Search Frictions and Efficiency in Decentralized Transport Markets

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Abstract

In this paper we explore efficiency and optimal policy in decentralized transportation markets that suffer from search frictions, such as taxicabs, trucks and bulk shipping. We illustrate the impact of two externalities: the well-known thin/thick market externalities and what we call pooling externalities. We characterize analytically the conditions for efficiency, show how they translate into efficient pricing rules, as well as derive the optimal taxes for the case where the planner is not able to set prices. We use our theoretical results to explore welfare loss and optimal policy in dry bulk shipping. We find that the constrained efficient allocation achieves 6% welfare gains, while the first-best allocation corresponding to the frictionless world, achieves 14% welfare gains. This suggests that policy can achieve substantial gains, even if it does not alleviate search frictions, e.g. through a centralizing platform. Finally, we demonstrate that simple policies designed to mimic the optimal taxes perform well.

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1 Introduction

The transportation sector is indispensable for economic growth and social development. With both people and goods covering larger distances than ever before, the sector has witnessed a newfound and growing interest by policymakers. In many transport markets interactions between carriers and customers occur in a decentralized manner. This is for instance the case in the markets for taxis, trucks and bulk shipping among others. In these markets, search frictions may result in unrealized trade, thus posing the question of whether the sector operates efficiently, and if not, what policies can be employed.

In this paper, we study efficiency and optimal policy in decentralized transport markets. In particular, taking search frictions as given, we dissect the sources of inefficiency that distort the market equilibrium allocation and characterize analytically the conditions for (constrained) efficiency. We derive both the set of efficient pricing rules, as well as optimal taxes, that the planner can employ to achieve the efficient allocation. We then use our theoretical results to explore welfare loss and optimal policy in dry bulk shipping.

Our starting point is a dynamic spatial search model for decentralized transport markets, in the spirit of Lagos (2000) and Brancaccio et al. (2020a) (henceforth BKP). There is a network of locations at different distances to each other. In each location, carriers (e.g. ships, taxis) and customers (e.g. exporters, passengers) participate in a random matching process. Carriers that get matched, transport their customer to their desired destination for a price, and restart there. Carriers that do not get matched, decide whether to wait at their current location or travel empty elsewhere to search. Customers that get matched, obtain a valuation from arriving at their destination, while customers that do not, wait another period. Finally, every period, potential customers decide whether to start searching for a carrier, as well as their destination, thus replenishing the customer pool seeking transportation. We do not impose restrictions on the price setting mechanism, nor the structure of the demand system, in order to nest different modes of transportation (in taxis prices are regulated, while in shipping they are negotiated).

Studying efficiency in this setup is not straightforward due to the dynamic nature of decision-making and the spatial network; yet, we are able to obtain analytical results. In particular, we provide a characterization of the market equilibrium allocation that allows us to directly compare it to the constrained efficient one, i.e., the allocation where the planner cannot directly overcome the constraints imposed by search frictions. This comparison allows us to identify the different types of externalities that can result in this setting and derive conditions for each one to be internalized.

We show that search frictions create two types of externalities. First, as is well-known, they generate thin/thick market externalities: when choosing whether to search, agents affect the matching probabilities faced by other agents both in the same and in the opposite side of the market. If agents' search decisions do not internalize this effect, the overall *number* of agents searching may be distorted away from the efficient one.

Thin/thick market externalities are internalized in equilibrium if and only if the private returns from searching are equal to the social returns. This amounts to the so-called "Hosios (1990) conditions" on surplus sharing: these conditions, which are well-known to characterize efficiency in search models of labor markets with homogeneous workers, require the share of the surplus which is appropriated by agents on each side of the market to be equal to the elasticity of the matching function with respect to the same side.

Second, search frictions generate what we call "pooling externalities": a carrier needs to restart its search once it has dropped off the customer at their destination; however customers may fail to internalize the impact of their destination choice on the distribution of carriers over space. Hence the *composition* of customers searching for transport to different destinations, and thus the composition of trips realized, may be distorted away from the efficient one.

Customers internalize pooling externalities in equilibrium if and only if prices are such that, carriers receive the same surplus regardless of the customer they match with. This condition for efficiency replicates the no-arbitrage condition obtained in a frictionless world, where competition among carriers ensures that prices coincide with the opportunity cost of each trip, until in equilibrium carriers are indifferent among different customers. In our frictional setup, separate markets for each customer type (e.g. for each destination) are missing: if carriers could compete for a specific customer type, so that heterogeneous customers were not pooled together, in equilibrium carriers would be indifferent across customers. Absent this condition, the price paid by customers for a trip does not reflect its social value and the share of destinations with high social value is too low in equilibrium.

The two efficiency conditions combined characterize analytically the efficient pricing rule, which is useful if a central authority is able to set prices, as in the case of taxicabs. In many markets, however, the planner is not able to directly control prices, but he may be able to impose taxes or subsidies. We show that, when prices are set via Nash bargaining, the planner can achieve efficiency using these instruments and we derive their optimal values. We consider a tax on searching carriers, a tax on searching customers and a tax on trips. The search tax (on either side) is set to equate the private value of an additional agent searching to its social value and forces agents to internalize the thin/thick market externalities. Taxes on trips are used to target the pooling externalities. The optimal trip tax depends on the deviation of the trip's social surplus from the average social surplus across destinations, so that a customer entering a route with social surplus higher (lower) than the average is subsidized (taxed). The planner can restore efficiency by taxing trips and one of the two searching sides.

We apply these results to study empirically the dry bulk shipping industry. A number of features of dry bulk shipping, such as information frictions and port infrastructure, can hinder the matching of ships and exporters. We begin by leveraging a rich dataset of vessel movements and bulk shipping prices to document the presence of search frictions. In particular, we propose a novel test to argue that these frictions lead to unrealized potential trade. The test is based on weather shocks at sea that exogenously shift ship arrivals at port: in a frictionless world, in regions with more ships than exporters, the change in the number of ships should not affect matching, since the short side of the market is always matched. Here, instead, matches are indeed affected by weather shocks. In addition, the law of one price does not hold: shipping prices exhibit substantial dispersion within a time-origin-destination triplet, also consistent with frictions. Finally, at any given time, in most countries there are simultaneous arrivals of empty ships that load and departures of empty ships, even though ships are homogeneous. This also suggests wastefulness.

We proceed to estimate the model using the dry bulk shipping data. We use the estimates obtained in BKP for the ship parameters; and we introduce external trade data to estimate the exporter parameters, including the bargaining coefficients.

We first test whether the observed equilibrium is efficient by checking whether the conditions for efficiency hold in the data. Perhaps not surprisingly, we find that neither condition is satisfied, suggesting that the market does not operate efficiently.

Next, we turn to the welfare analysis. We compare, (i) the market equilibrium; (ii) the constrained efficient outcome, i.e., the market allocation under the efficient prices; (iii) the first-best, i.e., the allocation in a world without search frictions. We find that total welfare in the market equilibrium allocation is 6.3% lower than the constrained efficient allocation and 14% lower than the frictionless benchmark. Moreover, trade volume and net trade value are substantially higher under constrained efficiency (by 13.5% and 11.7% respectively), as well as under the first-best (by 36.5% and 42.7% respectively). This suggests that the externalities have a substantial impact on world trade.

These results relay an important message: under the optimal policy, the market is able to achieve about 44% of the first-best welfare gains. If the first-best allocation is interpreted as a benchmark achievable by a platform that eradicates search frictions (like Uber/Lyft), our results imply that policy-makers can improve efficiency substantially through simple policies, such as taxes or subsidies, without resorting to some form of centralization. This is important because centralization may be infeasible in practice, or may come with market power if provided by private firms.

We next delve into the different role of the two externalities. We find that both externalities contribute to distorting the equilibrium market allocation. However, they have a qualitatively different impact on the economy. Thin/thick market externalities have a large impact on trade volume, as they distort the numbers of searching agents and therefore the total number of matches formed. In contrast, pooling externalities have an impact on trade value, as they distort the composition of exports and favor destinations with low social value.

In order to correct thin/thick market externalities, we find that exporters need to be subsidized: at the observed equilibrium, the entry of an additional exporter has a substantial positive externality on matching rates, but prices remain too high to achieve the socially efficient number of exporters. On the other hand, correcting the pooling externalities requires subsidizing routes with high social surplus; i.e., trips with high exporter revenue, short trip duration and/or trips to destinations with a high option value for the ship (many customers, high value matches, low travel costs to other locations etc).

