b}]$ for any $a, b \in \mathcal{L}^2$. Completeness of this space is with respect to the norm $\|a\| \equiv (a, a)^{1/2}$. For any collection of random variables in $\mathcal{L}^2$,

$$\{y_{i,t}\}, \quad \text{with } i \in \mathcal{I}_y \subseteq \mathbb{Z} \text{ and } t \in \mathbb{Z},$$

we let $\mathcal{H}(y)$ denote the closed subspace spanned by the variables $y_{i,t}$ for all $i \in \mathcal{I}_y$ and $t \in \mathbb{Z}$. Similarly, $\mathcal{H}_t(y)$ denotes the closed subspace spanned by these variables over all i but only up through date t.

Fundamentals are summarized by a scalar discrete-time process $\{x_t\}$. As in the previous subsection, this process is taken to be mean-zero, stationary, Gaussian, and purely non-deterministic.[5] The fact that $\{x_t\}$ is a scalar process is not restrictive; we can imagine a number of different scalar processes, each capturing changes in one particular fundamental. In that case it will be possible to apply the results from this section to each fundamental one at a time.

Agents' beliefs about fundamentals are summarized by a collection of random variables $\{\hat{x}_{i,t}\}$, with $i, t \in \mathbb{Z}$, where $\hat{x}_{i,t}$ represents the forecast of the fundamental realization x_{t+i} as of time t. Under rational expectations, $\hat{x}_{i,t}$ is equal to the mathematical expectation of x_{t+i} with respect to a particular date-t information set. We

[5]The results in this section can be extended, in an appropriate limiting sense, to processes that are stationary only after suitable differencing. We will examine one such case in Section (4.3).

assume that $\{x_t\}$ and $\{\hat{x}_{i,t}\}$ jointly form a Gaussian system; that is, the vector formed by any finite subset of these random variables is Gaussian. This allows us to summarize agents' entire conditional distribution over fundamentals at each date solely by their conditional expectations across different horizons.

A "representation of fundamentals and beliefs" means a specification of the fundamental process $\{x_t\}$ and the collection of agents' conditional expectations about that process at each point in time $\{\hat{x}_{i,t}\}$. A typical assumption is that agents' information set is equal to $\mathcal{H}_t(x)$, so $\hat{x}_{i,t} \in \mathcal{H}_t(x)$ for all $t \in \mathbb{Z}$. In this case, the process $\{x_t\}$ is itself sufficient to describe both the fundamental and agents' beliefs about it. A key departure in models of belief-driven fluctuations due to advance information is that agents may have more information than what is reflected in $\mathcal{H}_t(x)$ alone. Therefore, throughout the paper we maintain the assumption that $\mathcal{H}_t(x) \subseteq \mathcal{H}_t(\hat{x})$ for all $t \in \mathbb{Z}$.

Definition 1. In a "news representation" of fundamentals and beliefs, the process $\{x_t\}$ is related to a collection of independent, stationary Gaussian processes $\{a_{i,t}\}$ with $i \in \mathcal{I}_a \subseteq \mathbb{Z}_+$ by the summation

$$x_t = \sum_{i \in \mathcal{I}} a_{i,t-i} \quad \text{for all } t \in \mathbb{Z},$$

where agents' date-t information set is $\mathcal{H}_t(a)$.

The idea behind this representation is that agents observe parts of the fundamental realization x_t prior to date t. The variable $\epsilon^a_{i,t} \equiv a_{i,t} - E[a_{i,t}|\mathcal{H}_{t-1}(a)]$ is called the "news shock" associated with horizon i whenever $i > 0$. By convention, for $i = 0$ the variable $\epsilon^a_{0,t}$ is referred to as the "surprise shock." An important aspect of this definition is that all of the news shocks are correlated both with fundamentals and agents' beliefs. This is because any increase in fundamentals that agents observe in advance must, other things equal, generate a one-for-one increase in fundamentals at some point in the future.

Definition 2. In a "noise representation" of fundamentals and beliefs, there is a collection of signal processes $\{s_{i,t}\}$ with $i \in \mathcal{I}_s \subseteq \mathbb{Z}_+$ of the form

$$s_{i,t} = m_{i,t} + v_{i,t} \quad \text{for all } t \in \mathbb{Z},$$

where $m_{i,t} \in \mathcal{H}(x)$, $v_{i,t} \perp \mathcal{H}(x)$, and agents' date-$t$ information set is $\mathcal{H}_t(s)$, which satisfies $\mathcal{H}_t(s) = \mathcal{H}_t(\hat{x})$.

The idea behind this representation is that agents may receive signals about the fundamental realization x_t prior to date t, but those signals are contaminated with noise. The variable $\epsilon^v_{i,t} \equiv v_{i,t} - E[v_{i,t}|\mathcal{H}_{t-1}(v)]$ is called the "noise shock" associated with signal i. The variable $\epsilon^x_t \equiv x_t - E[x_t|\mathcal{H}_{t-1}(x)]$ is called the "fundamental shock." An important aspect of this definition is that all of the noise shocks are completely independent of fundamentals, but because agents cannot separately observe $m_{i,t}$ and $v_{i,t}$ at date t, their beliefs are still affected by noise. The condition that $\mathcal{H}_t(s) = \mathcal{H}_t(\hat{x})$ simply rules out redundant or totally uninformative signals.

With these definitions in hand, we are ready to state the main result of the paper. Its proof is in the Appendix.

Theorem 1. *Fundamentals and beliefs always have both a news representation and a noise representation. Moreover, the news representation is unique.*

This theorem clarifies the sense in which news and noise representations of fundamentals and beliefs are really just two sides of the same coin. It is possible to view the same set of data from either perspective. The proof is constructive, which means that it also provides an explicit computational method for passing from one representation to the other.

The only asymmetric aspect of the theorem involves the uniqueness of the two representations. Any particular news representation will be compatible with several different noise representations. This is the same sort of asymmetry present between signal models representations and innovations representations in the literature on state-space models. In general there exist infinitely many signal models with the same innovations representation. We explain in the subsequent sections, however, that this multiplicity of noise representations does not pose much of a problem.

An implication of Theorem (1) is that any model *economy* with a news representation of fundamentals and beliefs has an observationally equivalent version with a noise representation of fundamentals and beliefs, and vice versa. This is because the equivalence of fundamentals and beliefs implies the equivalence of any endogenous processes that are functions of them. To make this statement more precise, we first define here what we mean by an endogenous process, and then present this statement as a proposition. The proof of the proposition, together with all remaining proofs, are contained in the Online Appendix.

Definition 3. Given a fundamental process $\{x_t\}$ and a collection of forecasts $\{\hat{x}_{i,t}\}$ satisfying $\mathcal{H}_t(x) \subset \mathcal{H}_t(\hat{x})$, a process $\{c_t\}$ is "endogenous" with respect to $\{x_t\}$ if

$$c_t \in \mathcal{H}_t(\hat{x}) \quad \text{for all } t \in \mathbb{Z}.$$

Proposition 2. *If two different representations of fundamentals and beliefs are observationally equivalent, then they imply observationally equivalent dynamics for any endogenous process.*

The stipulation in Definition (3) that endogenous processes be linearly related to agents' forecasts of fundamentals is not restrictive. Proposition (2) holds even if we generalize the definition of an endogenous process $\{c_t\}$ to require only that c_t be measurable with respect to agents' date-t information set for all $t \in \mathbb{Z}$ (the proof provided in the Online Appendix establishes this stronger result). Together with Theorem (1), this means that as long as fundamentals and beliefs form a Guassian system, any *non-linear* economy that allows for belief-driven fluctuations can be equivalently written with either news or noise shocks.

Throughout the rest of the paper, however, we will retain the restriction of linearity in Definition (3). This is because the definitions of many objects of economic interest, such as variance decompositions, are typically defined only for linear models. Therefore, it is most natural to present our results in terms of endogenous variables that can be expressed as linear functions of agents' forecasts. Furthermore, all of the quantitative models we consider in Section (4) rely on linear-approximate equilibrium dynamics.

3 The Importance of Pure Beliefs

A central question in the literature on belief-driven fluctuations is: how important are *purely* belief-driven fluctuations? That is, fluctuations due to changes in beliefs that cannot be explained by any actual change in economic fundamentals. Perhaps surprisingly, it turns out that no existing quantitative study in this literature has answered this question. Some studies report the importance of news shocks, which combine the contribution due to fundamentals with the contribution purely due to beliefs. Others include noise shocks and news shocks in the same model, and as a result, do not isolate the contribution of either one. In this section we argue that

Theorem (1) provides a way to determine the importance of pure beliefs as a driver of fluctuations.

The first subsection explains the problem with using news shocks to determine the importance of pure beliefs, and the second subsection clarifies the problems that arise when attempting to include both news and noise shocks in the same model. To keep things clear, the discussion of both of these issues is framed in terms of the simple example from Section (2.1). The third subsection establishes a result regarding the uniqueness of variance decompositions. The fourth subsection discusses how to further decompose the contribution of fundamental shocks into parts due to past, present, and future fundamental shocks.

3.1 The Problem with News Shocks

In the context of dynamic linear models, the importance of a set of exogenous shocks can be determined by performing a variance decomposition. This entails computing the model-implied variance of an endogenous process under the assumption that all shocks other than those in the set of interest are counterfactually equal to zero almost surely, and comparing that variance to the unconditional variance of the process. More nuanced versions include only considering variation over a certain range of spectral frequencies, or variation in forecast errors over a certain forecast horizon.

The problem with using news shocks to determine the importance of pure beliefs is that news shocks mix changes that are due to fundamentals and changes that are purely due to beliefs. This is because a news shock is an anticipated change in fundamentals. Expectations change at the time the news shock is realized, but then fundamentals change in the future when the anticipated change actually occurs. Of course, agents' expectations may not always be fully borne out in future fundamental realizations, due to other unforeseen disturbances. Nevertheless, the anticipated shock is borne out on average, which is to say that news shocks are related to future fundamentals on average.

A stark way to see this point is to consider the importance of pure beliefs for driving fundamentals. Because fundamentals are exogenous, they are obviously not driven by beliefs at all. However, in the simple news representation from Section (2.1), for example, news shocks can be responsible for an arbitrarily large part of the fluctuations in the fundamental process $\{x_t\}$. Recall that in that example, $x_t =$

$a_{0,t} + a_{1,t-1}$. Therefore, the fraction of the variation in $\{x_t\}$ due to news shocks, $\{a_{1,t}\}$ is given by:

$$\frac{\text{var}[x_t | a_{0,t} = 0]}{\text{var}[x_t]} = \frac{\text{var}[a_{1,t}]}{\text{var}[x_t]} = \frac{\sigma^2_{a,1}}{\sigma^2_{a,0} + \sigma^2_{a,1}}. \tag{5}$$

As $\sigma^2_{a,1}$ increases relative to $\sigma^2_{a,0}$, this fraction approaches one, in which case news shocks would explain all the variation in $\{x_t\}$.

To disentangle the importance of pure beliefs from fundamentals in models with news shocks, we can use Theorem (1). Specifically, we can write down an observationally equivalent noise representation of the news model, and then use a variance decomposition to compute the share of variation attributable to noise shocks. Because these shocks are independent of fundamentals at all horizons, they capture precisely those changes in beliefs that cannot be explained by fundamentals. That is, noise shocks are pure belief shocks.

Returning to the example from Section (2.1), we have already shown that an observationally equivalent noise representation involves $x_t \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2_x)$ with $\sigma^2_x \equiv \sigma^2_{a,0} + \sigma^2_{a,1}$. Therefore, the fraction of variation in $\{x_t\}$ due to noise shocks is:

$$\frac{\text{var}[x_t | x_t = 0]}{\text{var}[x_t]} = 0,$$

which is the correct answer to the question of how much beliefs contribute to the fluctuations of fundamentals. This example illustrates the more general point that in order to determine the importance of pure beliefs, one should perform variance decompositions in terms of noise shocks rather than news shocks.

The fact that variance decompositions in terms of news shocks are not appropriate for determining the importance of pure beliefs has lead some researchers to conclude that there is a fundamental problem with using variance decompositions for that purpose.[6] We would like to suggest that the problem is not with variance decompositions as such; rather, the problem is with the type of shock one considers. It is noise shocks, not news shocks, that are the appropriate shocks for isolating the independent contribution of beliefs. Once that distinction has been made, traditional variance decompositions can be performed as usual.

[6] For example, Sims (2016) p.42 describes the problem of identifying the importance of pure beliefs (which both he and Barsky et al. (2015) call "pure news") as a fundamental limitation of the traditional variance decomposition.

3.2 Mixing News and Noise Shocks

In some cases, researchers have constructed representations of fundamentals and beliefs that seem to include both news and noise shocks at the same time (e.g. Blanchard et al., 2013; Barsky and Sims, 2012). A simple example is:

$$\begin{aligned} x_t &= \mu_{t-1} + \eta_t \\ s_t &= \mu_t + \xi_t \end{aligned} \qquad \begin{bmatrix} \eta_t \\ \mu_t \\ \xi_t \end{bmatrix} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, \begin{bmatrix} \sigma_\eta^2 & 0 & 0 \\ 0 & \sigma_\mu^2 & 0 \\ 0 & 0 & \sigma_\xi^2 \end{bmatrix}\right). \tag{6}$$

At each date t, agents observe $\{x_\tau, s_\tau\}$ for all $\tau \leq t$. The shock μ_t looks like a news shock because it affects agents' beliefs at date t (through the signal s_t), but does not affect fundamentals until the following period. Similarly, the shock η_t looks like a surprise shock because it affects agents' beliefs and the fundamental at the same time. Finally, the shock ξ_t looks like a noise shock because it affects agents' beliefs but is independent of fundamentals.

The problem with this type of representation, at least from the perspective of isolating the importance of pure beliefs, is that while ξ_t can generate non-fundamental fluctuations in beliefs, so can certain combinations of η_t and μ_t. To see this, notice that in the limit case $\xi_t = 0$, we have that $s_t = \mu_t$ and this representation collapses to a news representation with $a_{0,t} \equiv \eta_t$ and $a_{1,t} \equiv \mu_t$. We have already seen in Proposition (1) that such a news representation has an observationally equivalent noise representation with (non-zero) noise shocks. Therefore $\xi_t = 0$ does not mean that beliefs do not have an independent role to play as a driver of fluctuations.[7]

Of course, Theorem (1) implies that the representation in (6), which is neither news or noise representation, still has an observationally equivalent noise representation. The following proposition presents the mapping from one representation to the other.

