

ANALYSIS QUALIFYING EXAM SYLLABUS, 2013

Real Analysis (text: *Real Analysis: Modern Techniques and their applications* by G. Folland)

- Measure and Integration: General measures, outer measures, Lebesgue measure, measurable functions, integration, product measures, Fubini's theorem.
- Signed measures: Hahn and Jordan decomposition theorems, Radon-Nikodym derivatives, complex measures.
- Functional Analysis: Hilbert spaces, Banach spaces, Hahn-Banach theorem, Baire-Category theorem.
- L^p Spaces: Holder inequality, Minkowski inequality, L^p - L^q duality.
- Radon Measures: Riesz representation theorem
- Haar measure

Complex Analysis (text: *Complex Analysis* by L. Ahlfors)

- The field of complex numbers, the algebraic operations and n -th roots, their geometric interpretation, conjugation, modulus and argument, stereographic projection.
- Complex functions, limits and continuity, analytic functions, Cauchy-Riemann equations, basics on harmonic functions, conjugate harmonic functions, polynomials, rational functions.
- Sequences and series of complex functions, uniform convergence, power series, radius of convergence, Abel's theorem.
- Elementary functions: exponential, logarithm, trigonometric and hyperbolic trig functions.
- Analytic functions as mappings: length, angles and areas, arcs and closed curves, conformal mappings, analytic functions in regions, linear fractional transformations, the cross ratio, definition of a Riemann surface, elementary Riemann surfaces in the sense of Ahlfors Chapter 3 (at the end.)
- Complex line integrals, rectifiable arcs, Cauchy's theorem for a rectangle, Cauchy's theorem for a disc, homology of 1-cycles, simple-connectedness, winding numbers, locally exact differentials, Cauchy's theorem in a general region (with proof), multiply connected regions, homology bases, periods.
- Analytic functions defined by line integrals, Cauchy's integral formulas for the values of an analytic function and its derivatives, Morera's theorem, Cauchy's integral estimates, Liouville's theorem, Fundamental Theorem of Algebra.
- Local properties of analytic functions: classification of singularities, Taylor's Theorem, zeros and poles, the local geometry of an analytic map, the Maximum Principle,

Schwarz's lemma, residues, the Residue Theorem, the Argument Principle, Rouché's theorem, use of residues to evaluate definite integrals.

- Further properties of harmonic functions: the mean-value property, Poisson's formula, Schwarz's Theorem on harmonic functions, Schwarz's reflection principle for harmonic functions and for analytic functions.
- Weierstrass's Theorem on a series of analytic functions, Hurwitz's theorem on non-vanishing of a limit function, Taylor Series, Laurent Series.
- Partial fraction representation of meromorphic functions on \mathbb{C} with given principle parts and examples, infinite product representation of entire functions with given zeros and examples, the Gamma function and its basic properties, Stirling's formula.
- Equicontinuity, normal families of functions, Arzela's theorem, normal families of analytic functions characterized by local boundedness.
- The Riemann mapping theorem and its proof, behavior of the mapping at the boundary, use of the Schwarz reflection principle, conformal mappings of polygons, the Schwarz-Christoffel formula, its application to rectangles.
- Periodic functions: singly periodic functions, Fourier series, doubly periodic (elliptic) functions: the module of periods M , unimodular transformations, the canonical basis for the period module, poles and residues of an elliptic function, the Weierstrass theory of elliptic functions (\wp -, ζ - and σ - functions, the differential equation, how \mathbb{C}/M is an algebraic curve), the modular function (denoted by Ahlfors $\lambda(\tau)$.)
- Analytic continuation, analytic function elements, the Riemann surface of a general analytic function, the monodromy theorem, Picard's theorem.