Finally, although the efficient prices and optimal taxes that restore efficiency have known expressions, they may not be feasible to implement in practice, either because the planner does not have access to all instruments; or because they may be too complex or computationally challenging. We thus consider simple policies that are designed to mimic the optimal taxes but may be more easily implementable. We find that a destination-specific tax (customs tax) performs relatively well, as it can achieve 44% of welfare gains achieved under the optimal taxes. In contrast, a tax that is a function of distance achieves no welfare gains. This suggests that a pricing scheme based on distance, such as the one used in taxis, is far from efficient. Explicitly targeting origin and destination is essential in order to correct for the different sources of externalities.

Related Literature

This paper broadly relates to four strands of literature: search and matching; transportation; international trade; and industry dynamics.

First, our work naturally relates to the search and matching literature; see Diamond (1982), Mortensen (1982) and Pissarides (1985) for the canonical DMP labor market model, as well as Rogerson et al. (2005) for a survey.[1] More specifically, our paper relates to the literature on efficiency of search models. Hosios (1990) considers efficiency in markets with random search and Nash bargaining. He shows that these markets are generically inefficient and derives the well-known "Hosios condition" that restores efficiency. Acemoglu and Shimer (1999) show that the Hosios condition does not guarantee efficiency when firms are heterogeneous, and that efficiency is achieved in models of directed search and posted wages (see also Moen, 1997). In a follow-up paper, Acemoglu (2001) shows that with random search and heterogeneous firms, labor market policies, such as unemployment benefits or minimum wages, can potentially improve welfare.[2]

Our paper extends the existing literature by further investigating the externalities that distort the equilibrium allocation and deriving explicit conditions for efficiency with random search and heterogeneous agents on one side of the market. In particular, our main theorem shows that efficiency is restored if two conditions hold: first, the Hosios (1990) conditions that ensures that the *number* of matches in every market is optimal;[3] second, a no arbitrage/indifference condition that ensures that the *composition* of

[1] In addition to labor markets, the search and matching framework has been used in other decentralized markets, see Weill (2020) for over-the-counter financial markets, Burdett and Coles (1997) and Shimer and Smith (2000) for marriage markets and Lagos and Wright (2005) for monetary exchange.

[2] More recently, Bilal (2020) extends the results in Acemoglu (2001) to the spatial context and shows that there are too many low productivity jobs in high productivity locations.

[3] Extending the literature, we also show how to restore efficiency in the case of decreasing returns to scale in the matching technology. In addition, compared to Hosios (1990), we consider a more general setup with a network of interconnected markets and without the Nash bargaining assumption on pricing.

matches in every market is optimal. This latter condition is novel. In addition, we derive theoretically the set of policy instruments (both efficient pricing rules, and taxes/subsidies) that can restore efficiency.

Second, our paper contributes to a large and rapidly growing literature on transportation. Our model builds on Lagos (2000) (and Lagos, 2003). More recently, Frechette et al. (2019) and Buchholz (2020) study search frictions and regulation frictions in NYC taxicabs. In particular, Buchholz (2020) relies on a similar framework, and numerically implements tariff pricing changes in order to explore whether welfare improvements can be achieved. Frechette et al. (2019) investigate the welfare impact of changes in the number of active medallions, as well as the introduction of an "Uber-like" platform.

In addition, a series of papers study different aspects of efficiency in urban transportation; for instance, Shapiro (2018) and Liu et al. (2019) explore the welfare improvements from different centralizing formats; Ghili and Kumar (2020) investigate demand and supply imbalances in ride-sharing platforms; Ostrovsky and Schwarz (2018) focus on carpooling and self-driving cars; Kreindler (2020) studies optimal congestion pricing; Cao et al. (2018) explore competition in bike-sharing platforms; while several papers study platform pricing (e.g. Bian, 2020, Ma et al., 2018, Castillo, 2019).

Third, since our empirical application involves oceanic transportation, we relate to a literature studying transportation in the context of international trade; e.g. Koopmans (1949), Hummels and Skiba (2004), Fajgelbaum and Schaal (2019), Asturias (2018), Brooks et al. (2018), Cosar and Demir (2018), Holmes and Singer (2018), Wong (2018), Allen and Arkolakis (2019), Ducruet et al. (2019), Lee et al. (2020) and BKP. We also relate to a literature in international trade studying the role of frictions, such as Eaton et al. (2016), and Krolikowski and McCallum (2018) who consider search frictions between importers and exporters and Allen (2014) who investigates information frictions. In our prior work, BKP, we explore the role of the transportation sector in world trade and spell out the impact of endogenous trade costs. Although we rely on the model setup and empirical strategy employed there, our focus here is entirely different, as this paper considers search frictions and efficiency.

Finally, we relate to the literature on industry dynamics (Hopenhayn, 1992, Ericson and Pakes, 1995), while our empirical methodology borrows from the literature on the estimation of dynamic setups (e.g. Rust, 1987, Bajari et al., 2007, Pakes et al., 2007; applications include Ryan, 2012, Collard-Wexler, 2013, Kalouptsidi, 2014, 2018). Related to this paper, a small literature lying in the intersection of search and industry dynamics, has explored trading frictions in decentralized markets (e.g. Gavazza, 2011, 2016 for real assets and Brancaccio et al., 2020b for over-the-counter financial markets).

The rest of the paper is structured as follows: Section 2 presents the model. Section 3 provides the efficiency and optimal policy results. Section 4 describes the dry bulk shipping industry and the data used, presents evidence for search frictions and outlines the estimation of the model. Section 5 presents our welfare analysis. Section 6 concludes. The (Online) Appendix contains all proofs and additional theoretical results, evidence on random search in shipping, details on the estimation procedure, data and computation, as well as additional tables and figures.

2 Model

We introduce a model of decentralized transport markets that focuses on the interaction between carriers (e.g. ships, taxis, trucks) and customers (e.g. exporters, passengers).

2.1 Environment

Time is discrete and the horizon is infinite. There are I locations, $i \in \{1, 2, ..., I\}$. There are two types of agents: customers and carriers. Both are risk neutral and have discount factor β. Variables with superscript s refer to carriers and e to customers, in line with our empirical exercise of ships and exporters.

There is a measure S of homogeneous carriers in the economy.[4] At the beginning of every period, a carrier is either in some region i, or traveling full or empty, from some location i to some location j. Carriers at i can either search or remain inactive. The per-period payoff of staying inactive is set equal to 0 at each location, while searching carriers incur a per-period search cost c_i^s. Carriers traveling from i to j incur a per period traveling cost c_{ij}^s. The duration of a trip between location i and location j is stochastic: a traveling carrier arrives at j in the current period with probability d_{ij}, so that the average duration of the trip is $1/d_{ij}$.[5]

[4]A constraint on the fleet size is consistent with most applications of interest, and can be due to either regulatory constraints (e.g. fixed number of medallions) or time to build.

[5]It is straightforward to have deterministic trip durations instead. Our specification, however, preserves tractability and allows for some variability e.g. due to weather/traffic shocks, without affecting the steady state properties of the model.

Customers can only be delivered to their destination by carriers and each carrier can carry at most one customer. Following the search and matching literature, we model the number of matches that take place every period in region i, m_i, using a matching function, whereby

$$m_i = m_i(s_i, e_i) \leq \min\{s_i, e_i\}$$

where s_i is the measure of unmatched carriers in region i and e_i is the number of unmatched customers in region i. $m_i(s_i, e_i)$ is increasing and concave in both arguments. We allow for the possibility that $m_i(s_i, e_i) < \min\{s_i, e_i\}$ creating the potential for unrealized trade: two agents searching in the same location might fail to meet, due to impediments such as information frictions or physical constraints. As Petrongolo and Pissarides (2001) note, "[...] the matching function [...] enables the modeling of frictions [...] with a minimum of added complexity. Frictions derive from information imperfections about potential trading partners, heterogeneities, the absence of perfect insurance markets, slow mobility, congestion from large numbers, and other similar factors."