Proposition 3. *The representation of fundamentals and beliefs in* (6) *is observationally equivalent to the noise representation in* (2) *if and only if:*

$$\sigma_x^2 = \sigma_\mu^2 + \sigma_\eta^2 \quad \text{and} \quad \frac{\sigma_v^2}{\sigma_x^2} = \frac{\sigma_\mu^2(\sigma_\eta^2 + \sigma_\xi^2) + \sigma_\eta^2\sigma_\xi^2}{\sigma_\mu^4}.$$

[7] In two of the quantitative models we consider in the next section, this distinction is particularly stark; the contribution of pure beliefs turns out to *increase* as $\sigma_\xi^2 \to 0$.

To see how the process $\{\xi_t\}$ understates the importance of pure beliefs, consider the endogenous variable $\hat{x}_t = E_t[x_{t+1}]$. Under representation (6), $\hat{x}_t = \frac{\sigma_\mu^2}{\sigma_\mu^2+\sigma_\xi^2}(\mu_t + \xi_t)$, so the contribution of the process $\{\xi_t\}$ is

$$\frac{\text{var}[\hat{x}_t|\mu_t = \eta_t = 0]}{\text{var}[\hat{x}_t]} = \frac{\sigma_\xi^2}{\sigma_\mu^2 + \sigma_\xi^2}.$$

On the other hand, in the observationally equivalent noise representation implied by Proposition (3), $\hat{x}_t = \frac{\sigma_x^2}{\sigma_x^2+\sigma_v^2}(x_{t+1} + v_t)$. Therefore, the contribution of $\{v_t\}$ is

$$\frac{\text{var}[\hat{x}_t|x_t = 0]}{\text{var}[\hat{x}_t]} = \frac{\sigma_v^2}{\sigma_x^2 + \sigma_v^2} = \frac{\sigma_\mu^2\sigma_\eta^2}{(\sigma_\mu^2 + \sigma_\eta^2)(\sigma_\mu^2 + \sigma_\xi^2)} + \frac{\sigma_\xi^2}{\sigma_\mu^2 + \sigma_\xi^2},$$

where the second equality uses the parametric restrictions from Proposition (3). Because the first term in this expression is positive, it follows that $\{\xi_t\}$ understates the importance of pure beliefs for explaining variations in $\{\hat{x}_t\}$. It is also easy to see how the importance of pure beliefs can be strictly positive even as $\sigma_\xi^2 \to 0$.

3.3 Different Noise Representations

So far we have argued that it is possible to use a noise representation to separate fluctuations that are due to actual changes in fundamentals versus those that are due purely to changes in beliefs. First, one can rewrite any representation of fundamentals and beliefs as a noise representation using the constructive procedure from Theorem (1). Then, one can use a variance decomposition to determine the share of variation in any endogenous variable that is attributable to noise shocks. And this share represents the contribution purely due to non-fundamental changes in beliefs.

But is the variance decomposition in terms of noise shocks unique? As we pointed out in the discussion of Theorem (1), any representation of fundamentals and beliefs is compatible with infinitely many different noise representations. Fortunately, it turns out that all observationally equivalent noise representations deliver the same answer regarding the importance of pure beliefs for any endogenous process. For variance decompositions, the fact that noise representations are not unique is not a problem.

Proposition 4. *In any noise representation of fundamentals and beliefs, the variance decomposition of any endogenous process in terms of noise and fundamentals is uniquely determined over any frequency range.*

An immediate corollary of this proposition is that the variance decomposition of agents' errors in forecasting an endogenous process is also uniquely determined for any forecast horizon. This is because the forecast errors are themselves endogenous processes to which Proposition (4) applies.

Corollary 1. *In any noise representation of fundamentals and beliefs, the forecast error variance decomposition of any endogenous process in terms of noise and fundamentals is uniquely determined for any horizon, and over any frequency range.*

3.4 Past, Present, and Future Fundamentals

Our discussion in this section has focused on the distinction between the relative contributions of fundamental shocks and non-fundamental noise shocks. However, it is also possible to further decompose the contribution of fundamental shocks into parts separately due to past, present, and future fundamental shocks. Even if news shocks don't capture the contribution of noise shocks, maybe they capture something like the sum of the contribution of noise shocks and future fundamental shocks.

While that intuition seems sensible enough, it turns out to be incorrect. But before explaining why, we first show how to separately determine the contribution of past, present, and future fundamental shocks. Recall the i.i.d. noise model from Section (2.1),

$$s_t = x_{t+1} + v_t, \quad \begin{bmatrix} x_t \\ v_t \end{bmatrix} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}\right),$$

and consider an endogenous variable that depends on past, present, and expected future fundamentals with weights ϕ_{-1}, ϕ_0, and ϕ_1 respectively:

$$c_t = \phi_{-1} x_{t-1} + \phi_0 x_t + \phi_1 E_t[x_{t+1}]. \tag{7}$$

Solving the signal-extraction problem to obtain the optimal forecast, we have

$$c_t = \phi_{-1} x_{t-1} + \phi_0 x_t + \phi_1 \kappa x_{t+1} + \phi_1 \kappa v_t, \tag{8}$$

where $\kappa \equiv \sigma_x^2/(\sigma_x^2 + \sigma_v^2)$ is the gain parameter.

Since the fundamental and noise processes are both i.i.d., we can decompose the variance of c_t into four parts,

$$\text{var}[c_t] = \underbrace{\phi_{-1}^2 \sigma_x^2}_{\text{past fundamentals}} + \underbrace{\phi_0^2 \sigma_x^2}_{\text{present fundamentals}} + \underbrace{\phi_1^2 \kappa^2 \sigma_x^2}_{\text{future fundamentals}} + \underbrace{\phi_1^2 \kappa^2 \sigma_v^2}_{\text{noise}}, \tag{9}$$

where the sum of the first three parts equals the total contribution of fundamentals. Notice from this equation that even when there are no noise shocks ($\sigma_v = 0$), the contribution of future fundamentals is not necessarily equal to zero. In that case, $\kappa = 1$, so the share of the variance of c_t due to future fundamentals would be $\phi_1^2/(\phi_{-1}^2+\phi_0^2)$.

More generally, we can use the noise representation of fundamentals and beliefs to uniquely decompose any endogenous variable as

$$\begin{aligned} c_t &= \underbrace{\sum_{j=-\infty}^{\infty} \alpha_j \epsilon_{t-j}^x}_{\text{fundamentals}} + \underbrace{\sum_{j=0}^{\infty} \beta_j \epsilon_{t-j}^v}_{\text{noise}} \\ &= \underbrace{\sum_{j=1}^{\infty} \alpha_j \epsilon_{t-j}^x}_{\text{past fundamentals}} + \underbrace{\alpha_0 \epsilon_t^x}_{\text{present fundamentals}} + \underbrace{\sum_{j=-\infty}^{-1} \alpha_j \epsilon_{t-j}^x}_{\text{future fundamentals}} + \underbrace{\sum_{j=0}^{\infty} \beta_j \epsilon_t^v}_{\text{noise}}, \end{aligned} \tag{10}$$

where $\{\epsilon_t^x\}$ are the fundamental shocks, and $\{\epsilon_t^v\}$ are the noise shocks. Because the shocks are i.i.d., the variance of c_t is equal to the sum of the variances in each of the four terms on the right, just as in equation (9). The uniqueness of the decomposition in equation (10) is summarized in the following proposition.

Proposition 5. *In any noise representation of fundamentals and beliefs, the share of the variance of any endogenous process due to past, present, or future fundamental shocks is uniquely determined.*

Going back to equation (7), we can show that the contribution of news shocks is not equal to the sum of the contribution of future fundamental shocks and noise shocks. Consider the contribution of news shocks for c_t in the special case that $\phi_{-1} = \phi_1 = 0$ and $\phi_0 = 1$, so that $c_t = x_t$. In this case, the contribution of future fundamentals and noise are both zero; all that matters for c_t is the current fundamental realization. But we have already seen in equation (5) that news shocks can be arbitrarily important for explaining fluctuations in $\{x_t\}$. This is because past news shocks eventually show up as changes in current fundamentals. Therefore, news shocks can be very important even when both noise shocks and future fundamentals are not.

On the other hand, if the contribution of news shocks is small, that does tell us that the contribution of both future shocks and noise shocks must be small as well. To see this, we can use equation (9) and Proposition (1) to write the part of the variance of c_t due to future fundamentals shocks and noise shocks in terms of the corresponding parameters from the observationally equivalent news representation:

$$\phi_1^2\kappa^2\sigma_x^2 + \phi_1^2\kappa^2\sigma_v^2 = \phi_1^2\frac{\sigma_{a,1}^4}{\sigma_{a,0}^2+\sigma_{a,1}^2} + \phi_1^2\frac{\sigma_{a,0}^2\sigma_{a,1}^2}{\sigma_{a,0}^2+\sigma_{a,1}^2}.$$

As the variance of news shocks, $\sigma_{a,1}^2$, approaches zero, this expression also approaches zero (term by term). From this we can conclude that a large contribution of news shocks is necessary but not sufficient for there to be a large contribution of either future fundamental shocks or noise shocks.

One difference relative to Proposition (4) is that Proposition (5) does not apply "over any frequency range." It only applies to *unconditional* variance decompositions; that is, to decompositions across all frequencies $\lambda \in [-\pi, \pi]$. The distinction between past, present, and future makes sense in the time domain, but not in the frequency domain. Either we can look at the contribution of fundamentals over different time ranges or frequency ranges, but not both at the same time.

Finally, it is worth noting that the extent to which an endogenous process depends on future fundamental shocks depends on both the physical economic environment and agents' information structure. In equation (8), the weight of c_t on x_{t+1} depends both on ϕ_1 and κ. If the economic model is not sufficiently "forward-looking," so $\phi_1 \to 0$, then the share of future fundamentals will be small. Perhaps less intuitively, if $\kappa \to 0$ then the share of future fundamentals will also be small. Even if the model is forward-looking, so $\phi_1 > 0$, future fundamental shocks can still be unimportant for current actions if the only information agents have about future fundamentals is completely contained in current and past fundamentals. Note that this is true even if the model is *purely* forward looking; that is, when $\phi_{-1} = \phi_0 = 0$ and $\phi_1 > 0$.

4 Quantitative Analysis

In this section, we use Theorem (1) and Proposition (4) to empirically quantify the importance of pure beliefs in driving business-cycle fluctuations. Because several models of belief-driven fluctuations have already been constructed and estimated in the literature, we take a meta-analytic perspective. We select three prominent theoretical models that have been estimated in the literature and compute the importance of pure beliefs implied by each of those models for different macroeconomic variables (e.g. output, investment, etc.). The three models are the model of news shocks from

Schmitt-Grohé and Uribe (2012), the model of news and animal spirits from Barsky and Sims (2012), and the model of noise shocks from Blanchard et al. (2013).

These three models are different in several respects. First, they incorporate different physical environments, including differences in preferences, frictions and market structure. Second, the three models are estimated on different data and with different sample periods. Third, the authors make different assumptions about the information structure faced by agents. While agents in all three models observe current fundamentals and receive advance information about future fundamentals, Schmitt-Grohé and Uribe (2012) take a pure news perspective while the Barsky and Sims (2012) and Blanchard et al. (2013) offer somewhat different perspectives on combining news and noise within a single model.

Perhaps not surprisingly given the scope of these differences, the authors above come to very different conclusions. Schmitt-Grohé and Uribe (2012) conclude that news shocks explain about one half of aggregate fluctuations, but do not take an explicit stance on the importance of independent fluctuations in beliefs. Barsky and Sims (2012) also conclude that news shocks are important, and that noise shocks explain essentially none of the variation in any variable. However, Blanchard et al. (2013) conclude that noise shocks play a crucial role in business cycle dynamics, especially for consumption.

In principle, it is possible that these different conclusions are largely a result of the different "normalizations" the authors take with respect to noise shocks. Indeed, our analysis indicates that all authors have (implicitly or explicitly) underestimated the actual of role of pure beliefs in their estimated models. For Schmitt-Grohé and Uribe (2012) and Barsky and Sims (2012), we find that the role of noise rises from being essentially zero to being small but non-trivial, generally between 3% and 11% at the business cycle frequency. Surprisingly, even Blanchard et al. (2013) underestimate the role of noise shocks in driving their economy, with pure beliefs about productivity driving endogenous variables more than productivity itself.

While our results indicate that noise shocks are more important than previously reported, they do not fully explain the degree of disagreement regarding the independent contribution of beliefs. To understand the remaining differences, we perform a series of exercises, including re-estimating different versions of these models after swapping information structures.

4.1 Schmitt-Grohé and Uribe (2012)

The first model comes from Schmitt-Grohé and Uribe (2012), and was constructed to determine the importance of news shocks for explaining aggregate fluctuations in output, consumption, investment, and employment. The main result of their paper is that news shocks account for about half of the predicted aggregate fluctuations in those four variables. As we have seen in the previous section, however, news shocks mix fluctuations due to beliefs and fundamentals. As a result, exactly what this model implies about the importance of pure beliefs is still an unanswered question.

The model is a standard real business cycle model with six modifications: investment adjustment costs, variable capacity utilization with respect to the capital stock, decreasing returns to scale in production, one period internal habit formation in consumption, imperfect competition in labor markets, and period utility allowing for a low wealth effect on labor supply. Fundamentals comprise seven different independent processes, which capture exogenous variation in stationary and non-stationary neutral productivity, stationary and non-stationary investment-specific productivity, government spending, wage markups, and preferences. The model is presented in more detail in Online Appendix (B.1).

Each of the seven exogenous fundamentals follows a law of motion:

$$x_t = \rho_x x_{t-1} + \epsilon^a_{0,t} + \epsilon^a_{4,t-4} + \epsilon^a_{8,t-8}, \qquad \begin{bmatrix} \epsilon^a_{0,t} \\ \epsilon^a_{4,t} \\ \epsilon^a_{8,t} \end{bmatrix} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, \begin{bmatrix} \sigma^2_{a,0} & 0 & 0 \\ 0 & \sigma^2_{a,4} & 0 \\ 0 & 0 & \sigma^2_{a,8} \end{bmatrix}\right). \tag{11}$$

where $0 < \rho_x < 1$. The model is estimated by likelihood-based methods on a sample of quarterly U.S. data from 1955:Q2 to 2006:Q4. The time series used for estimation are: real GDP, real consumption, real investment, real government expenditure, hours, utilization-adjusted total factor productivity, and the relative price of investment.

A variance decomposition shows that news shocks turn out to be very important. The first column of Table (1) shows the share of business-cycle variation in the level of four endogenous variables that is attributable to surprise shocks $\{\epsilon^a_{0,t}\}$, and the second column shows the share attributable to the news shocks $\{\epsilon^a_{4,t}\}$ and $\{\epsilon^a_{8,t}\}$ combined. We define business cycle frequencies as the components of the endogenous process with periods of 6 to 32 quarters, and we focus on variance decompositions over these frequencies to facilitate comparison across the different models in this section. Our results are consistent with the authors' original findings (see their Table V).

However, to determine the contribution of beliefs relative to fundamentals, we would like to construct a noise representation that is observationally equivalent to representation (11). One such noise representation is in the following proposition.

Proposition 6. *The representation of fundamentals and beliefs in system* (11) *is observationally equivalent to the noise representation*

$$\begin{aligned} x_t &= \rho_x x_{t-1} + \epsilon_t^x \\ s_{4,t} &= \epsilon_{t+4}^x + v_{4,t} \\ s_{8,t} &= \epsilon_{t+8}^x + v_{8,t}, \end{aligned} \qquad \begin{bmatrix} \epsilon_t^x \\ v_{4,t} \\ v_{8,t} \end{bmatrix} \overset{iid}{\sim} \mathcal{N}\left(0, \begin{bmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_{v,4}^2 & 0 \\ 0 & 0 & \sigma_{v,8}^2 \end{bmatrix}\right)$$

with the convention that $s_{0,t} \equiv x_t$, and where

$$\begin{aligned} \sigma_x^2 &= \sigma_{a,0}^2 + \sigma_{a,4}^2 + \sigma_{a,8}^2 \\ \sigma_{v,4}^2 &= \frac{1}{\sigma_{a,4}^2}\sigma_{a,0}^2(\sigma_{a,0}^2 + \sigma_{a,4}^2) \\ \sigma_{v,8}^2 &= \frac{1}{\sigma_{a,8}^2}(\sigma_{a,0}^2 + \sigma_{a,4}^2)(\sigma_{a,0}^2 + \sigma_{a,4}^2 + \sigma_{a,8}^2). \end{aligned}$$

We can use the noise representation in Proposition (6) with the same parameter estimates as before, and re-compute the variance decomposition of the seven observable variables in terms of fundamental shocks and noise shocks. This decomposition is unique by Proposition (4). There is no need to re-estimate the model because observational equivalence implies that the likelihood function is the same under both representations. The third column of Table (1) shows the share of variation attributable to fundamental shocks $\{\epsilon_t^x\}$, and the fourth column shows the share attributable to the noise shocks $\{v_{4,t}\}$ and $\{v_{8,t}\}$ combined.

The main result is that nearly all of the variation in output, consumption, investment, and hours is due to fundamentals. In terms of differences across the endogenous variables, it is interesting that real investment growth is affected the least by news shocks, but it is affected the most by noise shocks. At the same time, hours worked is affected the most by news shocks and the least by noise shocks. But based on the fact that 89% or more of the variation in every series is attributable to fundamental changes, we conclude that beliefs are not an important independent source of fluctuations through the lens of this model.

Variable	Surprise	News	Fundamental	Noise
Output	57	43	94	6
Consumption	50	50	95	5
Investment	55	45	89	11
Hours	16	84	97	3

Table 1: Variance decomposition (%) in the model of Schmitt-Grohé and Uribe (2012) over business cycle frequencies of 6 to 32 quarters. All variables are in levels. Estimated model parameters are set to their posterior median values.

4.2 Barsky and Sims (2012)

The second model comes from Barsky and Sims (2012). It was constructed to determine whether measures of consumer confidence change in ways that are related to macroeconomic aggregates because of noise (i.e. "animal spirits") or news. The main result of the paper is that changes in consumer confidence are mostly driven by news and not noise. Noise shocks account for negligible shares of the variation in forecast errors of consumption and output, while news shocks account for over half of the variation in long-horizon forecast errors. However, as we saw in Section (3.2), including both news and noise shocks in the same model can be problematic when it comes to isolating the importance of pure beliefs.

The model is a standard New-Keynesian DSGE model with real and nominal frictions: one period internal habit formation in consumption, capital adjustment costs (as opposed to investment adjustment costs, according to which costs are a function of the growth rate of investment rather than the level of investment relative to the existing capital stock), and monopolistic price setting with time-dependent price rigidity. Fundamentals comprise three different independent processes, which capture exogenous variation in non-stationary neutral productivity, government spending, and monetary policy. More details are in Online Appendix (B.2).

Agents only receive advance information about productivity, and not about the other two fundamentals. So it is only pure beliefs about productivity that can play an independent role in driving fluctuations. Letting x_t denote the growth rate of productivity (in deviations from its mean), and using our notation from Section (3.2), the process $\{x_t\}$ is assumed to follow a law of motion of the form:

$$\begin{aligned} x_t &= \mu_{t-1} + \eta_t \\ \mu_t &= \rho\mu_{t-1} + \epsilon_t^\mu \\ s_t &= \mu_t + \xi_t \end{aligned} \qquad \begin{bmatrix} \epsilon_t^\mu \\ \eta_t \\ \xi_t \end{bmatrix} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, \begin{bmatrix} \sigma_\mu^2 & 0 & 0 \\ 0 & \sigma_\eta^2 & 0 \\ 0 & 0 & \sigma_\xi^2 \end{bmatrix}\right), \tag{12}$$

where $0 < \rho < 1$. Barsky and Sims (2012) refer to ϵ_t^μ as a news shock, η_t as a surprise shock, and ξ_t as a noise (animal spirits) shock.[8] However, these definitions are not consistent with the definitions in our paper. To avoid any confusion we will use asterisks to indicate the terminology of Barsky and Sims (2012). So we refer to ϵ_t^μ as a news* shock, η_t as a surprise* shock, and ξ_t as a noise* shock.

The model is estimated by minimizing the distance between impulse responses generated from simulations of the model and those from estimated structural vector autoregressions. The vector autoregressions are estimated on quarterly U.S. data from 1960:Q1 to 2008:Q4. The time series used to estimate the vector autoregression are real GDP, real consumption, CPI inflation, a measure of the real interest rate, and a measure of consumer confidence from the Michigan Survey of Consumers (E5Y).

A variance decomposition shows that news* shocks are much more important than noise* shocks. The first column of Table (2) shows the share of business-cycle variation in the level of four endogenous variables that is attributable to surprise* shocks $\{\eta_t\}$, the second shows the share attributable to news* shocks $\{\epsilon_t^\mu\}$, and the third shows the share attributable to noise* shocks $\{\xi_t\}$. Due to the presence of exogenous government spending and monetary policy shocks, the rows do not sum to 100%; the residual represents the combined contribution of these two additional fundamental shocks. These results are consistent with the authors' original findings, which are stated in terms of the variance decompositions of forecast errors over different horizons, but across all frequency ranges (see their Table 3).

To properly isolate the independent contributions of beliefs, we would again like to construct a noise representation that is observationally equivalent to representation (12). The following proposition presents one such noise representation.

Proposition 7. *The representation of fundamentals and beliefs in system* (12) *is*

[8] While these authors refer to signal noise as "animal spirits," they also use the term "pure noise" to refer to statistical measurement error. We are only concerned with noise in the first sense.

observationally equivalent to the noise representation

$$
\begin{aligned}
x_t &= -\rho \frac{\sigma_\eta^2}{\sigma_\mu^2}\left[m_t - \left(\frac{1+\delta^2}{\delta}\right) m_{t-1} + m_{t-2}\right] \\
m_t &= (\rho+\delta)m_{t-1} - \rho\delta m_{t-2} + \epsilon_t^m \\
s_t &= m_t + v_t \\
v_t &= \delta v_{t-1} + \epsilon_t^v - \beta\epsilon_{t-1}^v \\
\begin{bmatrix} \epsilon_t^m \\ \epsilon_t^v \end{bmatrix} &\overset{iid}{\sim} \mathcal{N}\left(0, \begin{bmatrix} \delta\sigma_\mu^4/(\rho\sigma_\eta^2) & 0 \\ 0 & \delta\sigma_\xi^2/\beta \end{bmatrix}\right),
\end{aligned}
$$

with the convention that $s_{0,t} \equiv x_t$*, and where*

$$
\delta = \frac{1}{2\rho}\left(1+\rho^2+\frac{\sigma_\mu^2}{\sigma_\eta^2} - \left[\left(1+\rho^2+\frac{\sigma_\mu^2}{\sigma_\eta^2}\right)^2 - 4\rho^2\right]^{1/2}\right)
$$

$$
\beta = \frac{1}{2\rho}\left(1+\rho^2+\frac{\sigma_\mu^2(\sigma_\eta^2+\sigma_\xi^2)}{\sigma_\eta^2\sigma_\xi^2} - \left[\left(1+\rho^2+\frac{\sigma_\mu^2(\sigma_\eta^2+\sigma_\xi^2)}{\sigma_\eta^2\sigma_\xi^2}\right)^2 - 4\rho^2\right]^{1/2}\right).
$$

Using the noise representation in this proposition, we can re-compute the variance decomposition of the endogenous processes in terms of fundamental shocks and noise shocks. The fourth column of Table (2) shows the share of variation attributable to fundamental productivity shocks, and the fifth column shows the share attributable to productivity noise shocks. Again, the rows do not sum to 100% due to the presence of government spending and monetary policy shocks. Conceptually, the contribution of these shocks should also be included under the heading of fundamental shocks, but for comparison with the first three columns, we only include fundamental productivity shocks in the fourth column.

As in the model of Schmitt-Grohé and Uribe (2012), nearly all of the variation in output, consumption, investment, and hours is due to fundamentals. The contribution of noise shocks is larger than the contribution of noise* shocks, for all variables. However, the bulk of the contribution of news* shocks turns out to be due to fundamentals rather than noise.

To further highlight the difference between noise and noise* shocks, we plot in Figure (1) both the noise and noise* shares of consumption for different values of the standard deviation of noise shocks, σ_ξ. The striking result is that the noise share

Variable	Surprise*	News*	Noise*	Fundamental	Noise
Output	53	37	0	89	1
Consumption	61	34	1	89	9
Investment	40	43	1	80	4
Hours	62	14	0	75	3

Table 2: Variance decomposition (%) in the model of Barsky and Sims (2012) over business cycle frequencies of 6 to 32 quarters. All variables are in levels, and estimated parameters are set to their point-estimated values. The rows do not sum to 100% because of other non-technology fundamental processes. Asterisks refer to the authors' terminology.

of consumption is monotonically decreasing in σ_ξ. This means that removing noise* shocks altogether, by taking $\sigma_\xi \to 0$, actually leads to a *larger* noise share.

The intuition for this result is that the noise share of agents' forecasts (and their actions) is a hump-shaped function of the relative size of noise shocks. When noise shocks are very small, agents' signal is very precise, and noise shocks do not affect their forecasts very much. At the other extreme, when noise shocks are very large, agents' signal is very imprecise, so they rationally ignore it. The maximum contribution of noise shocks occurs is achieved for an intermediate size of these shocks.

In this model, noise is generated explicitly by the noise* shocks $\{\xi_t\}$, but also implicitly by the two shocks $\{\eta_t\}$ and $\{\epsilon_t^\mu\}$. The left panel of Figure (1) indicates that at the estimated parameter values, the combined level of noise is large enough that agents have already begun to pay less attention to the signal. By eliminating noise* shocks, the signal becomes more informative and agents to rely on it more. This allows the remaining noise coming from $\{\eta_t\}$ and $\{\epsilon_t^\mu\}$ to affect their forecasts to a greater extent.

4.3 Blanchard, L'Huillier, and Lorenzoni (2013)

The third model we consider comes from Blanchard et al. (2013), and was constructed "to separate fluctuations due to changes in fundamentals (news) from those due to temporary errors in agents' estimates (noise)."[9] The main quantitative result of

[9]This quotation is taken from the article's abstract (not printed with the article), which can be found on the AEA's website: `https://www.aeaweb.org/articles?id=10.1257/aer.103.7.3045`.

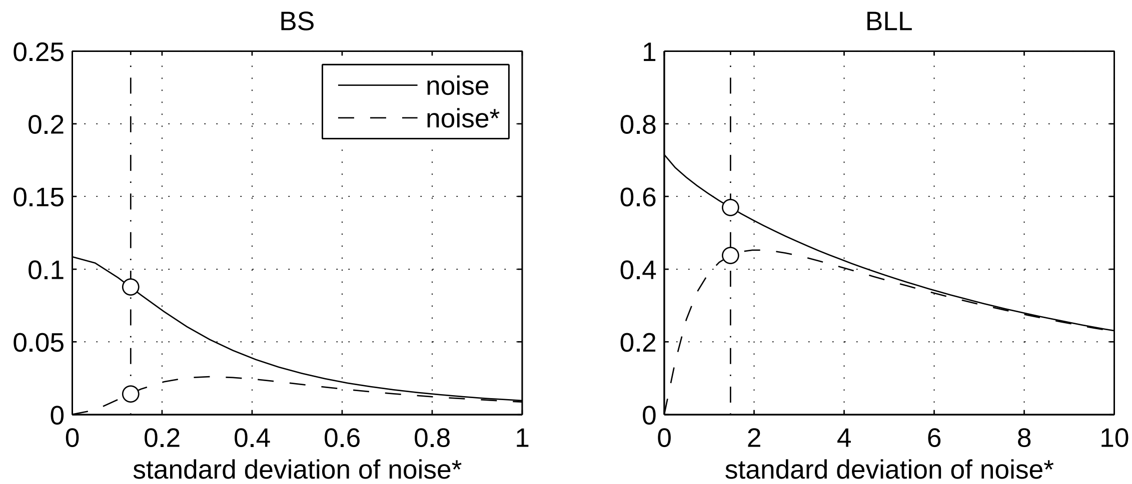

Figure 1: Noise versus noise*. This figure plots the noise and noise* shares of consumption over business cycle frequencies of 6 to 32 quarters, for different values of the variance of noise* shocks. The vertical dash-dotted line marks the estimated value of this parameter; the white circles correspond to the consumption noise shares reported in Tables (2) and (3). (The asterisk denotes the authors' original terminology.)

their paper is that noise shocks explain a sizable fraction of short-run consumption fluctuations. However, it turns out that what the authors call "noise" shocks do not fully isolate fluctuations due to temporary errors in agents' estimates. So we can investigate what this model implies about the importance of pure beliefs.