Since search is random, the probability according to which customers searching at i meet a carrier is $\lambda_i^e = m_i(s_i, e_i)/e_i$, which is the same for all customers. Similarly the probability according to which carriers searching at i meet a customer is $\lambda_i^s = m_i(s_i, e_i)/s_i$.

When a carrier and a customer meet, if they both accept to match, the customer pays a price τ_{ij} upfront and the carrier begins its trip immediately to j. We are agnostic for now as to what the price mechanism is in the market. This allows us to nest several different practices in different markets; for instance prices are fixed by regulation in taxicabs, while prices are bilaterally negotiated in bulk shipping.

Carriers that remain unmatched decide whether to stay in their current region or travel empty to a different region where they wait for a match. Customers that remain unmatched wait in their current region. Inactive carriers restart the following period in the same region.

Finally, every period, at each location i, a large pool of potential customers decide whether to enter and search for a carrier, in order to be transported to a destination $j \neq i$, subject to an entry cost κ_{ij}. Denote by e_{ij} the endogenous measure of customers in i who search for transportation to j. The total measure of customers searching at location i is $e_i = \sum_{j \neq i} e_{ij}$, while G_{ij} is the share of demand routed from i to j, i.e.,

$$\forall i,j: G_{ij} \equiv e_{ij}/e_i.$$

Once they have entered, customers pay a per-period search cost c^e_{ij}.[6]

Upon matching with a carrier, customers obtain a valuation from being transported from origin i to destination j. We model customer valuations via the function, $w: \mathbb{R}_+^{I\times I} \to \mathbb{R}_+^{I\times I}$, where $w_{ij}(q)$ is the valuation of the marginal customer on route ij, and q is the matrix with typical element q_{ij} denoting the quantity transported every period (i.e. the measure of accepted matches) on route ij. This can be thought of as an inverse demand curve for transportation services, before customer entry and search costs. For example, consider customers with heterogeneous valuations for transportation (e.g. passengers looking for taxis with different value of time): when q_{ij} matches are formed on route ij, $w_{ij}(q)$ describes the valuation of the q_{ij}-th (i.e. the marginal) consumer entering route ij.[7] As a simpler case, if valuations are homogeneous so that all customers obtain w_{ij} on route ij, the marginal customer naturally also obtains w_{ij}.

Consistent with this interpretation, w is the gradient of a concave and differentiable function $W: q \mapsto \mathbb{R}_+$, which is interpreted as the total customer value from transportation, as a function of the total quantity transported, q.

2.2 Behavior and equilibrium

We consider the steady state of our industry model. In a steady state equilibrium, customers and carriers respond optimally to their expectations of the endogenous market variables, which are consistent with agents' behavior (rational expectations) and constant over time. Market clearing can be achieved either by price adjustment, or rationing (captured by the adjustment of the meeting probabilities faced by the agents at each location), or a mix of these two mechanisms.[8]

We begin by describing the optimal behavior of carriers and customers facing a *given* tuple $\tau, \lambda^s, \lambda^e, G$. We then endogenize these variables to achieve market clearing.

[6] By convention we set $c^e_{ii} = 0$, $\kappa_{ii} = 0$, and $c^s_{ii} = 0$.

[7] In addition, valuations might depend on quantities through general equilibrium effects; for instance in a general equilibrium trade model, traded goods' prices depend on traded quantities.

[8] For instance in the case of taxicabs, where prices are regulated, wait times determine the entry decisions of customers and equilibrate demand and supply. In the case of bulk shipping, both prices and wait times clear the market.

Carrier optimality Let V_{ij}^s denote the value of a carrier that begins the period traveling from i to j (empty or loaded), V_i^s the value of a carrier that begins the period in location i, and U_i^s the value of a carrier that remained unmatched at i at the end of the period. In everything that follows, we suppress the dependence of the value functions on the state of the economy, given our focus on a steady state; in Appendix E we consider out of steady state dynamics. Given prices and meeting rates we have,

$$V_{ij}^s = -c_{ij}^s + d_{ij}\beta V_j^s + (1 - d_{ij})\,\beta V_{ij}^s \tag{1}$$

In words, a carrier that is traveling from i to j: pays the per period cost of traveling c_{ij}^s; with probability d_{ij} it arrives at destination j where it begins unmatched with value V_j^s; with the remaining probability $1 - d_{ij}$, the carrier does not arrive and keeps traveling.

A carrier that starts the period in region i obtains:

$$V_i^s = \max\left\{-c_i^s + \lambda_i^s \sum_{j\neq i} G_{ij} \max\left\{\tau_{ij} + V_{ij}^s, U_i^s\right\} + (1 - \lambda_i^s)\,U_i^s, \beta V_i^s\right\}.$$

In words, if the carrier decides to search, it pays the per period search cost c_i^s; with probability $\lambda_i^s G_{ij}$ it meets a customer heading to destination j, in which case it accepts to match if and only if its value, inclusive of the price τ_{ij}, is higher than its outside option, U_i^s, otherwise it receives the outside option U_i^s. With probability $1 - \lambda_i^s$, the carrier does not meet a customer and receives the value of being unmatched. If the carrier remains inactive, it obtains a flow payoff of zero and restarts the following period at the same location.

Defining the carrier meeting surplus as,

$$\Delta_{ij}^s = \max\left\{\tau_{ij} + V_{ij}^s - U_i^s, 0\right\} \tag{2}$$

the carrier's value V_i^s can be written as follows,

$$V_i^s = \max\left\{-c_i^s + \lambda_i^s \sum_{j\neq i} G_{ij}\Delta_{ij}^s + U_i^s, \beta V_i^s\right\}. \tag{3}$$

Next, if the carrier remains unmatched, it chooses where to search: it can either keep waiting at i or travel empty to another location. The unmatched carrier value function is equal to:

$$U_i^s = \max_j V_{ij}^s \tag{4}$$

where we set $V_{ii}^s \equiv \beta V_i^s$. In words, if the carrier stays in region i, at the beginning of the next period it will be waiting at i; otherwise if the carrier chooses another region $j \neq i$ it begins its trip towards j.

Having defined all the carrier value functions, we now characterize their optimal behavior in terms of the three decisions they make (whether to search in the beginning of the period, whether to accept a match and where to search if unmatched at the end of the period). First, carriers search only when it is profitable to do so, so that from equation (3),

$$s_i > 0 \rightarrow V_i^s = -c_i^s + \lambda_i^s \sum_{j \neq i} G_{ij} \Delta_{ij}^s + U_i^s. \tag{5}$$

Second, carriers do not reject any match yielding a strictly positive surplus, and they accept only matches yielding a positive surplus:[9]

$$q_{ij} < \lambda_i^s s_i G_{ij} \rightarrow \Delta_{ij}^s = 0 \tag{6}$$

$$q_{ij} > 0 \rightarrow \Delta_{ij}^s = \tau_{ij} + V_{ij}^s - U_i^s \tag{7}$$

Third, carriers choose where to search when unmatched. Denote by b_{ij} the measure of carriers who decide to relocate empty from i to j (and let b_{ii} be the measure that decides to remain in i); optimality requires that b_{ij} is positive only if option j achieves the maximum value across all possible choices:

$$b_{ij} > 0 \rightarrow U_i^s = V_{ij}^s. \tag{8}$$

Finally, it must be the case that, whenever the measure of inactive carriers is greater than zero, there is some location where some carriers would rather not search at all. Since $q_{ij} + b_{ij}$ carriers depart from i towards j every period, traveling for $1/d_{ij}$ periods on average, the total measure of active carriers in

[9]Note that we allow for the number of meetings (i.e., $m_{ij} \equiv G_{ij} m_i = s_i \lambda_i^s G_{ij}$) to be higher than the number of realized matches (i.e., q_{ij}) since agents can reject a match upon meeting. However, in equilibrium generically no rejections occur, since a customer would not enter the market, only to have a match with a carrier rejected later. Therefore $q_{ij} = m_{ij}$.

steady state is given by $\sum_{ij}(q_{ij}+b_{ij})/d_{ij}$ (setting $d_{ii}=1$). Hence this condition can be written as,

$$\sum_{ij}\frac{q_{ij}+b_{ij}}{d_{ij}}<S\to\exists i:\,V_i^s=0. \tag{9}$$

Customer optimality We now turn to the value functions of customers; we begin with existing customers and then consider customer entry. If a customer meets a carrier they can either agree to form a match, in which case the customer pays price τ_{ij} and receives its valuation, or the customer can revert to its outside option and stay unmatched. Hence the meeting surplus of the marginal customer with valuation $w_{ij}(q)$ is given by,

$$\Delta_{ij}^e=\max\left\{w_{ij}(q)-\tau_{ij}-\beta U_{ij}^e,0\right\}, \tag{10}$$

where U_{ij}^e is its value of searching for a carrier in i with destination j:

$$\begin{aligned}U_{ij}^e&=-c_{ij}^e+\lambda_i^e\left(\Delta_{ij}^e+\beta U_{ij}^e\right)+(1-\lambda_i^e)\,\beta U_{ij}^e\\&=-c_{ij}^e+\lambda_i^e\Delta_{ij}^e+\beta U_{ij}^e.\end{aligned} \tag{11}$$

In words, the customer pays the cost c_{ij}^e while searching; then with probability λ_i^e it meets a carrier and receives the meeting surplus on top of its outside option, while with the remaining probability it remains unmatched and receives its outside option.