The model is a standard New Keynesian DSGE model with real and nominal frictions: one-period internal habit formation in consumption, investment adjustment costs, variable capital capacity utilization, and monopolistic price and wage setting with time-dependent price rigidities. Fundamentals comprise six different independent processes, which capture exogenous variation in non-stationary neutral productivity, stationary investment-specific productivity, government spending, wage markups, final good price markups, and monetary policy. For more details, see Online Appendix (B.3).

Agents only receive advance information about productivity, and not about the other five fundamentals. So it is only pure beliefs about productivity that can play an independent role in driving fluctuations. Let x_t denote the level of productivity, which is observed by agents in the economy, and let s_t denote the additional informative signal that agents receive. Then the processes $\{s_t\}$ and $\{x_t\}$ are assumed to evolve according to a system of the form

$$
\begin{aligned}
x_t &= \mu_t + \eta_t \\
s_t &= \mu_t + \xi_t \\
\Delta\mu_t &= \rho\Delta\mu_{t-1} + \epsilon_t^\mu \\
\eta_t &= \rho\eta_{t-1} + \epsilon_t^\eta
\end{aligned}
\qquad
\begin{bmatrix} \epsilon_t^\mu \\ \epsilon_t^\eta \\ \xi_t \end{bmatrix} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, \begin{bmatrix} \sigma_\mu^2 & 0 & 0 \\ 0 & \sigma_\eta^2 & 0 \\ 0 & 0 & \sigma_\xi^2 \end{bmatrix}\right), \tag{13}
$$

with the parameter restriction that $\rho\sigma_\mu^2 = (1-\rho)^2\sigma_\eta^2$.[10]

The authors refer to ϵ_t^μ as a permanent productivity shock, ϵ_t^η as a transitory productivity shock, and ξ_t as a noise shock. Taken together, they refer to ϵ_t^μ and ϵ_t^η as news shocks, because they are both correlated with future productivity. Again, because these definitions are not consistent with the ones in our paper, we will use asterisks to indicate the authors' terminology in contrast to ours.

The model is estimated using likelihood-based methods on a sample of quarterly U.S. data from 1954:Q3 to 2011:Q1. The time series used for estimation are real GDP, real consumption, real investment, employment, the federal funds rate, inflation as measured by the implicit GDP deflator, and wages.

A variance decomposition reveals that noise* shocks are important, especially for consumption. The first column of Table (3) shows the share of business-cycle variation in the level of output, consumption, investment, and hours that is attributable to news* shocks, $\{\epsilon_t^\mu\}$ and $\{\epsilon_t^\eta\}$, and the second column shows the share attributable to noise* shocks $\{\xi_t\}$. Due to the presence of the other five fundamental shocks, the rows do not sum to 100%; the residual represents the combined contribution of these additional fundamental shocks. These results are consistent with the authors' original findings, which are stated in terms of the variance decompositions of forecast errors over different horizons (see their Table 6).

However, to properly isolate the independent contribution of beliefs, we can derive a noise representation that is observationally equivalent to representation (13). The following proposition presents one such noise representation.

Proposition 8. *The representation of fundamentals and beliefs in system* (13) *is*

[10]This restriction ensures that productivity is a random walk.

observationally equivalent to the noise representation

$$
\begin{aligned}
x_t &= -\frac{\rho}{(1-\rho)^2}m_{t+1} + \frac{(1+\rho^2)}{(1-\rho)^2}m_t - \frac{\rho}{(1-\rho)^2}m_{t-1} \\
s_t &= m_t + v_t \\
m_t &= (1+2\rho)m_{t-1} - \rho(2+\rho)m_{t-2} + \rho^2 m_{t-3} + \epsilon_t^m \\
v_t &= 2\rho v_{t-1} - \rho^2 v_{t-2} + \epsilon_t^v - (\delta+\bar{\delta})\epsilon_{t-1}^v + \delta\bar{\delta}\epsilon_{t-2}^v \\
\begin{bmatrix} \epsilon_t^m \\ \epsilon_t^v \end{bmatrix} &\overset{iid}{\sim} \mathcal{N}\left(0, \begin{bmatrix} (1-\rho)^2\sigma_\mu^2 & 0 \\ 0 & \rho^2\sigma_\xi^2/(\delta\bar{\delta}) \end{bmatrix}\right),
\end{aligned}
$$

with the convention that $s_{0,t} = x_t$*, and where*[11]

$$
\delta = \frac{1}{2\rho}\left(1 + \rho^2 + \rho^{1/2}\frac{\sigma_\mu}{\sigma_\xi}i - \left[\left(1 + \rho^2 + \rho^{1/2}\frac{\sigma_\mu}{\sigma_\xi}i\right)^2 - 4\rho^2\right]^{1/2}\right).
$$

Using the noise representation in this proposition, we can re-compute the variance decomposition of the endogenous processes in terms of fundamental shocks and noise shocks. The fourth column of Table (3) shows the share of variation attributable to fundamental productivity shocks and the fifth column shows the share attributable to productivity noise shocks. Again, the rows do not sum to 100% due to the presence of fundamental processes other than productivity.

Variable	News*	Noise*	Fundamental	Noise
Output	34	22	26	29
Consumption	40	44	27	57
Investment	6	3	4	5
Hours	17	29	7	39

Table 3: Variance decomposition (%) in the model of Blanchard et al. (2013) over business cycle frequencies of 6 to 32 quarters. All variables are in levels, and estimated parameters are set to their posterior median values. The rows do not sum to 100% because of other non-technology fundamental processes.

In contrast to both the Schmitt-Grohé and Uribe (2012) and Barsky and Sims (2012) models, we find that a sizable fraction of the variation in output, consumption, and hours worked can be attributed to noise shocks. For example, nearly 60%

[11] In the definition of δ, $i \equiv \sqrt{-1}$ is the imaginary unit, and $\bar{\delta}$ denotes the complex conjugate of δ. Both $\delta + \bar{\delta}$ and $\delta\bar{\delta}$ are real numbers.

of the variation in consumption is due to noise shocks. This is more than 10% larger than the share Blanchard et al. (2013) originally attributed to independent fluctuations in beliefs. A result of similar magnitude is true for output and hours worked. It is interesting that for all variables in the table, noise about productivity is in fact more important than productivity itself. This cannot be seen from the original decomposition.

Moreover, the right panel of Figure (1) indicates that, as in the model of Barsky and Sims (2012), the noise share of consumption is maximized when the size of noise* shocks is zero. This emphasizes the fact that variance decompositions in terms of noise* shocks can be a misleading measure of the importance of pure beliefs.

4.4 Future Fundamentals

Across all three of the models we consider, fundamental shocks appear to play a relatively large role. This is especially true in the models of Schmitt-Grohé and Uribe (2012) and Barsky and Sims (2012). Are fundamentals important because agents are correctly anticipating future fundamental changes before they occur, or because they are merely reacting to past fundamental changes? To answer this question, we can use the decomposition in equation (10) to compare the importance of current and past fundamental shocks relative to future fundamental shocks.

As we described in Section (3.4), it is only possible to consider decompositions in terms of past, present, and future fundamental shocks if the endogenous process under consideration is stationary. Each of the three models in this section exhibits trend growth in output, consumption, and investment. One option would be to first de-trend these processes using a frequency-domain filter (e.g. band-pass filter) and then perform the past versus future decomposition. However, this would not be a good idea, because frequency filters of this type scramble up the dependence across time periods. As a result, they can introduce spurious dynamic relationships that are not part of the underlying economic model.

Therefore, we propose to use a flexible exponential de-trending procedure that preserves the distinction between past and future shocks. For a difference-stationary process $\{y_t\}$, we define the stochastic trend $\bar{y}_t(\theta)$ to be an exponential moving average of past values,

$$\bar{y}_t(\theta) = (1-\theta)y_{t-1} + \theta\bar{y}_{t-1}(\theta),$$

where $\theta \in [0,1)$. We then define the de-trended process $\{\tilde{y}_t(\theta)\}$ as $\tilde{y}_t(\theta) \equiv y_t - \bar{y}_t(\theta)$.

The parameter θ controls the extent to which the trend depends on past values. When $\theta = 0$, $\tilde{y}_t(\theta) = \Delta y_t$, so the de-trended process is the first-differenced version of the original process. As $\theta \to 1$, $\tilde{y}_t(\theta) \to y_t$. By varying θ, we can therefore consider a range of different hypotheses regarding the stochastic trend. Because the filter is one-sided for any θ (unlike most frequency-domain filters), it preserves the notions of past, present, and future defined by the original process $\{y_t\}$.[12]

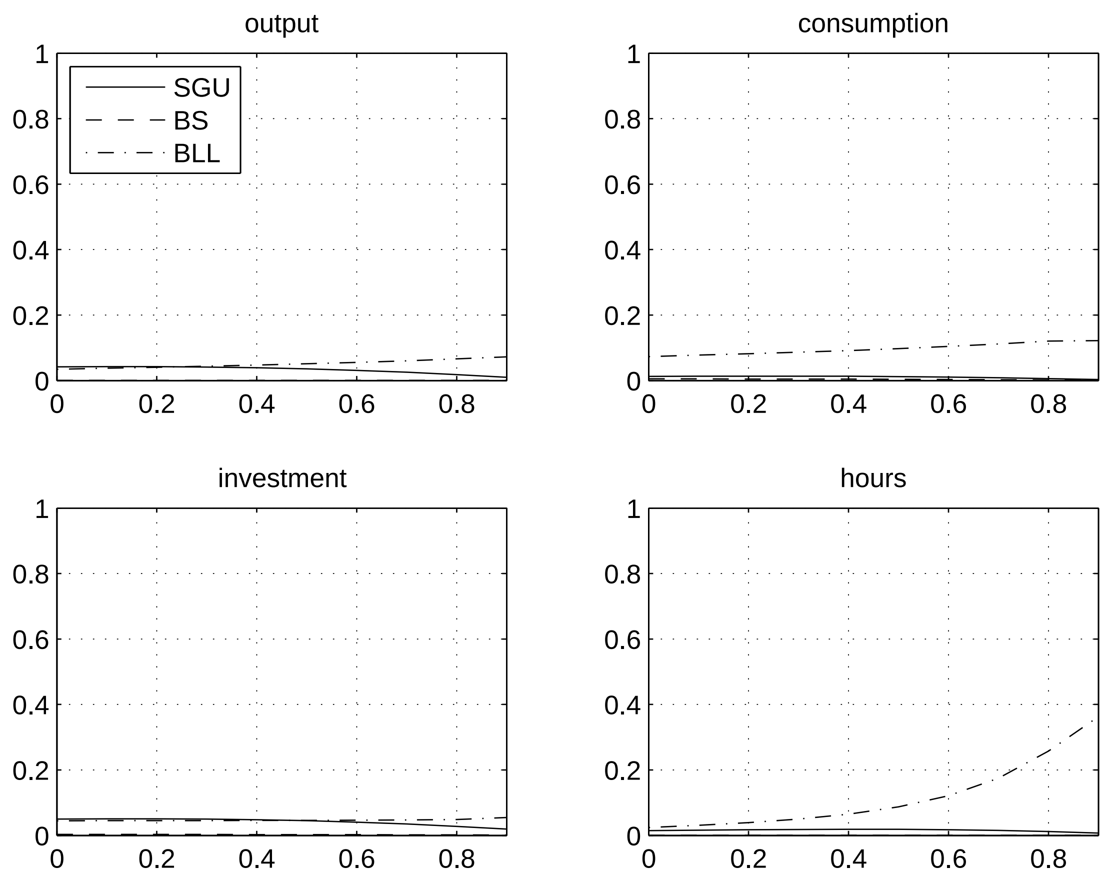

Figure 2: Fraction of the fundamental share due to future fundamental shocks, as a function of the de-trending parameter $\theta \in [0,1)$. $\theta = 0$ corresponds to a decomposition in (log) first differences, and $\theta \to 1$ corresponds to a decomposition in (log) levels.

Figure (2) plots the fraction of the fundamental share due to future fundamental shocks, for each of the three models considered in this section. We plot this fraction

[12] In this respect, our proposal is similar to the procedure recently suggested by Hamilton (2017).

for a range of different de-trended versions of the endogenous variables, corresponding to a different values of θ. As in the previous decompositions in this section, we focus only on fundamentals about which agents receive some advance information. That means that for the models of Barsky and Sims (2012) and Blanchard et al. (2013), we focus only on productivity, while in the model of Schmitt-Grohé and Uribe (2012) we include all seven fundamentals.[13]

The consistent result across all three models is that the bulk of the contribution of fundamentals comes from current and past — not future — fundamental shocks. In some cases, it is difficult to see that there are actually three lines in each subplot. This is because one of the lines is visually indistinguishable from zero. In the model of Barsky and Sims (2012), endogenous variables are the least sensitive to future shocks (on average across θ), followed by the model of Schmitt-Grohé and Uribe (2012) and then Blanchard et al. (2013).

This result may seem surprising considering that news* shocks are fairly important in all three models. How can it be that news* shocks are so important, but future fundamental shocks are not? As discussed in Section (3.4), two conditions must be satisfied for future fundamental shocks to be an important driver of current actions. First, agents' actions must depend to a sufficient degree on their expectations of future fundamentals. Second, agents must receive signals that provide substantial information about future fundamentals, above and beyond what they can infer from current and past fundamentals.

While the different models deliver the same conclusion regarding the importance of future fundamentals, they do so for very different reasons. The model of Schmitt-Grohé and Uribe (2012) is not very forward-looking, so the first condition is not met. This can be seen in the forecast error variance decompositions from Figure (3), which report the share of news shocks in explaining the variance of forecast errors in various endogenous variables as a function of the forecast horizon. Most of the contribution of news shocks occurs only *after* the 4-quarter-ahead and 8-quarter-ahead news shocks actually materialize. This is the reason the news shares look like step functions with jumps just after 4 and 8 quarters.[14]

The models of Barsky and Sims (2012) and Blanchard et al. (2013) are more

[13] Agents only receive advance information about productivity in the first two models, so including other non-productivity fundamentals would only reduce the future fundamental share.

[14] This same observation is made by Sims (2016).

forward-looking, but as we will discuss in more detail in Section (4.5) below, agents' signals do not provide substantial information about future fundamentals above and beyond what they can already infer from observing current and past productivity. That is, the second condition is not met. In the Barsky and Sims (2012) model, agents can already forecast future productivity very well based on current and past productivity realizations alone, and have relatively little need for the signal. In the Blanchard et al. (2013) model, agents rely on their signal much more, but that signal is quite noisy. Indeed, the fact that the signal is noisy is important for helping that model generate a large role for noise shocks.

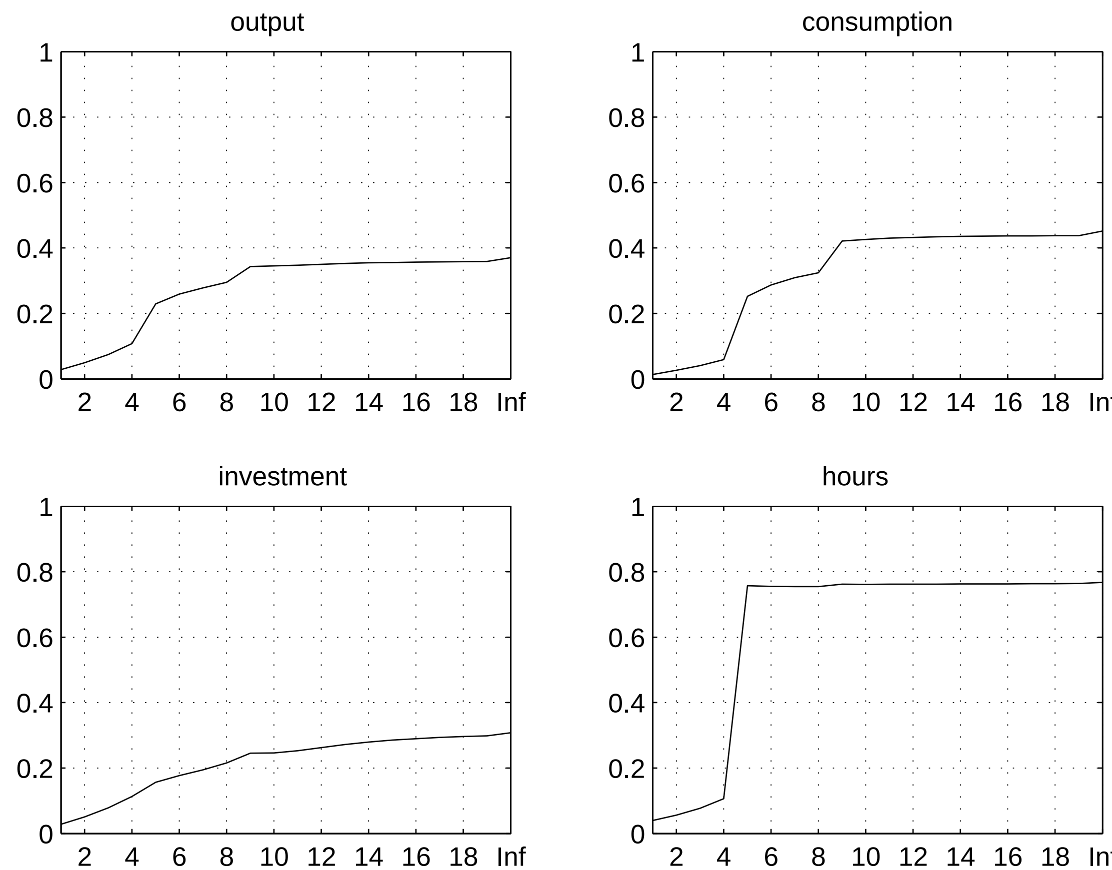

Figure 3: Forecast error variance decomposition in the model of Schmitt-Grohé and Uribe (2012). The line represents the share of news shocks in explaining the forecast error variance at each horizon. The decomposition is performed in growth rates ($\theta = 0$) and in terms of unconditional variances.

4.5 Understanding the Differences

How is it that the three models we consider in this section, especially the rather similar models of Barsky and Sims (2012) and Blanchard et al. (2013), deliver such different results regarding the importance of noise shocks? The existing literature has offered two separate explanations, one that emphasizes differences in information structures and another that emphasizes differences in physical economic environments. Beaudry and Portier (2014) argue that the key difference is that agents in the model of Blanchard et al. (2013) face a more difficult inference problem, which leads them to make larger and more persistent forecast errors. By contrast, Barsky and Sims (2012) argue that the key difference is that Blanchard et al. (2013) estimate a very accommodative monetary policy rule and a high degree of price rigidity, which work together to allow expectational shocks to propagate to the real side of the economy.

In this section we perform several exercises to better understand the reasons why these models disagree about the importance of noise shocks. We focus exclusively on the models of Barsky and Sims (2012) and Blanchard et al. (2013), since those are the most similar. We will argue that, at least with respect to these models, both the "right" information structure and the "right" physical environment are needed. Neither one alone is sufficient to generate an large role for noise shocks.

First, we present in Figure (4) some prima facie evidence that the disagreement is not just due to differences in information structure. If we replace the information structure in the Barsky and Sims (2012) model with the information structure from Blanchard et al. (2013), keeping all parameters at their original estimated values, the noise share of consumption does not change by much. This suggests that having the right information structure alone is not enough. However, having the right information structure is still important. If we replace the information structure in the Blanchard et al. (2013) model with the information structure from Barsky and Sims (2012), the noise share of consumption falls dramatically.

What is it about the information structure of Blanchard et al. (2013) that makes it amenable to a high consumption noise share? With this information structure, agents have to rely a good deal on their noisy signal in order to forecast future productivity. With the Barsky and Sims (2012) information structure, on the other hand, agents can forecast future productivity fairly well from the past history of productivity alone. As a result, they rely less on the noisy signal.

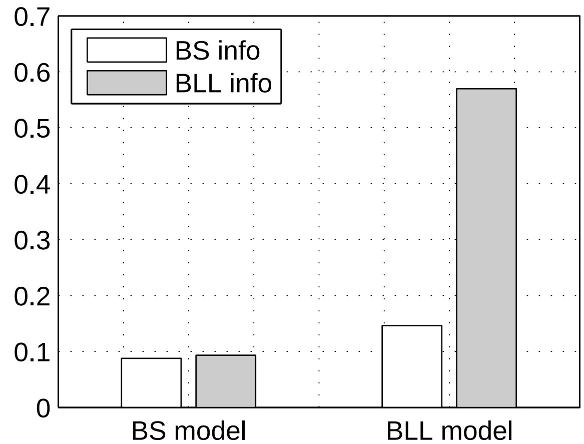

Figure 4: Swapping information structures. This figure plots the noise share of consumption over business cycle frequencies of 6 to 32 quarters, for each of four different combinations of model and information structure. All parameters are fixed at the authors' original estimated values.

The left panel of Figure (5) shows the standard deviation of productivity forecast errors in both models, with and without signals; larger forecast error standard deviations mean that agents are making larger mistakes. Agents with Blanchard et al. (2013) information have a harder time forecasting productivity, even with their signal. But the additional benefit from receiving the signal is larger for these agents compared to those with Barsky and Sims (2012) information. For those agents, forecasts with and without the signal are basically the same. The result of these differences can be seen in the right panel of Figure (5): long-horizon productivity forecasts are affected by noise to a much greater extent under Blanchard et al. (2013) information.

Of course, a large noise share in long-horizon productivity forecasts only translates into a large noise share in consumption if agents' consumption decisions depend on long-horizon forecasts to a sufficient degree. In New Keynesian models, one way to achieve this is to have very rigid prices and a relatively unresponsive monetary policy rule. When prices cannot adjust and nominal rates remain unchanged, real rates don't move much in response to changes in beliefs about the future (whether justified or not). Instead, permanent-income logic implies that consumption must respond. In fact, as Blanchard et al. (2013) show, in a limiting case of their model with perfectly rigid prices and a policy rule that does not respond to output, $c_t = \lim_{j\to\infty} E_t[a_{t+j}]$

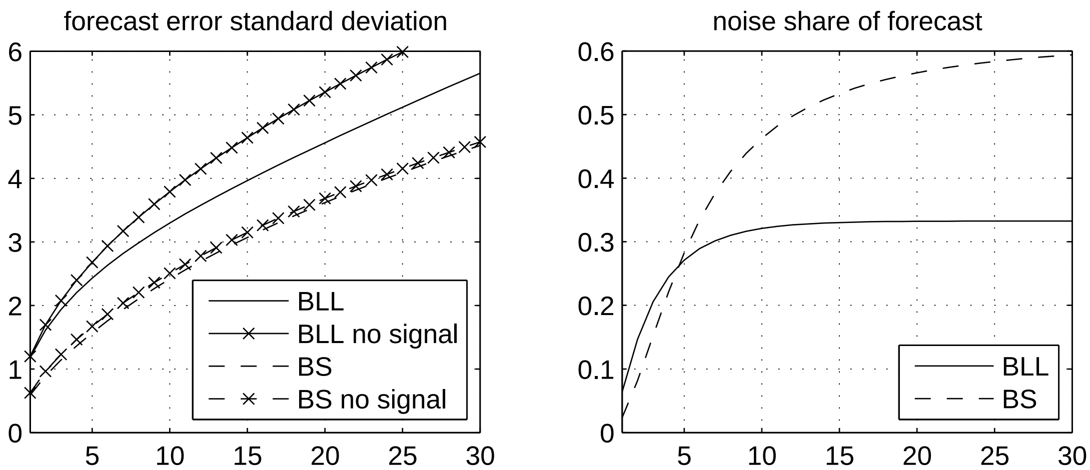

Figure 5: Comparing information structures. *Left*: standard deviation of j-quarter ahead productivity forecast errors, $a_{t+j} - E_t[a_{t+j}]$, with and without noisy signals. *Right*: fraction of j-quarter ahead productivity forecasts, $E_t[a_{t+j}]$, attributable to noise shocks over business cycle frequencies of 6 to 32 quarters.

up to a first order approximation. In this limiting case, current consumption only depends on agents' infinite-horizon forecast.

This is the line of reasoning emphasized by Barsky and Sims (2012). And as they suggest, it is true that for "extreme" parameter values it would be possible to achieve a higher consumption noise share. But that does not provide much by way of an explanation for why the *estimates* disagree. It may be possible for both models to achieve a higher noise share, but apparently only one estimated model actually does. Since the difference between possible and actual parameter configurations ultimately depends on the data, we consider that next.

To understand why one set of estimates delivers a high noise share while the other does not, we perform an estimation exercise. We first level the playing field by removing incidental differences between the two original estimation exercises (e.g. differences in the number of shocks, sample period, estimation procedure, and data series), and then re-estimate both models using maximum likelihood.[15] To build confidence that our changes are incidental, we first verify that we replicate the disagreement between the two models. The top entry in the first column of Table (4) reports that in our version of the Barsky and Sims (2012) model, the noise share of

[15]The details are described in Online Appendix (C).

consumption is 17%, and the bottom entry of the second column reports that in our version of the Blanchard et al. (2013) model, the noise share of consumption is 51%. This is close to what we found under the authors' original estimates.

	BS model	BLL model	BLL flex wage
BS info	17	35	16
	(3626)	(2041)	(2308)
BLL info	13	51	18
	(3654)	(2035)	(2273)

Table 4: Estimation results. This table reports the noise share of consumption over business cycle frequencies of 6 to 32 quarters, estimated under different combinations of model and information structure. The numbers in parentheses are the BIC values associated with each of the estimated models.

Next, we swap information structures and re-estimate both models. The resulting consumption noise shares are reported in the bottom entry of the first column and the top entry of the second column in Table (4). These results re-confirm the prima facie evidence we presented in Figure (4), that having the right information structure is important but not sufficient to generate a high noise share. In fact, the physical environment appears somewhat more important after estimation, since our version of the Blanchard et al. (2013) model still delivers a noise share of 35% when estimated with the Barsky and Sims (2012) information structure.

Of the remaining differences in the physical environments, it turns out that the most important from the perspective of the importance of noise is whether nominal wages are perfectly flexible or not. To show this, we estimate our version of Blanchard et al. (2013) model after removing the nominal wage rigidities; the results are in the third column of Table (4). When wages are flexible, the Blanchard et al. (2013) model delivers results that are much more in line with the Barsky and Sims (2012) model. Under either information structure, the noise share of consumption is less than 20%.

The reason noise shocks are so much more important when wages are sticky is that nominal wage rigidities help the model to generate positive business cycle co-movement in response to noise shocks. Figure (6) plots the impulse responses of output, consumption, investment, and hours in our estimated version of the Blanchard et al. (2013) model in response to a one standard deviation noise shock (baseline).

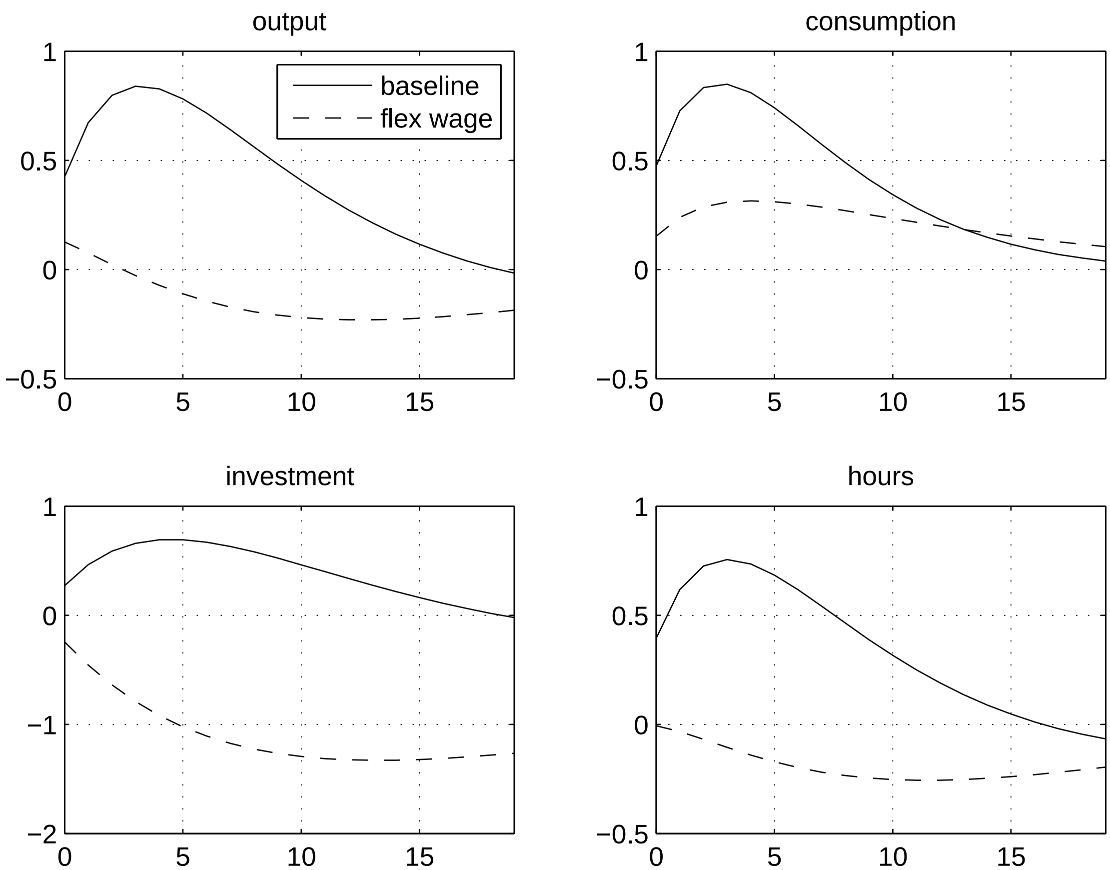

Figure 6: Impulse responses to a one standard-deviation noise shock. Here, the "flex wage" model is our the baseline estimated model when the wage rigidity parameter is taken to its flexible wage limit. All other parameters are the same in both models, and are equal to our baseline estimates (cf. Online Appendix (C)).