Similarly to carriers, optimality requires that the customer does not reject any match yielding a strictly positive surplus, and that it accepts only matches yielding positive surplus:

$$q_{ij}<\lambda_i^e e_{ij}\to\Delta_{ij}^e=0 \tag{12}$$

$$q_{ij}>0\to\Delta_{ij}^e=w_{ij}(q)-\tau_{ij}-\beta U_{ij}^e \tag{13}$$

Finally, the measure of customers searching on each route ij is pinned down by a free entry condition for the marginal customer:

$$U_{ij}^e-\kappa_{ij}\leq 0,\ \text{with equality if } e_{ij}>0. \tag{14}$$

We adopt the convention that customers in i choosing i do not enter, and normalize the payoff in that case to zero.

Feasible allocations An allocation for the transportation economy consists of a tuple (s, E, q, b) where $s = [s_1, \ldots, s_I]$ denotes the measure of carriers waiting in each region, $E \in \mathbb{R}_+^{I \times I}$, with typical element e_{ij}, denotes the measure of customers waiting for transport on each route ij, $q \in \mathbb{R}_+^{I \times I}$ denotes the measure of new matches formed on each route, and $b \in \mathbb{R}_+^{I \times I}$ denotes the measure of carriers departing empty on each route. Equivalently, we will sometimes denote an allocation by (s, e, G, q, b), where $e = [e_1, \ldots, e_I] = \left[\sum_j e_{1j}, \ldots, \sum_j e_{Ij}\right]$ denotes the measure of customers waiting in each region. This will be useful when we want to emphasize the implications of search behavior on the share of waiting customers in location i headed towards j, captured by the matrix G. The first triplet (s, e, G) captures search activities; hence we will often refer to it as a search allocation, in contrast with the last pair (q, b), capturing transportation and relocation activities.

An allocation is feasible if it satisfies: (i) the set of constraints defining a steady state, equations (15) and (16) below; (ii) the total fleet constraint, equation (17) below; and (iii) the constraints on the transported quantities imposed by the meeting technology, equation (18) below. Thus, the following feasibility constraints must hold:

$$\sum_j (q_{ij} + b_{ij}) = \sum_j (q_{ji} + b_{ji}), \ \forall i \tag{15}$$

$$s_i = \sum_j (q_{ij} + b_{ij}), \ \forall i \tag{16}$$

$$\sum_{ij} \frac{q_{ij} + b_{ij}}{d_{ij}} \leq S \tag{17}$$

$$q_{ij} \leq m_i (s_i, e_i) G_{ij}, \ \forall ij \tag{18}$$

The first set of constraints requires that the measure of carriers departing at any given location i equals the measure of arrivals, so that flows into i are equal to the flows out of i. Equation (16) requires that the measure of carriers searching at each location (s_i) must equal those that will be matched ($\sum_j q_{ij}$), those that will be unmatched and choose to remain (b_{ii}) and those that will be unmatched and decide to travel elsewhere empty ($\sum_{j\neq i} b_{ij}$). Equation (17) imposes the fleet constraint. Finally, constraints (18) require that the number of matches does not exceed the number of meetings between carriers and customers.

Equilibrium We now define the equilibrium of the transportation economy, which is a tuple (s, E, q, b, τ) consisting of an allocation (s, E, q, b) and prices, $\tau \in \mathbb{R}_+^{I\times I}$.

Definition 1. An outcome (s, E, q, b, τ) is a steady state equilibrium under prices τ if:

1. (s, E, q, b) satisfies the feasibility constraints (15)-(18).
2. (s, q, b) satisfies the carrier optimality conditions (1)-(9) given τ, λ^s and G.
3. E, q satisfies the customer optimality and free entry conditions (10)-(14) given τ, λ^e.
4. The perceived meeting probabilities are consistent with the true ones, i.e., for all i, j, $\lambda_i^s = m_i(s_i, e_i)/s_i$, $\lambda_i^e = m_i(s_i, e_i)/e_i$ and $G_{ij} = e_{ij}/e_i$.

(s, E, q, b) is an equilibrium allocation if there exists a price matrix τ such that (s, E, q, b, τ) is an equilibrium outcome.

3 Efficiency

In this section we present our efficiency results. In Section 3.1 we present the social planner's optimization problem and we provide a theorem that allows us to compare the solution of that problem to the market equilibrium allocation. In Section 3.2 we discuss the presence of two externalities and provide our main theorem that states the conditions for efficiency. We also discuss the efficient pricing rules. In Section 3.3 we derive taxes and subsidies that restore efficiency, when prices are set via Nash bargaining.

3.1 Comparing the market equilibrium to the efficient allocation

In this section, we characterize the set of equilibrium allocations as agents become patient, so that $\beta \to 1$, and compare them to the social planner's solution. Focusing on the case of patient agents simplifies the dynamic problem at hand without sacrificing its essential features. Patient (in the limit) agents only care about their average payoff under the steady state and not about the transition dynamics. In Appendix E we demonstrate that our efficiency results hold with discounting, as well as out of steady state.

We begin by defining the limits of equilibrium allocations as $\beta \to 1$:

Definition 2. (s, E, q, b, τ) is a limit equilibrium outcome if there exists a sequence $(s^n, E^n, q^n, b^n, \tau^n, \beta^n)_{n\geq 0}$ such that: (i) for each n, $(s^n, E^n, q^n, b^n, \tau^n)$ is an equilibrium outcome for the economy populated by agents with discount factor β^n; and (ii) as $\beta^n \to 1$, $(s^n, E^n, q^n, b^n, \tau^n) \to (s, E, q, b, \tau)$. (s, E, q, b) is a limit equilibrium allocation if there exists a price matrix τ such that (s, E, q, b, τ) is a limit equilibrium outcome.

The main task now is to compare the limit equilibrium allocation to the social planner's solution, who wishes to maximize total welfare. When agents do not discount future payoffs, the (constrained) efficient steady state allocation is a solution to the following problem,

$$\max_{s,E,q,b\geq 0} W(q) - \sum_{ij} q_{ij}\kappa_{ij} - \sum_{ij} (q_{ij} + b_{ij}) \frac{c_{ij}^s}{d_{ij}} - \sum_i s_i c_i^s - \sum_{ij} e_{ij} c_{ij}^e \tag{19}$$
$$\text{s.t. feasibility constraints (15)-(18)}$$

In words, the social planner maximizes the per-period welfare corresponding to (s, E, q, b): every period, q_{ij} customers depart on each route ij, generating gross customer value equal to $W(q)$; matched customers are replaced by a pool of new entrants of equal measure who pay the entry cost κ_{ij}; $q_{ij} + b_{ij}$ carriers begin traveling on route ij for $1/d_{ij}$ periods on average, paying a per-period traveling cost c_{ij}^s, while at every location i, s_i unmatched carriers pay the search cost c_i^s, and e_i unmatched customers incur the search cost c_{ij}^e. The planner is subject to the set of steady state feasible allocations (15)-(18). Note that since we focus on constrained efficiency, the planner is subject to the same frictions as the market.