In addition, we also plot the responses of this model in the limit as wage rigidities vanish, keeping all other parameters at their estimated values (flex wage).[16] Only in the baseline case with sticky wages do all four aggregates increase together in response to a noise shock.

The noise shock makes agents (mistakenly) expect higher future productivity. This has two conflicting effects on hours worked. On the one hand, households feel wealthier and want to consume more and work less. On the other hand, the expected marginal products of capital and labor are higher, which makes firms want more of

[16]Alternatively, we could have plotted the responses in the estimated flexible wage version of the model used to generate the third column of Table (4); the same patterns hold.

both. When wages are flexible, the first effect dominates; households increase their wages enough that in equilibrium hours begin to fall. Since labor and capital are complementary, and there are investment adjustment costs, equilibrium investment falls on impact. When wages are sticky, however, the second effect dominates; households expect to be working more and therefore increase investment on impact. In either case, as time passes agents begin to learn that the shock was noise, and eventually reverse their actions and return back to the original steady state.

Lastly, we also report in Table (4) the Bayesian information criterion (BIC) values associated with each estimated model. Smaller values indicate better fit, adjusted for the number of free parameters. According to this criterion, we find that our version of the Blanchard et al. (2013) model fits better than our version of the Barsky and Sims (2012) model, regardless of the information structure or the nature of wage setting. The best fitting model is also the one in which noise shocks play the largest role.

5 Conclusion

Models with news and noise are intimately related. In fact, as we have argued here, there is a precise sense in which they are identical. The missing link is the observation that they are really just two different ways of describing the joint dynamics of exogenous economic fundamentals and agents' beliefs about them. This link is formalized by Theorem (1).

The observational equivalence of news and noise representations also raises important questions regarding the applicability of semi-structural empirical methods, such as structural vector autoregression (VAR) analysis, to models of belief-driven fluctuations. Some have argued that, while it may be possible to use these methods to analyze models with news shocks, it is never possible to use them to analyze models with noise shocks.[17] The reason is that news shocks can be expressed as a function of current and past observables (at least with a rich enough dataset), but noise shocks cannot. Noise representations are not "invertible." If an econometrician could recover noise shocks from current and past observables, so could the agents in the model. But then, the agents would rationally ignore the noise shocks and they would never affect any of the agents' actions.

[17] This is the main methodological argument of Blanchard et al. (2013).

How can news and noise representations be observationally equivalent if it is only possible to use semi-structural methods to analyze models with news shocks and not models with noise shocks? The answer, as it turns out, is that "invertibility" is not a necessary condition for using these methods. What matters is not whether shocks can be recovered from the current and past history of observables, but simply whether shocks can be recovered from the observables. This weaker condition, which we refer to as "recoverability," is satisfied in any noise representation if and only if it is satisfied in its observationally equivalent news representation. We discuss these issues in more detail in Chahrour and Jurado (2017).

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Appendix

Proof of Proposition (1)**.** Let $\hat{x}_t \equiv E_t[x_{t+1}]$ denote agents' expectations of the fundamental at date $t+1$ given their information at t. The observable processes are $\{x_t\}$ and $\{\hat{x}_t\}$. Expectations at horizons greater than one are spanned by these two processes.

The two representations are observationally equivalent if and only if the spectral density of the observable data is the same. In this case, the data consists of the process $\{d_t\}$ with $d_t \equiv (x_t, \hat{x}_t)'$ for all $t \in \mathbb{Z}$. Equating the spectral density implied by each representation,

$$f_d(\lambda) = \underbrace{\frac{1}{2\pi}\begin{bmatrix} \sigma_{a,0}^2 + \sigma_{a,1}^2 & \sigma_{a,1}^2 e^{-i\lambda} \\ \sigma_{a,1}^2 e^{i\lambda} & \sigma_{a,1}^2 \end{bmatrix}}_{\text{news}} = \underbrace{\frac{1}{2\pi}\begin{bmatrix} \sigma_x^2 & \left(\frac{\sigma_x^4}{\sigma_x^2+\sigma_v^2}\right) e^{-i\lambda} \\ \left(\frac{\sigma_x^4}{\sigma_x^2+\sigma_v^2}\right) e^{i\lambda} & \left(\frac{\sigma_x^4}{\sigma_x^2+\sigma_v^2}\right) \end{bmatrix}}_{\text{noise}}.$$

This equality holds if and only if the relations in Proposition (1) are satisfied. $\square$

Proof of Theorem (1)**.** To prove the first part, note that because $\mathcal{H}_{t-1}(\hat{x}) \subset \mathcal{H}_t(\hat{x})$ for all $t \in \mathbb{Z}$, it is possible to decompose $\mathcal{H}_t(\hat{x})$ into an orthogonal family of subspaces

$$\mathcal{H}_t(\hat{x}) = \bigoplus_{i=0}^{\infty} \mathcal{D}_{t-i}(\hat{x}),$$

where $\mathcal{D}_t(\hat{x}) \equiv \mathcal{H}_t(\hat{x}) \ominus \mathcal{H}_{t-1}(\hat{x})$ (cf. Rozanov, 1967, ch. 2). This means that $x_t \in \mathcal{H}_t(\hat{x})$ has a unique representation of the form

$$x_t = \sum_{i=0}^{\infty} w_{i,t-i}, \tag{14}$$

where the random variable $w_{i,t-i}$ represents the projection of x_t onto $\mathcal{D}_{t-i}(\hat{x})$ for any $i \in \mathbb{Z}_+$. By the orthogonality of the sequence of subspaces $\{\mathcal{D}_t(\hat{x})\}$, the process $\{w_{i,t}\}$ is uncorrelated over time for each $i \in \mathbb{Z}_+$.

While equation (14) looks almost like a news representation, it does not satisfy Definition (1) because it may be that $w_{i,t} \not\perp w_{j,t}$ for some $i \neq j$. Therefore, we use a version of the Gram-Schmidt orthogonalization procedure (cf. Luenberger, 1969, ch. 3) to transform these into an orthogonal sequence of shocks. Specifically, we define:

$$\epsilon_{0,t}^a = w_{0,t} \quad \epsilon_{i,t}^a = w_{i,t} - \sum_{j=0}^{i-1} \phi_{i,j} \epsilon_{j,t}^a \quad \text{for } i > 0,$$

where $\phi_{i,j} \equiv \langle w_{i,t}, \epsilon^a_{j,t} \rangle / \|\epsilon^a_{j,t}\|^2$ is the projection coefficient. Define the index set $\mathcal{I}_a$ to be the set of indices $i \in \mathbb{Z}_+$ such that $\|\epsilon^a_{i,t}\| > 0$. The collection of orthogonal shocks $\epsilon^a_{i,t}$ with $i \in \mathcal{I}_a$ is uniquely determined because the collection of input shocks $w_{i,t}$ with $i \in \mathbb{Z}_+$ is unique. Substituting the orthogonalized shocks into equation (14), x_t can be uniquely rewritten as:

$$x_t = \sum_{i=0}^{\infty} \sum_{j \leq i} \phi_{i,j} \epsilon^a_{j,t-i} = \sum_{j \in \mathcal{I}_a} \sum_{i=j}^{\infty} \phi_{i,j} \epsilon^a_{j,t-i} = \sum_{j \in \mathcal{I}_a} a_{j,t-j}.$$

The second equality rearranges the indexes on the double summation, and the third equality introduces the definition $a_{j,t-j} \equiv \sum_{i=j}^{\infty} \phi_{i,j} \epsilon^a_{j,t-i}$. The fact that the orthogonalized shocks are also uncorrelated over time implies that $a_{j,t} \perp a_{k,\tau}$ for all $j \neq k$ and $t, \tau \in \mathbb{Z}$. Therefore, this defines the unique news representation when agents' date-t information set is $\mathcal{H}_t(a)$.

What remains is to prove that the expectations implied by this news representation are in fact equal to $\{\hat{x}_{i,t}\}$ for any $i \in \mathbb{Z}$. Under rational expectations, the i-step ahead expectation of x_t at date t under the original noise representation is equal to the orthogonal projection of x_{t+i} onto $\mathcal{H}_t(\hat{x})$: $\hat{x}_{i,t} = E[x_{t+i} | \mathcal{H}_t(\hat{x})]$. By the uniqueness of orthogonal projections,

$$w_{i,t} = \hat{x}_{i,t} - \hat{x}_{i+1,t-1},$$

where $w_{i,t}$ was defined in equation (14). Therefore, $\mathcal{H}_t(w) = \mathcal{H}_t(\hat{x})$. But then because $\mathcal{H}_t(a) = \mathcal{H}_t(w)$ by construction, it follows that $\mathcal{H}_t(a) = \mathcal{H}_t(\hat{x})$. So expectations are indeed the same under both representations, $\hat{x}_{i,t} = E[x_{t+i} | \mathcal{H}_t(\hat{x})] = E[x_{t+i} | \mathcal{H}_t(a)]$, which completes the proof of the first part of the theorem.

To prove the second part, we start from the (unique) news representation and define

$$s_{i,t} \equiv a_{i,t} \quad \text{for all } i \in \mathcal{I}_a.$$

Because $\mathcal{H}(x) \subset \mathcal{H}(a)$, there exist unique elements $m_{i,t} \in \mathcal{H}(x)$ and $v_{i,t} \in \mathcal{H}(s) \ominus \mathcal{H}(x)$ such that $s_{i,t} = m_{i,t} + v_{i,t}$. This defines a noise representation when agents' date-t information set is $\mathcal{H}_t(s)$. What remains is to prove that the expectations implied by this noise representation are the same as the ones implied by the original news representation. Because $\mathcal{H}_t(s) = \mathcal{H}_t(a)$ by construction, and $\mathcal{H}_t(a) = \mathcal{H}_t(\hat{x})$ by the definition of a news representation, it follows that $\mathcal{H}_t(s) = \mathcal{H}_t(\hat{x})$ and therefore expectations are the same, $\hat{x}_{i,t} = E[x_{t+i} | \mathcal{H}_t(\hat{x})] = E[x_{t+i} | \mathcal{H}_t(s)]$. This completes the proof of the second part of the theorem. $\square$

Appendix for Online Publication

A Proofs

Proof of Proposition (2)**.** By rational expectations, $\mathcal{H}_t(x) \subseteq \mathcal{H}_t(\hat{x})$, and the fact that $\{\hat{x}_{i,t}\}$ forms a Gaussian system, it follows that agents' date$-t$ information is fully summarized by the random variables $\hat{x}_{i,\tau}$ across all i and $\tau \leq t$.

We can let $\mathcal{F}_t(\hat{x})$ denote the smallest σ-algebra generated by these variables. That is, $\mathcal{F}_t(\hat{x})$ is generated by cylinder sets of the form

$$\mathcal{A}_t \equiv \{\omega \in \Omega : \hat{x}_{i_1,t_1} \in \mathcal{G}_1, \ldots, \hat{x}_{i_n,t_n} \in \mathcal{G}_n\},$$

where Ω denotes the space of elementary events, $\mathcal{G}_1, \ldots, \mathcal{G}_n$ are arbitrary Borel sets in $\mathbb{R}$, the indices $t_1, \ldots, t_n$ assume values in the set $\{\tau \in \mathbb{Z} : \tau \leq t\}$, and the indices $i_1, \ldots, i_n$ assume values in $\mathbb{Z}$. By construction, the sequence of σ-algebras $\{\mathcal{F}_t(\hat{x})\}$ is uniquely determined by the forecasts $\{\hat{x}_{i,t}\}$. If two representations of fundamentals and beliefs imply the same dynamics for $\{\hat{x}_{i,t}\}$, they imply the same information structure $\{\mathcal{F}_t(\hat{x})\}$. Therefore, the conditional distribution function of any stochastic process $\{c_t\}$, such that c_t is measurable with respect to $\mathcal{F}_t(\hat{x})$ for each $t \in \mathbb{Z}$, is also the same. $\square$

Proof of Proposition (3)**.** As in the proof of Proposition (1), we can equate the spectral density of $\{d_t\}$ with $d_t \equiv (x_t, \hat{x}_t)'$ under each representation. In this case,

$$f_d(\lambda) = \underbrace{\frac{1}{2\pi}\begin{bmatrix} \sigma_\eta^2 + \sigma_\mu^2 & \left(\frac{\sigma_\mu^4}{\sigma_\mu^2+\sigma_\xi^2}\right) e^{-i\lambda} \\ \left(\frac{\sigma_\mu^4}{\sigma_\mu^2+\sigma_\xi^2}\right) e^{i\lambda} & \frac{\sigma_\mu^4}{\sigma_\mu^2+\sigma_\xi^2} \end{bmatrix}}_{\text{system (6)}} = \underbrace{\frac{1}{2\pi}\begin{bmatrix} \sigma_x^2 & \left(\frac{\sigma_x^4}{\sigma_x^2+\sigma_v^2}\right) e^{-i\lambda} \\ \left(\frac{\sigma_x^4}{\sigma_x^2+\sigma_v^2}\right) e^{i\lambda} & \frac{\sigma_x^4}{\sigma_x^2+\sigma_v^2} \end{bmatrix}}_{\text{noise}}.$$