Comparing the socially optimal allocation to the market one is not straightforward, since neither one has a closed-form expression. Indeed, the market equilibrium allocation solves a nonlinear system of equalities and inequalities, as described in Definition 1 (agent optimality conditions and value functions, feasibility constraints and rational expectations constraints), while the efficient allocation solves the planner's constrained optimization Problem (19) above. Nonetheless, the following theorem establishes that the market allocation can be found by solving an optimization problem that is remarkably similar, in form, to the planner's problem.

Theorem 1. *If (s, E, q, b) is a limit equilibrium allocation then it solves*

$$\max_{s,E,q,b\geq 0} W(q) - \sum_{ij} q_{ij}\kappa_{ij} - \sum_{ij}(q_{ij}+b_{ij})\frac{c_{ij}^s}{d_{ij}} - \sum_i s_i c_i^s - \sum_{ij} e_{ij}c_{ij}^e \tag{20}$$

s.t. feasibility constraints (15)-(17)

$$\forall i, j : q_{ij} \leq \lambda_i^s s_i G_{ij} \tag{21}$$

$$\forall i, j : q_{ij} \leq \lambda_i^e e_{ij}. \tag{22}$$

where the perceived probabilities λ^s, λ^e and G are taken as given and are consistent with the true ones (i.e. they satisfy condition 4 in Definition 1).

Theorem 1 characterizes market equilibrium allocations as solutions to Problem (20), the "market problem". As in the planner Problem (19), the objective function is equal to total welfare. Moreover, both the market and the planner face the steady state constraints (15)-(16), and the total fleet constraint (17). However, when it comes to the matching constraints, Problems (19) and (20) differ. Indeed, the social planner faces constraint (18), which treats the meeting rates λ^s, λ^e and the destination shares G as endogenous objects that are functions of s, e; in contrast, constraints (21) and (22) in the market Problem (20) treat these objects as exogenous constants.

The proof of Theorem 1, provided in Appendix A, rests heavily on duality. In particular, the dual variables of the market Problem (20) are linked to the carrier and customer value functions. This, in turn allows us to show that the carrier optimality conditions, equations (1)-(9), and the customer optimality conditions, (10)-(14), are equivalent in the limit to the first order conditions of the market Problem (20).[10]

Importantly, when comparing the market Problem (20), to the planner Problem (19), the only difference is that the latter internalizes the effect of search behavior on the endogenous meeting probabilities and destination shares. The market's failure to optimize with respect to these variables is the unique potential source of inefficiency in the economy.

[10]Caution is needed however when limits are taken as the discount factor goes to one, because the value functions per se may diverge. The desired correction is obtained by subtracting a reference value function from the remaining ones. Detailed arguments are found in the Appendix A.

3.2 Externalities and efficient prices

In contrast to a frictionless world, in an economy with search frictions prices may fail to balance demand and supply efficiently. As suggested by the comparison of the market Problem (20) to the planner Problem (19), the inefficiency arises from the effect of agents' decisions on the meeting probabilities and destination shares. This hints at the presence of two externalities, one related to the matching rates λ^s, λ^e and one to the destination shares G.

First, when choosing whether to join the search pool, agents may not internalize the effect that their entry has on the matching opportunities faced by other agents. Indeed, an extra carrier (customer) makes it easier for customers (carriers) to find a match and harder for other carriers (customers) to find a match. These are known as "thin/thick market externalities" in the search and matching literature.

Second, when choosing their destination, customers do not internalize the effect that this choice has on the distribution of carriers over space: a carrier will have to take the customer to his destination, and restart its search there. The customer only cares about its private surplus of the trip, whereas the planner also cares about the carrier's surplus, which depends on the destination. The random matching process creates what we call "pooling externalities": as heterogeneous customers are *pooled* together in the matching process, prices may fail to fully capture the social value of a match between a carrier and a customer. This distorts the customers' destination decisions and the resulting destination shares G.

We now formalize this intuition. Define the social value of a search allocation s, e, G by

$$V^p(s,e,G) \equiv \max_{q,b\geq 0} W(q) - \sum_{ij} q_{ij}\kappa_{ij} - \sum_{ij}(q_{ij}+b_{ij})\frac{c^s_{ij}}{d_{ij}} - \sum_i s_i c^s_i - \sum_i e_i \sum_j G_{ij} c^e_{ij} \qquad (23)$$

$$\text{s.t. feasibility constraints (15)-(18)}$$

This problem essentially solves for the carriers' optimal relocation decisions (b) and the decision of whether to accept a match or not (q), while taking as given the entry decisions of carriers (s) and customers (e), as well as customers' destination decisions (G).

The social planner Problem (19) is equivalent to,[11]

$$\max_{s,e,G\geq 0} V^p\left(s,e,G\right), \quad \text{s.t. } \sum_j G_{ij}{=}1\,\forall\text{i and} \sum_i s_i \leq\text{S} \tag{24}$$

Intuitively, since the only source of inefficiency results from agents' search behavior, it is useful to "optimize out" the other variables (i.e. q, b) in order to focus on the impact of the main variables of interest, s, e, G.

Definition 3. At a search allocation (s, e, G):

- Carriers internalize thin/thick market externalities if

$$s \in \arg\max_{s'\geq 0} V^p\left(s', e, G\right) \text{ s.t. } \sum_i s_i \leq S. \tag{25}$$

- Customers internalize thin/thick market externalities if

$$e \in \arg\max_{e'\geq 0} V^p\left(s, e', G\right). \tag{26}$$

- Customers internalize pooling externalities if

$$G \in \arg\max_{G'\geq 0} V^p\left(s, e, G'\right) \text{ s.t. } \sum_j G_{ij} = 1\,\forall i. \tag{27}$$

Our next theorem states three conditions that determine how the meeting surpluses must be shared between carriers and customers in order for the externalities to be internalized in equilibrium. For every $i \in I$, we denote by $\eta_i^s = d\ln m_i\left(s_i, e_i\right)/d\ln s_i$ and $\eta_i^e = d\ln m_i\left(s_i, e_i\right)/d\ln e_i$, the elasticities of the matching function with respect to the measure of carriers and customers searching at i, respectively. To avoid delving into corner solutions arising in trivial cases, we assume that the equilibrium is such that there is a positive measure of customers and carriers searching at each location ($s_i, e_i > 0\ \forall i$) and that $\sum_i s_i < S$.[12] Let $\bar{\Delta}^s$ and $\bar{\Delta}^e$ denote the carrier and customer limit surpluses associated with the limit equilibrium outcome, (s, e, G, q, b, τ), when $\beta \to 1$. In addition, let $\bar{\Delta}_{ij} \equiv \bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e$, $\forall i, j$ denote the limit

[11]Note that for every feasible solution s, e, G of this problem there exists a pair $q, b \geq 0$ such that the resulting allocation is steady state feasible: simply set $q = 0$, $b_{ij} = 0$ for $i \neq j$ and $b_{ii} = s_i\ \forall i$.

[12]If $s_i = 0$ or $e_i = 0$ for some i, then the efficiency conditions must hold only at locations with positive number of carriers and customers. The second is a non-triviality assumption which precludes having no carriers traveling at all, since $\sum_i s_i = S$ implies that $(q_{ij}, b_{ij}) = (0, 0)$ for every ij such that $d_{ij} < 1$.

social surplus. For a formal definition, see Appendix A.1.

Theorem 2. *Let (s, e, G, q, b, τ) be a limit equilibrium outcome. Suppose that Problem (23) admits a unique optimal solution.*[13] *Then:*

(i) Carriers internalize thin/thick market externalities if and only if[14]

$$\forall i \in I: \frac{\sum_j G_{ij} \bar{\Delta}^s_{ij}}{\sum_j G_{ij} \bar{\Delta}_{ij}} = \eta^s_i. \tag{28}$$

(ii) Customers internalize thin/thick market externalities if and only if

$$\forall i \in I: \frac{\sum_j G_{ij} \bar{\Delta}^e_{ij}}{\sum_j G_{ij} \bar{\Delta}_{ij}} = \eta^e_i. \tag{29}$$

(iii) Customers internalize pooling externalities if and only if

$$\bar{\Delta}^s_{ij} = \max_{k \neq i} \bar{\Delta}^s_{ik} \tag{30}$$

for every ij such that $G_{ij} > 0$.

(iv) (s, e, G, q, b) is efficient only if conditions (i)-(iii) hold jointly.

The proof, provided in Appendix A.3, first establishes that the function $V^p(s, e, G)$ is concave. Therefore the supergradients with respect to each of the arguments (s, e, G) are well-defined at every search allocation and in fact V^p is differentiable almost everywhere in its domain. Then we demonstrate, through the use of the dual variables associated with a limit equilibrium allocation, that the resulting first order conditions coincide with the conditions internalizing the three externalities.

Conditions (i) and (ii) of Theorem 2 recast the familiar Hosios (1990) conditions requiring that the share of the surplus appropriated by agents on each side of the market equals the elasticity of the matching function with respect to the measure of agents on that side of the market. As discussed above, when agents (either customers or carriers) choose whether to search for a match, they do not take into account the effect that this decision has on other agents' meeting probabilities, generating the well-known thin/thick

[13] This is a technical condition which is generally satisfied, for example, when the function W is strictly concave.

[14] Formally, this condition is necessary only when $V^p(s, e, G)$ is differentiable in s, which is the case almost everywhere. A similar disclaimer applies to statement (ii), (iii) and (iv), where necessity relies on differentiability with respect to e, G and (s, e, G), respectively.

market externalities. Conditions (28) and (29) have a similar flavor as the standard Coasian conditions in the presence of externalities, where the private value of an action must be equal to its social value. Indeed, we can rewrite equation (28) as

$$\lambda_i^s \sum_j G_{ij} \bar{\Delta}_{ij}^s = \frac{dm_i(s_i, e_i)}{ds_i} \sum_j G_{ij} \bar{\Delta}_{ij}.$$

The left-hand-side captures the per-period expected private return of a carrier entering market i, which equals the expected carrier surplus from matching ($\sum_j G_{ij} \bar{\Delta}_{ij}^s$) multiplied by its matching probability (λ_i^s). The right-hand-side captures the per-period expected social return from an additional carrier entering i, which equals the expected social surplus from an additional match ($\sum_j G_{ij} \bar{\Delta}_{ij}$) multiplied by the marginal increase in the number of matches ($\frac{dm_i(s_i,e_i)}{ds_i}$).

Condition (iii) of Theorem 2 deals with the pooling externalities. The inefficiency here arises because customer type-specific (in this case destination-specific) markets are missing. When heterogeneous customers are pooled together, carriers cannot compete among themselves to serve a given type of customer; this grants carriers market power, creating a wedge between prices and the carriers' opportunity cost of a trip. It is useful to compare our setup with a frictionless environment. In that case, competition among carriers ensures that prices coincide with their opportunity cost. In equilibrium, therefore, carriers are indifferent among serving different types of customers. Similarly, in a world with search frictions, but where carriers can direct their search to a specific customer type (i.e. a model of directed search which is efficient, see Moen, 1997), in equilibrium a similar no-arbitrage condition would make carriers indifferent across destinations.[15] In our setup, the planner essentially restores this indifference condition in Condition (iii) of Theorem 2. A consequence of pooling externalities is that the share of destinations with high social value is too low in equilibrium.

Moreover, conditions (i) and (ii) imply that a necessary condition for efficiency is that the matching function exhibits constant returns to scale; indeed, if we add equations (28) and (29), the left-hand-side is equal to one, and thus the elasticities must add to one as well. Corollary 3 in Appendix A.5 demonstrates that under non-constant returns to scale in matching, efficiency can still be restored, via a tax or subsidy

[15] In a directed search world, there is a separate market for each customer type/destination; carriers enter different markets, until in equilibrium they are (ex ante) indifferent across different choices. Markets of more desirable destinations entail longer waiting times for carriers and vice versa.

which however creates a wedge between the price paid by the customer and the one received by the carrier.

Efficient prices Condition (iv) of Theorem 2 provides a characterization of the efficient pricing rule:

Corollary 1. *Let a limit equilibrium outcome (s, e, G, q, b, τ) be efficient. Then we have $\eta_i^s = 1 - \eta_i^e$ and the equilibrium prices are such that, for every ij such that $G_{ij} > 0$:*

$$\tau_{ij} = w_{ij}(q) - \kappa_{ij} - \eta_i^e \sum_j G_{ij} \bar{\Delta}_{ij} - \left(\bar{\Delta}_{ij} - \sum_j G_{ij} \bar{\Delta}_{ij} \right). \tag{31}$$

To gain some intuition for this pricing rule, we can show that (31) can be rewritten as follows:

$$\forall i, j: \; (1 - \eta_i^s) \bar{\Delta}_{ij}^s = \eta_i^s \left[\bar{\Delta}_{ij}^e - \left(\bar{\Delta}_{ij} - \sum_j G_{ij} \bar{\Delta}_{ij} \right) \right], \tag{32}$$

where the terms $\bar{\Delta}_{ij}^s$ and $\bar{\Delta}_{ij}^e$ depend on the price τ_{ij}. Relationship (32) is reminiscent of a surplus sharing condition under Nash bargaining (i.e. the equilibrium condition that determines prices when agents Nash bargain in a decentralized fashion), where however we have (i) replaced the bargaining coefficients with the respective matching function elasticities (this amounts to satisfying the Hosios condition under Nash bargaining, see Section 3.3); and (ii) adjusted the outside option of the customer by the deviation of route ij's social surplus from the average social surplus across destinations. By adjusting the outside option of the customer, we ensure that customers fully internalize the social value of their destination in their decision-making. If the customer has chosen a destination whose social surplus is higher than the mean from origin i, he should enjoy a higher outside option (and thus a lower price), and vice versa.

Corollary 1 suggests that if a central authority could post prices, they should choose them according to (31). For instance, in the case of taxicabs, prices are regulated by central agencies. In practice, they are roughly set equal to a tariff plus a fee proportional to distance. Corollary 1 indicates that this pricing rule is unlikely to be efficient, since the efficient prices should be origin-destination specific. Naturally, implementing the efficient prices may not be straightforward in practice, as there are not many examples of markets where prices can be fully regulated. Thus, in the next section we consider optimal policy when prices are bilaterally negotiated, as is often the case in decentralized markets.

3.3 Optimal policy under Nash bargaining

In this section we consider the problem of a planner who cannot directly control prices, but can use taxes/subsidies to restore efficiency in the market. We show that the planner can indeed achieve efficiency using such instruments and we derive their optimal values.

Suppose that the planner can impose a tax/subsidy h^q on loaded trips, h^s on searching carriers, and h^e on searching customers. In other words, searching carriers in region i pay h_i^s in addition to their waiting cost c_i^s every period they search; customers searching in i pay h_i^e in addition to their cost c_{ij}^e every period they search; finally, there is a one-time tax h_{ij}^q on every new match (as illustrated below which side pays the tax does not matter).

We focus on a specific price mechanism, that of Nash bargaining, which is a commonly employed model used to capture bilateral negotiations. We can extend the definition of equilibrium to accommodate Nash bargaining and taxes in a straightforward manner: (s, e, G, q, b, τ) is an equilibrium outcome under taxes h and Nash bargaining, if carriers and customers behave optimally given h, τ, λ^s, λ^e and G; the feasibility constraints are satisfied; λ^s, λ^e and G are consistent with the allocation; and finally, prices are determined by the usual surplus sharing condition,

$$(1 - \gamma_i)\,\bar{\Delta}_{ij}^s = \gamma_i \bar{\Delta}_{ij}^e \tag{33}$$

where γ_i is the carrier bargaining coefficient at i (see Appendix A.6 for further details).

Corollary 2 derives the tax scheme h that resolves the two externalities:

Corollary 2. *Let (s, e, G, q, b, τ) be a limit equilibrium outcome under taxes h and Nash bargaining. Then:*

(i) Thin/thick market externalities are internalized if and only if for every i

$$\gamma_i \sum_j G_{ij}\bar{\Delta}_{ij} - \left(\frac{h_i^s}{\lambda_i^s} + \gamma_i \sum_j G_{ij} h_{ij}^q\right) = \eta_i^s \sum_j G_{ij}\bar{\Delta}_{ij} \tag{34}$$

and similarly,

$$(1 - \gamma_i) \sum_j G_{ij}\bar{\Delta}_{ij} - \left(\frac{h_i^e}{\lambda_i^e} + (1 - \gamma_i) \sum_j G_{ij} h_{ij}^q\right) = \eta_i^e \sum_j G_{ij}\bar{\Delta}_{ij}. \tag{35}$$

(ii)Pooling externalities are internalized if and only if for all ij

$$h_{ij}^q - \sum_j G_{ij} h_{ij}^q \leq -\frac{\gamma_i}{1-\gamma_i}\left(\bar{\Delta}_{ij} - \sum_j G_{ij}\bar{\Delta}_{ij}\right) \tag{36}$$

with equality if $G_{ij} > 0$.