This equality holds if and only if the relations in Proposition (3) are satisfied. $\square$

Proof of Proposition (4)**.** Consider an arbitrary noise representation of fundamentals and beliefs and an arbitrary endogenous process $\{c_t\}$. Using the structure of signals in a noise representation, $\mathcal{H}(s) = \mathcal{H}(m) \oplus \mathcal{H}(v)$. Because $v_{i,t} \in \mathcal{H}(s) \ominus \mathcal{H}(x)$ for all $i \in \mathcal{I}_s$, the uniqueness of orthogonal decompositions implies that $\mathcal{H}(m) = \mathcal{H}(x)$. Therefore, $\mathcal{H}(s) = \mathcal{H}(x) \oplus \mathcal{H}(v)$. Furthermore, the definition of noise shocks implies that $\mathcal{H}(\epsilon^v) = \mathcal{H}(v)$, so

$$\mathcal{H}(s) = \mathcal{H}(x) \oplus \mathcal{H}(\epsilon^v). \tag{15}$$

By the endogeneity of $\{c_t\}$ and the rationality of expectations, $c_t \in \mathcal{H}(s)$ for all $t \in \mathbb{Z}$. Combining this with Equation (15), it follows that for each c_t, there exist two unique elements $a_t \in \mathcal{H}(x)$ and $b_t \in \mathcal{H}(\epsilon^v)$ such that

$$c_t = a_t + b_t. \tag{16}$$

To consider variance decompositions at different frequencies, let $f_y(\lambda)$ denote the spectral density function of a stochastic process $\{y_t\}$. Then because $a_t \perp b_t$ for all $t \in \mathbb{Z}$, it follows that

$$f_c(\lambda) = f_a(\lambda) + f_b(\lambda),$$

where the functions $f_a(\lambda)$ and $f_b(\lambda)$ are uniquely determined by the processes $\{a_t\}$ and $\{b_t\}$. These functions in turn uniquely determine the share of the variance of $\{c_t\}$ due to noise shocks in any frequency range $\underline{\lambda} < \lambda < \overline{\lambda}$, which is equal to

$$\frac{\int_{\underline{\lambda}}^{\overline{\lambda}} f_b(\lambda) d\lambda}{\int_{\underline{\lambda}}^{\overline{\lambda}} f_c(\lambda) d\lambda}.$$

The share due to fundamentals is equal to one minus this expression. □

Proof of Proposition (5)**.** Beginning with the decomposition of $\mathcal{H}(s)$ in equation (15), we can further decompose $\mathcal{H}(x)$ uniquely into the sum of subspaces $\mathcal{D}_t(x) \equiv \mathcal{H}_t(x) \ominus \mathcal{H}_{t-1}(x)$,

$$\mathcal{H}(s) = \left(\bigoplus_{j=-\infty}^{\infty} \mathcal{D}_{t-j}(x) \right) \oplus \mathcal{H}(\epsilon^v).$$

By definition, each fundamental shock $\epsilon_t^x \equiv x_t - E[x_t | \mathcal{H}_{t-1}(x)]$ forms a basis in the space $\mathcal{D}_t(x)$. Since $c_t \in \mathcal{H}(s)$ for all $t \in \mathbb{Z}$, it follows that for each c_t, there exists a unique sequence of projection coefficients $\{\alpha_j\}$ such that

$$c_t = \sum_{j=-\infty}^{\infty} \alpha_j \epsilon_{t-j}^x + b_t,$$

where $\alpha_j \equiv E[c_t \epsilon_{t-j}^x]/\text{var}[\epsilon_t^x]$ and $b_t \perp \mathcal{H}(x)$. The shares of the variance of $\{c_t\}$ due to past, present, and future fundamental shocks are therefore uniquely determined, and are given by

$$\underbrace{\frac{\sum_{j=1}^{\infty} \alpha_j^2 \text{var}[\epsilon_t^x]}{\text{var}[c_t]}}_{\text{past}}, \quad \underbrace{\frac{\alpha_0^2 \text{var}[\epsilon_t^x]}{\text{var}[c_t]}}_{\text{present}}, \quad \text{and} \quad \underbrace{\frac{\sum_{j=-\infty}^{-1} \alpha_j^2 \text{var}[\epsilon_t^x]}{\text{var}[c_t]}}_{\text{future}}.$$

□

Proof of Corollary (1)**.** Consider an arbitrary noise representation of fundamentals and beliefs, and an endogenous process $\{c_t\}$. By the rationality of expectations, agents' best forecast of c_{t+h} as of date t is equal to

$$\hat{c}_{h,t} = E[c_{t+h}|\mathcal{H}_t(s)] = E[c_{t+h}|\mathcal{H}_t(\hat{x})].$$

Therefore, $\hat{c}_{h,t} \in \mathcal{H}_t(\hat{x})$. This means that the forecast error $w_t^h \equiv c_t - \hat{c}_{h,t-h}$ also satisfies $w_{h,t} \in \mathcal{H}_t(\hat{x})$. Therefore, $\{w_t^h\}$ is an endogenous process. By Proposition (4), the variance decomposition of this process in terms of noise and fundamentals is uniquely determined over any frequency range. Moreover, this result is true for any forecast horizon $h \in \mathbb{Z}$ because h was chosen arbitrarily. □

Lemma 1. *Any news representation in which each process $\{a_{i,t}\}$ is i.i.d. over time is observationally equivalent to a noise representation with $x_t \overset{iid}{\sim} \mathcal{N}(0, \sigma_x^2)$ and*

$$s_{i,t} = x_{t+i} + v_{i,t}, \quad v_{i,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma_{v,i}^2),$$

where $v_{i,t} \perp x_\tau$ and $v_{i,t} \perp v_{j,\tau}$ for any $i \neq j \in \mathcal{I}_s$ and $t, \tau \in \mathbb{Z}$, if and only if

$$\sigma_x^2 = \sum_{i \in \mathcal{I}_s} \sigma_{a,i}^2 \quad \textit{and} \quad \sigma_{v,i}^2 = \frac{1}{\sigma_{a,i}^2} \left(\sum_{j<i} \sigma_{a,j}^2 \right) \left(\sum_{j \leq i} \sigma_{a,j}^2 \right) \quad \textit{for all } i \in \mathcal{I}_s.$$

Proof of Lemma (1)**.** The proof of this result is a straightforward generalization of the proof of Proposition (1). In a news representation with i.i.d. news processes, the joint spectral density of any two forecast processes $\{\hat{x}_{j,t}\}$ and $\{\hat{x}_{k,t}\}$ for $j, k \in \mathbb{Z}_+$ is equal to

$$f_{j,k}(\lambda) = \frac{1}{2\pi} \sum_{m \in \mathcal{M}} \sigma_{a,m}^2 e^{-i\lambda(k-j)}, \tag{17}$$

where $\mathcal{M}$ is defined as the set of indices $m \in \mathcal{I}_a$ such that $m \geq |k-j| + j$. In a noise representation of the type described in the proposition, the joint spectral density of any two forecast processes $\{\hat{x}_{j,t}\}$ and $\{\hat{x}_{k,t}\}$ for $j, k \in \mathbb{Z}_+$ is equal to

$$\begin{aligned} f_{0,0}(\lambda) &= \frac{1}{2\pi} \sigma_x^2 \\ f_{j,k}(\lambda) &= \frac{1}{2\pi} \sigma_x^2 \left[1 + \frac{1/\sigma_x^2}{\sum_{m \in \mathcal{M}} 1/\sigma_{v,m}^2} \right]^{-1} e^{-i\lambda(k-j)} \quad \text{for } j, k > 0. \end{aligned} \tag{18}$$

Equating the densities in (17) with those in (18), and recursively solving for the parameters of the noise representation delivers the relations stated in the lemma. □

Proof of Proposition (6)**.** Define the composite shock

$$\epsilon_t^x \equiv \epsilon_{0,t}^a + \epsilon_{4,t-4}^a + \epsilon_{8,t-8}^a. \tag{19}$$

The process $\{\epsilon_t^x\}$ is i.i.d. because $\{\epsilon_{i,t}^a\}$ is i.i.d. for each $i \in \mathcal{I}_a \equiv \{0, 4, 8\}$. agents' date-$t$ information set in representation (11) is $\mathcal{H}_t(\epsilon^a)$. But based on this information set, equation (19) defines a news representation for $\{\epsilon_t^x\}$ with i.i.d. news processes. Therefore, we can apply Lemma (1) to the composite shock process, which gives the relations stated in the proposition. □

Proof of Proposition (7)**.** According to representation (12), the two signals observed by agents in the economy are $s_{0,t} \equiv x_t$ and $s_{1,t} \equiv \mu_t + \xi_t$. Because $\mathcal{H}(x) \subset \mathcal{H}(s)$, there exist two unique elements $m_t \in \mathcal{H}(x)$ and $v_t \perp \mathcal{H}(x)$ such that:

$$s_{1,t} = m_t + v_t \quad \text{for all } t \in \mathbb{Z}. \tag{20}$$

The spectral density of $\{x_t\}$ is non-zero for almost all $\lambda \in [-\pi, \pi]$, which means that $\{m_t\}$ can be obtained from $\{x_t\}$ by a linear transformation of the form

$$m_t = \int_{-\pi}^{\pi} e^{i\lambda t} \varphi(\lambda) \Phi_x(d\lambda), \tag{21}$$

where Φ_x is the random spectral measure of $\{x_t\}$, and $\varphi(\lambda) = f_{s,x}(\lambda)/f_x(\lambda)$ is the spectral characteristic of the transformation (cf. Rozanov, 1967, ch. 2). Using the restrictions in the system (12), we have

$$\varphi(\lambda) = \frac{\sigma_\mu^2 e^{i\lambda}}{\sigma_\mu^2 + \sigma_\eta^2 |1 - \rho e^{-i\lambda}|^2} = \frac{\delta \sigma_\mu^2 e^{i\lambda}}{\rho \sigma_\eta^2 |1 - \delta e^{-i\lambda}|^2},$$

where $|\delta| < 1$ is equal to the expression stated in the proposition. Combining $\varphi(\lambda)$ with the spectral density of $\{x_t\}$, we can use equation (21) to obtain the spectral density of $\{m_t\}$,

$$f_m(\lambda) = \frac{1}{2\pi} \frac{\delta \sigma_\mu^4}{\rho \sigma_\eta^2} \left| \frac{1}{(1 - \rho e^{-i\lambda})(1 - \delta e^{-i\lambda})} \right|^2.$$

This corresponds to the law of motion presented in the proposition. From equation (21), it follows that the fundamental process $\{x_t\}$ can be obtained from $\{m_t\}$ by a linear transformation with spectral characteristic $\varphi(\lambda)^{-1}$. Finally, the definition of the noise process $\{v_t\}$ in equation (20) implies that

$$f_v(\lambda) = \frac{1}{2\pi} \frac{\sigma_\mu^2 \sigma_\eta^2}{\sigma_\mu^2 + \sigma_\eta^2 |1 - \rho e^{-i\lambda}|^2} + \sigma_\xi^2 = \frac{1}{2\pi} \sigma_\xi^2 \frac{\delta}{\beta} \left| \frac{1 - \beta e^{-i\lambda}}{1 - \delta e^{-i\lambda}} \right|^2,$$

where $|\beta| < 1$ is equal to the expression stated in the proposition. Because $\mathcal{H}_t(s)$ is unchanged from representation (12) for all $t \in \mathbb{Z}$, it follows that $\hat{x}_{j,t} \equiv E[x_{t+j}|\mathcal{H}_t(s)]$ is also unchanged for any $j \in \mathbb{Z}$. Therefore these two representations are observationally equivalent. $\square$

Proof of Proposition (8)**.** A complication in this case is that both fundamentals and the signal of future fundamentals are difference-stationary, rather than stationary processes. As a result, they do not have finite second moments, which is a prerequisite for working in $\mathcal{L}^2$. We handle this complication by defining a new processes $\{\tilde{x}_t(\theta)\}$ as the solution to the difference equation

$$\tilde{x}_t(\theta) = \theta\tilde{x}_{t-1}(\theta) + \Delta x_t, \quad \text{for all } t \in \mathbb{Z}, \tag{22}$$

where Δ is the first-difference operator; $\Delta x_t \equiv x_t - x_{t-1}$. This new process is stationary for each value of $\theta \in [0, 1)$, and admits the spectral representation

$$\tilde{x}_t(\theta) = \int_{\pi}^{\pi} e^{i\lambda t}(1 - \theta e^{-i\lambda})^{-1}\Phi_{\Delta x}(d\lambda),$$

where $\Phi_{\Delta x}$ is the random spectral measure of $\{\Delta x_t\}$. We define a new signal process $\{\tilde{s}_t(\theta)\}$ analogously, derive the noise representation in terms of $\{\tilde{x}_t(\theta)\}$ and $\{\tilde{s}_t(\theta)\}$ for an arbitrary value of θ, and then take limits as θ approaches one from below.