Before discussing the result, note that if the planner does not impose any taxes, so that $h = 0$, the conditions required to internalize the thin/thick market externalities (34) and (35) become the well-known Hosios (1990) conditions. Indeed, in that case equations (34) and (35) imply that

$$\gamma_i = \frac{d\ln m_i(s_i, e_i)}{d\ln s_i} \equiv \eta_i^s, \quad \text{and} \quad 1-\gamma_i = \frac{d\ln m_i(s_i, e_i)}{d\ln e_i} \equiv \eta_i^e \tag{37}$$

In addition, notice that, regardless of who pays the matching tax h_{ij}^q, we can think of $\gamma_i h_{ij}^q$ as the incidence of this tax on carriers and $(1-\gamma_i)\, h_{ij}^q$ as the incidence on customers, since Nash bargaining implies that the agents split the gross surplus from the match according to γ_i.[16] Moreover, a searching carrier in expectation pays h_i^s/λ_i^s while searching and a searching customer pays h_i^e/λ_i^e.

Condition (34) states that the private value of an additional carrier searching in i must equal its social value. Indeed, the left-hand-side of equation (34) consists of the share of the total surplus accruing to the carrier (which is equal to his bargaining coefficient times the total surplus) minus his tax incidence. The right-hand-side is the surplus the planner wants the carrier to receive, which equals the contribution of the extra carrier to total surplus. In other words, the tax incidence of the carrier must be set so that the original condition for thin/thick market externalities, equation (28) of Theorem 2, is satisfied. A similar intuition holds for equation (35).

Condition (36) determines the trip taxes that resolve the pooling externalities. At first glance, it has an intuitive explanation: destinations with a surplus above average should get a subsidy above average and vice versa. Next, note that the system of equations in (36) is not full rank; for simplicity we can

[16] To see this note that Nash bargaining implies that $\bar{\Delta}_{ij}^s = \gamma_i\bar{\Delta}_{ij} - \gamma_i h_{ij}^q$ and $\bar{\Delta}_{ij}^e = (1-\gamma_i)\,\bar{\Delta}_{ij} - (1-\gamma_i)\,h_{ij}^q$, so that agents split the gross surplus $\bar{\Delta}_{ij}$ according to γ_i, and similarly pay their share of the matching tax also according to γ_i. Indeed, the social surplus is now defined by $\bar{\Delta}_{ij} = \bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e + h_{ij}^q$ (as either $\bar{\Delta}_{ij}^s$ or $\bar{\Delta}_{ij}^e$ include $-h_{ij}^q$, $\bar{\Delta}_{ij}$ does not depend on h_{ij}^q) and therefore $\bar{\Delta}_{ij}^s = \gamma_i\left(\bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e\right) = \gamma_i\left(\bar{\Delta}_{ij} - h_{ij}^q\right)$. Note also that the definition of efficiency remains the same, except that we include the planner's revenue in the welfare.

set the planner revenue in region i, $\sum_j G_{ij} h_{ij}^q$, equal to zero.[17] Multiplying both sides by $-(1-\gamma_i)$, it is easy to see that Condition (36) requires that the subsidy on route ij that falls on the customer, $(1-\gamma_i)(-h_{ij}^q)$, is equal to the deviation of the carrier surplus, $\gamma_i \bar{\Delta}_{ij}$ from the average carrier surplus from i, $\gamma_i \sum_j G_{ij} \bar{\Delta}_{ij}$. Therefore, routes where the carrier surplus is high (low) are subsidized (taxed). By setting the customer tax/subsidy equal to the deviation of the carrier surplus, the planner forces the customer to fully internalize the impact of his destination decision on the carrier surplus.

Finally, note that if the planner can only use the search taxes h^s, h^e, he can correct the thin/thick market externalities.[18] Similarly if he can tax only matches but not search of any side of the market, then he can correct the pooling externalities (using equation (36) as discussed above). The planner can correct all externalities by taxing matches and either searching carriers or searching customers.[19]

4 Empirical application: dry bulk shipping

In this section we describe our empirical application using data from the dry bulk shipping industry. We begin in Section 4.1 with a description of the industry and the available data. In Section 4.2 we discuss search frictions in this market. In Section 4.3 we briefly discuss model estimation. With the exception of Section 4.2, this section follows closely BKP. Throughout the following sections, unless otherwise noted, we split ports into 15 geographical regions, depicted in Figure 6 of Appendix D.[20]

4.1 Industry description and data

Dry bulk shipping involves vessels designed to carry a homogeneous unpacked dry cargo, for individual shippers on non-scheduled routes. Bulk carriers operate much like taxi cabs: a specific cargo is transported individually by a specific ship, for a trip between a single origin and a single destination. Dry bulk shipping

[17] Condition (36) defines a linear system of equations in terms of the $I-1$ trip taxes h_{ij}^q for each location i. This system has multiple solutions as its rank equals $I-2$. Thus, to obtain a unique solution we would have to impose a linear constraint. Imposing the constraint $\sum_j G_{ij} h_{ij}^q = 0$ is natural as it implies that the budget is balanced in each location.

[18] He can do so by setting $h_i^e/\lambda_i^e = (1-\gamma_i)\sum_j G_{ij}\bar{\Delta}_{ij} - \eta_i^e \sum_j G_{ij}\bar{\Delta}_{ij}$ and $h_i^s/\lambda_i^s + h_i^e/\lambda_i^e = (1-\eta_i^e-\eta_i^s)\sum_j G_{ij}\bar{\Delta}_{ij}$.

[19] If he taxes matches and searching carriers, he sets $(1-\gamma_i) h_{ij}^q = (1-\gamma_i)\bar{\Delta}_{ij} + \sum_j G_{ij}\bar{\Delta}_{ij} - \bar{\Delta}_{ij} - \eta_i^e \sum_j G_{ij}\bar{\Delta}_{ij}$, if $G_{ij} > 0$ and $h_i^s/\lambda_i^s + \sum_j G_{ij} h_{ij}^q = (1-\eta_i^e-\eta_i^s)\sum_j G_{ij}\bar{\Delta}_{ij}$.

[20] To determine the regions, we employ a clustering algorithm that minimizes the within-region distance between ports. The regions are: West Coast of North America, East Coast of North America, Central America, West Coast of South America, East Coast of South America, West Africa, Mediterranean, North Europe, South Africa, Middle East, India, Southeast Asia, China, Australia, Japan-Korea. We ignore intra-regional trips and entirely drop these observations.

involves mostly commodities, such as iron ore, steel, coal, bauxite, phosphates, but also grain, sugar, chemicals, lumber and wood chips; it accounts for about half of total seaborne trade in tons (UNCTAD) and 45% of the total world fleet, which includes also containerships and oil tankers.[21]

There are four size categories of dry bulk carriers: Handysize (10,000–40,000 DWT), Handymax (40,000–60,000 DWT), Panamax (60,000–100,000 DWT) and Capesize (larger than 100,000 DWT). The industry is unconcentrated, consisting of a large number of small shipowning firms (see Kalouptsidi, 2014). Moreover, shipping services are largely perceived as homogeneous. In his lifetime, a shipowner will contract with hundreds of different exporters, carry a multitude of different products and visit numerous countries.

Trips are realized through individual contracts that are intermediated by a disperse network of brokers. Ships carry the cargo of a single customer at a time, who fills up the entire ship. In this paper, we focus on spot contracts and in particular the so-called "trip-charters", in which the shipowner is paid in a per day rate.[22]

We combine four data sets. The first is a data set of shipping contracts, from 2010 to 2016, collected by Clarksons Research. An observation is a transaction between a shipowner and a charterer for a specific trip. We observe the vessel, the charterer, the contract signing date, the loading and unloading dates, the price in dollars per day, as well as some information on the origin and destination.