The two signals observed by agents in the economy are $\tilde{s}_{0,t} \equiv \tilde{x}_t(\theta)$ and $\tilde{s}_{1,t} \equiv \tilde{s}_t(\theta)$. Because $\mathcal{H}(\tilde{x}) \subset \mathcal{H}(\tilde{s})$, there exist two unique elements $\tilde{m}_t(\theta) \in \mathcal{H}(\tilde{x})$ and $\tilde{v}_t(\theta) \perp \mathcal{H}(\tilde{x})$ such that:

$$\tilde{s}_t(\theta) = \tilde{m}_t(\theta) + \tilde{v}_t(\theta) \quad \text{for all } t \in \mathbb{Z}. \tag{23}$$

The spectral density of $\{\tilde{x}_t(\theta)\}$ is non-zero for almost all $\lambda \in [-\pi, \pi]$, which means that $\{\tilde{m}_t(\theta)\}$ can be obtained from $\{\tilde{x}_t(\theta)\}$ by a linear transformation of the form in equation (21), where in this case the spectral characteristic $\varphi(\lambda)$ is

$$\varphi(\lambda) = \rho\frac{\sigma_\mu^2}{\sigma_\eta^2}\frac{1}{|1 - \rho e^{-i\lambda}|^2}.$$

Combining this with the spectral density of $\{\tilde{x}_t(\theta)\}$, it follows that the spectral density of $\{\tilde{m}_t(\theta)\}$ is

$$f_{\tilde{m}}(\lambda; \theta) = \frac{1}{2\pi}\rho\frac{\sigma_\mu^4}{\sigma_\eta^2}\left|\frac{1}{(1 - \theta e^{-i\lambda})(1 - \rho e^{-i\lambda})^2}\right|^2.$$

By writing out the corresponding law of motion for $\{\tilde{m}_t(\theta)\}$ and then taking limits as θ approaches one from below, we obtain the law of motion for $\{m_t\}$ stated in the proposition. In a similar manner, we can obtain the law of motion for $\{x_t\}$ in terms of $\{m_t\}$ by using the spectral characteristic $\varphi(\lambda)^{-1}$. Finally, the definition of the noise process $\{\tilde{v}_t(\theta)\}$ in equation (23) implies that

$$f_v(\lambda;\theta) = \frac{1}{2\pi}\frac{\rho^2\sigma_v^2}{|\delta|^2}\left|\frac{(1-e^{-i\lambda})(1-\delta e^{-i\lambda})(1-\bar{\delta}e^{-i\lambda})}{(1-\theta e^{-i\lambda})(1-\rho e^{-i\lambda})^2}\right|^2,$$

where $|\delta| < 1$ is equal to the expression stated in the proposition. By letting θ tend to one from below, we obtain the law of motion for $\{v_t\}$. Because $\mathcal{H}_t(\tilde{s})$ is unchanged from representation (13) for each $\theta \in [0, 1)$ and all $t \in \mathbb{Z}$, it follows that

$$\hat{x}_{j,t} \equiv \lim_{\theta\to 1^-} E_t[\tilde{x}_{t+j}(\theta)|\mathcal{H}_t(\tilde{s})]$$

is also unchanged for any $j \in \mathbb{Z}$. Therefore these two representations are observationally equivalent. $\square$

B Quantitative Models

The following subsections provide a sketch of each of the three quantitative models considered in this paper. For more details, we refer the reader to the original articles and their supplementary material.

B.1 Model of Schmitt-Grohé and Uribe (2012)

A representative household chooses consumption $\{C_t\}$, labor supply $\{h_t\}$, investment $\{I_t\}$, and the utilization rate of existing capital $\{u_t\}$ to maximizes its lifetime utility,

$$E\left[\sum_{t=0}^{\infty}\beta^t\zeta_t\frac{(C_t - bC_{t-1} - \psi h_t^{\theta}S_t)^{1-\sigma}}{1-\sigma}\right],$$

subject to a standard sequence of constraints,

$$\begin{aligned} S_t &= (C_t - bC_{t-1})^{\gamma}S_{t-1}^{1-\gamma} \\ C_t + A_tI_t + G_t &= \frac{W_t}{\mu_t}h_t + r_tu_tK_t + P_t \\ K_{t+1} &= (1-\delta(u_t))K_t + z_t^I I_t\left[1 - \Phi\left(\frac{I_t}{I_{t-1}}\right)\right] \end{aligned}$$

Relative to the standard real business cycle model, this model features investment adjustment costs $\Phi(I_t/I_{t-1})$; variable capacity utilization, which increases the return on capital $r_t u_t$ at the cost of increasing its rate of depreciation through $\delta(u_t)$; one period internal habit formation in consumption, controlled by $0 < b < 1$; a potentially low wealth effect on labor supply, when $0 < \gamma < 1$ approaches its lower limit; and monopolistic labor unions, which effectively reduce the wage rate by an amount μ_t each period but rebate profits lump sum to the household through P_t.

Output is produced by a representative firm, which combines capital K_t, labor h_t, and a fixed factor of production L using a (potentially) decreasing returns to scale production function:

$$Y_t = z_t(u_t K_t)^{\alpha_k}(X_t h_t)^{\alpha_h}(X_t L)^{1-\alpha_k-\alpha_h}.$$

Market clearing requires that the goods and labor markets clear so that the aggregate resource constraint is satisfied: $C_t + A_t I_t + G_t = Y_t$. The seven fundamental processes capture exogenous variation in permanent and transitory neutral productivity $\{X_t, z_t\}$, permanent and transitory investment-specific productivity $\{A_t, z_t^I\}$, government spending $\{G_t\}$, wage markups $\{\mu_t\}$, and preferences $\{\zeta_t\}$.

B.2 Model of Barsky and Sims (2012)

A representative household chooses consumption $\{C_t\}$, labor supply $\{N_t\}$, and real holdings of riskless one-period bonds $\{B_t\}$ to maximize its lifetime utility,

$$E\left[\sum_{t=0}^{\infty}\beta^t\left(\ln(C_t - \kappa C_{t-1}) - \frac{N_t^{1+1/\eta}}{1+1/\eta}\right)\right]$$

subject to a standard flow budget constraint,

$$C_t + B_t = w_t N_t - T_t + (1 + r_{t-1})B_{t-1} + \Pi_t,$$

where r_t is the net nominally risk-free interest rate, w_t is the wage, T_t denotes lump-sum taxes, and Π_t is aggregate profits.

Final goods producers are competitive and take the price of intermediate goods, $P_t(j)$, as given and each have a production function of the form:

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}$$

Intermediate goods firms are monopolistically competitive and take the demands of final goods firms as given. They each have a production function of the form $Y_t(j) = A_t K_t(j)^\alpha N_t(j)^{1-\alpha}$. Each intermediate firm chooses a price for its own good, subject to the constraint that it will only be able to re-optimize its price each period with constant probability $1 - \theta$.

A continuum of capital producers produce new capital (to sell to intermediate firms) according to the production function

$$Y_t^k(\nu) = \phi\left(\frac{I_t(\nu)}{K_t(\nu)}\right) K_t(\nu),$$

where ϕ is an increasing and concave function. The aggregate capital stock evolves according to $K_t = \phi(I_t/K_t)K_{t-1} + (1-\delta)K_{t-1}$, where $0 < \delta < 1$ is the depreciation rate. The aggregate resource constraint is $Y_t = C_t + I_t + G_t$ (ignoring resources lost due to inefficient price dispersion). The monetary authority sets the one-period nominally risk-free rate of return according to a feedback rule of the (log-linear approximate) form:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\phi_\pi(\pi_t - \pi^*) + (1-\rho_i)\phi_y(\Delta Y_t - \Delta Y^*) + \varepsilon_{i,t}.$$

The three fundamental processes capture exogenous variation in permanent neutral productivity $\{A_t\}$, government spending $\{G_t\}$, and monetary policy $\{\varepsilon_{i,t}\}$

B.3 Model of Blanchard, L'Huillier, and Lorenzoni (2013)

Each household $j \in (0,1)$ chooses consumption $\{C_{j,t}\}$, investment $\{I_{j,t}\}$, nominally risk-free bond holdings $\{B_{j,t}\}$, and the rate of capital utilization $\{U_{j,t}\}$ to maximize its lifetime utility

$$E\left[\sum_{t=0}^{\infty} \beta^t \left(\ln(C_{j,t} - hC_{j,t-1}) - \frac{N_{j,t}^{1+\zeta}}{1+\zeta}\right)\right]$$

subject to a standard flow budget constraint. Each household is the monopoly supplier of labor type j, and chooses wages $\{W_{j,t}\}$ subject to the constraint that it can only re-optimize its wage each period with constant probability $1 - \theta_w$. Risk-sharing among households results in a common budget constraint, which is the same as if each household were to receive its pro rata share of the economy's total wage bill:

$$P_t C_t + P_t I_t + T_t + P_t \mathcal{C}(U_t)\bar{K}_{t-1} + B_t = R_{t-1}B_{t-1} + \Upsilon_t + \int_0^1 W_{j,t}N_{j,t}dj + R_t^k U_t \bar{K}_{t-1},$$

$$\bar{K}_t = (1-\delta)\bar{K}_{t-1} + D_t[1 - \mathcal{G}(I_t/I_{t-1})]I_t.$$

P_t is the price level, T_t is a lump sum tax, R_t is the gross nominally risk-free rate, Υ_t is aggregate profits, R_t^k is the capital rental rate, $0 < \delta < 1$ is the rate of depreciation, $\mathcal{G}(I_t/I_{t-1})$ represents investment adjustment costs, $\mathcal{C}(U_t)$ represents the marginal cost of increasing capacity utilization.

Final goods producers are competitive and take the price of intermediate goods as given, P_{jt}, and each have a production function of the form

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{1}{1+\mu_{pt}}} dj\right]^{1+\mu_{pt}}.$$

Intermediate goods firms are monopolistically competitive, each with a production function of the form $Y_{jt} = (K_{jt})^{\alpha}(A_t L_{jt})^{1-\alpha}$. Each intermediate firm chooses a price for its own good, subject to a $1-\theta_p$ probability of re-optimization each period.

Labor services are supplied to intermediate goods producers by competitive labor agencies that take wages as given, W_{jt}, and have a production function of the form

$$N_t = \left[\int_0^1 N_{jt}^{\frac{1}{1+\mu_{wt}}} dj\right]^{1+\mu_{wt}}.$$

Market clearing in the final goods market requires that $C_t + I_t + \mathcal{C}(U_t)\bar{K}_{t-1} + G_t = Y_t$, and in the labor market that $\int_0^1 L_{jt} dj = N_t$. Monetary policy follows the rule:

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\gamma_\pi \pi_t + \gamma_y \hat{y}_t) + q_t.$$

The six fundamental processes capture exogenous variation in permanent neutral productivity $\{A_t\}$, transitory investment-specific productivity $\{D_t\}$, price markups $\{\mu_{pt}\}$, wage markups $\{\mu_{wt}\}$, government spending $\{G_t\}$, and monetary policy $\{q_t\}$.

C Estimation Details

We estimate each model using quarterly data on log growth rates of real per-capita output, consumption, and hours, along with the log-levels of inflation and the nominal interest rate. The data span 1960:Q1 to 2017:Q2, with observations from 1954:Q3 to 1959:Q4 used to initialize the Kalman filter. Real variables are deflated by the implicit GDP price deflator, and put in per-capita terms using civilian non-institutional population age 16 and above. Consumption includes expenditure on non-durable goods and services. Inflation is measured by the log-change in the GDP deflator while the nominal interest rate is given by the effective federal funds rate. Data were downloaded from the St. Louis Federal Reserve Database, FRED, on October 25, 2017. Original downloaded data and data transformations can be seen in the online code accompanying this appendix.

For each model, we allow for shocks to the same four exogenous processes: productivity, noise, monetary policy, and government spending. The productivity and information blocks are described in Sections (4.2) and (4.3) of the main text. We allow both the government spending process and the exogenous component of monetary policy to follow first-order autoregressive laws of motion. We follow Barsky and Sims (2012) in fixing the parameters for the government spending process, $\rho_g = 0.95$ and $\sigma_g = 0.25$. (The original estimates of Blanchard et al. (2013) for these parameters are quite similar.) We estimate the parameters of the monetary policy process.

Since our approach targets five measured variables with only four fundamental shocks, we allow for small independent and identically distributed measurement error shocks in the observation of each series. In our estimation, we bound the variance of measurement error for each variable at 2.5% of that variable's unconditional variance in the data. Since this bound is attained in all six of our estimations, we do not report those parameters here. Our results are not sensitive to changing this bound.

For each combination of economic environment and information structure, we re-estimate the model using the method of maximum likelihood. Specifically, we search for the set of parameters that maximizes the log-likelihood function of the data using a robust global optimization routine that combines a genetic algorithm to discover many good initial parameter combinations with a hill-climbing routine that ensures our final answer is (at least) a local optimum. All of our results are robust to changing the random seed that underlies the initial points.

The following tables summarize our parameter estimates for each of the estimated models. In these tables, an asterisk indicates that the estimated parameter lies at, or very close, to the boundary of the parameter space, which we define before maximizing the likelihood function.

Economic parameters		BS info	BLL info
κ	habit	0.3145	0.0252
η	Frisch elasticity	4.9976*	4.9999*
γ	capital adj. cost	5.2093	3.4670
θ	Calvo price	0.9420	0.9309
ϕ_π	Taylor inflation	4.8073	4.8897
ϕ_y	Taylor output growth	0.0042	0.0484
ρ_i	interest smoothing	0.5072	0.4074
σ_i	s.d. policy shock	0.1343	0.1629
ρ_{ε_i}	autocorr. policy	0.9989*	0.9892
BS info parameters			
ρ	autocorr. growth	0.9231	
σ_μ	s.d. growth shock	0.2190	
σ_η	s.d. surprise* shock	0.8716	
σ_ξ	s.d. noise* shock	0.0001*	
BLL info parameters			
ρ	autocorr. growth		0.8581
σ_μ	s.d. growth shock		1.3638
σ_ξ	s.d. noise* shock		0.0010

Table 5: Estimated parameters for alternative versions of the Barsky and Sims (2012) model.

	Economic parameters	BS info	BLL info	BS info + flex wage	BLL info + flex wage
h	habit	0.8145	0.7066	0.6209	0.4726
ζ	inverse Frisch elasticity	0.2000*	0.2000*	0.2000*	0.2000*
ξ	cap. util. cost	0.5023	0.0079	0.4628	0.0010*
χ	inv. adj. cost	15.0000*	15.0000*	15.0000*	15.0000*
θ_p	Calvo price	0.8771	0.8645	0.8654	0.8929*
θ_w	Calvo wage	0.9013	0.8708	-	-
γ_π	Taylor inflation	4.2259	3.8640	1.0100*	1.0100*
γ_y	Taylor output gap	0.0010*	0.0010*	0.4742	0.4135
ρ_r	interest smoothing	0.4686	0.4540	0.2813	0.1127
σ_q	s.d. policy shock	0.3394	0.2792	0.3089	0.4112
ρ_q	autocorr. policy	0.9990*	0.9990*	0.9425	0.9481
	BS info parameters				
ρ	autocorr. growth	0.9166		0.8980	
σ_μ	s.d. growth shock	0.2553		0.4430	
σ_η	s.d. surprise* shock	0.9762		1.2358	
σ_ξ	s.d. noise* shock	0.0001*		0.0001*	
	BLL info parameters				
ρ	autocorr. growth		0.8911		0.8068
σ_μ	s.d. growth shock		1.3025		1.8876
σ_ξ	s.d. noise* shock		0.0001*		0.0001*

Table 6: Estimated parameters for alternative versions of the Blanchard et al. (2013) model.