Second, we use satellite AIS (Automatic Identification System) data from exactEarth Ltd for the ships in the Clarksons data set between July 2010 and March 2016. AIS transceivers on the ships automatically broadcast information, such as their position (longitude and latitude), speed, and level of draft (the vertical distance between the waterline and the bottom of the ship's hull), at regular intervals of at most six minutes. The draft is a crucial variable, as it allows us to determine whether a ship is loaded or not at any point in time.

Third, we augment the ship data sets above, with international trade data from Comtrade on export value and volume by country pair for bulk commodities.

[21]Bulk ships are different from containerships, which carry cargo (mostly manufactured goods) from many different cargo owners in container boxes, along fixed itineraries according to a timetable. It is not technologically possible to substitute bulk with container shipping.

[22]Trip-charters are the most common type of contract. Long-term contracts ("time-charters"), however, do exist: on average, about 10% of contracts signed are long-term. Interestingly, though, ships in long-term contracts, are often "relet" in a series of spot contracts, suggesting that arbitrage is possible.

Fourth, we use the ERA-Interim archive, from the European Centre for Medium-Range Weather Forecasts (CMWF), to collect global data on daily sea weather. This allows us to construct weekly data on the wind speed (in each direction) on a 6° grid across all oceans.

We provide a brief overview of the data and empirical regularities and we refer the interested reader to BKP for further details. Our final dataset stretches from 2012 to 2016 and involves 5,398 ships (about half the world fleet) and 12,007 shipping contracts with a known price, origin and destination.[23] The average trip price is 14,000 dollars per day (or 290,000 dollars for the entire trip), with substantial variation. Trips last on average 2.9 weeks. Contracts are signed close to the loading date, on average six days before. We have 393,058 ship-week observations at which the ship decides to either travel empty someplace (termed "ballast") or stay at its current location. Ships that do not sign a contract, remain in their current location with probability 77%, while the remaining ships ballast elsewhere. Clarksons reports the product carried in about 20% of the sample and the main commodity categories are grain (29%), ores (21%), coal (25%), steel (8%) and chemicals/fertilizers (6%). For summary statistics see Table 1 in BKP while for details on the construction of the final dataset see the Supplemental Material of BKP.

Finally, an important feature of this market revealed by the satellite data, is that most countries are either large net importers or large net exporters. For instance, Australia, Brazil and Northwest America, the world's biggest exporters of commodities, are rich in minerals, grain, coal, etc. At the same time, China and India, the world's biggest importers, require raw materials to grow further. As a result, commodities flow out of the former, towards the latter. The trade imbalances have implications for both ship ballast behavior and shipping prices. Indeed, at any point in time, 42% of ships are traveling without cargo. At the same time, prices are largely asymmetric and depend on the destination's trade imbalance: all else equal, the prospect of having to ballast after offloading is associated with higher shipping prices (see results in Table 7).

[23] We drop the first two years (until May 2012) of vessel movement data, as satellites are still launched at that time and the geographic coverage is more limited.

4.2 Search frictions in dry bulk shipping[24]

A number of features of dry bulk shipping, such as information frictions and port infrastructure, can hinder the matching of ships and exporters. In this section we argue that these frictions indeed lead to unrealized potential trade. Consider a geographical region, such as a country or a set of neighboring countries, where there are s ships available to pick up cargo and e exporters searching for a ship to transport their cargo. We define search frictions by the inequality:

$$m < \min\{s, e\} \tag{38}$$

where m is the number of matched ships and exporters. In other words, under frictions there is potential trade that remains unrealized; in contrast, in a frictionless world, the entire short side of the market gets matched, so that $m = \min\{s, e\}$. When inequality (38) holds, matches are often modeled via a matching function, $m = m(s, e)$, as is done in Section 2 above, and also extensively in the labor literature.

In this section, we present three facts consistent with frictions, as defined by (38). In particular, we (i) provide a direct test for inequality (38); (ii) we document wastefulness in ship loadings; (iii) we document substantial price dispersion. Then, we estimate the matching function $m = m(s, e)$ and gauge the degree of frictions.

Evidence of search frictions We begin with a simple test for search frictions. If we observed all variables s, e, m, it would be straightforward to test (38); this is essentially what is done in the labor literature, where the co-existence of unemployed workers and vacant firms is interpreted as evidence of frictions. However in our setup, we observe m (i.e. ships leaving loaded) and s, but not e; we thus need to adopt a different approach.

Assume there are more ships than exporters, i.e. $\min(s, e) = e$. We begin with this assumption, because our sample period is one of low shipping demand and severe ship oversupply due to high ship investment between 2005 and 2008 (see Kalouptsidi, 2014). If there are no search frictions, so that $m = \min(s, e) = e$, small exogenous changes in the number of ships should not affect the number of

[24]The material in this section was included in a previous working version of our paper "Geography, Transportation and Endogenous Trade Costs"; please see NBER Working Paper 23581.

matches. In contrast, if there are search frictions, an exogenous change in the number of ships changes the number of matches, through the matching function $m = m(s, e)$. We approximate an exogenous change in the number of ships, with unpredictable ocean weather conditions. The intuition is that wind affects the speed at which ships travel and therefore exogenously shifts the supply of ships at port. We therefore explore whether exogenously changing the number of ships in regions with a lot more ships than exporters affects the realized number of matches.[25] Since we do not observe exporters directly, to select periods in which there are more ships than exporters, for each region we consider weeks when there are at least twice as many ships waiting in port as matches. Table 1 presents the results. We find that indeed matches are affected by weather conditions in all but one region, consistent with the presence of search frictions.

Second, we document simultaneous arrivals and departures of empty ships. Indeed, the first two panels of Figure 1 display the weekly number of ships that arrive empty and load, as well as the number of ships that leave empty, in two net exporting countries: Norway and Chile. In Norway, several ships arrive empty and load, but almost no ship departs empty. In Chile, however, the picture is quite different: it frequently happens that an empty ship arrives and picks up cargo, while at the same time another ship departs empty. This is suggestive of wastefulness in Chile: why does the ship that depart empty, not pick up the cargo, instead of having another ship arrive from elsewhere to pick it up?

This pattern is observed in many countries. Indeed, the third panel of Figure 1 depicts the histogram of the bi-weekly ratios of outgoing empty ships over incoming empty and loading ships for net exporting countries. In the absence of frictions, one would expect this ratio to be close to zero. However, we see that the average ratio is well above zero. Moreover, this pattern is quite robust in a number of dimensions.[26]

Third, again inspired by the labor literature, we investigate dispersion in prices. In markets with no frictions, the law of one price holds, so that there is a single price for the same service. This does not hold

[25] We partition the globe into cells of $9° \times 9°$; for each cell we collect data on the wind speed in different directions, as well as wave period and height. To control for seasonality, we residualize the weather measurements for each cell on a quarter fixed effect. We follow Belloni et al. (2012) to select the relevant instruments in each region i. The potential regressors include all the weather measurements for cells in the sea, for one and two weeks prior to period t.

[26] This figure is robust to alternative market definitions, time periods and ship types. Capesize vessels exhibit somewhat larger mass towards zero, consistent with the somewhat higher concentration of ships and charterers, as well as the ships' ability to approach fewer ports. The figure is also similar if done by port rather than country. To control for repairs we remove stops longer than 6 weeks. Finally, we only consider as "ships arriving empty" the ships arriving empty and sailing full towards another region, and we consider as "ships leaving empty" ships sailing empty toward a different country; so movements to nearby ports are excluded. This definition also implies that refueling cannot explain the fact either- though there are very small differences in fuel prices across space anyway (less than 10%).

	N	Joint Significance	$\frac{s}{m}$
North America West Coast	193	0	2.706
North America East Coast	200	0	3.172
Central America	199	0.001	3.451
South America West Coast	198	0	2.913
South America East Coast	200	0	4.083
West Africa	200	0.001	5.862
Mediterranean	200	0	4.244
North Europe	200	0	3.577
South Africa	200	0	2.862
Middle East	200	0	3.86
India	200	0.34	8.58
South East Asia	200	0	3.334
China	200	0.038	6.194
Australia	187	0	2.457
Japan-Korea	200	0	5.311

Table 1: Test for search frictions. Regressions of the number of matches in each region on the unpredictable component of weather conditions in the surrounding seas. For each region we use weeks in which there are at least twice as many ships as matches. The first column reports the number of observations; the second column joint significance; and the third column the average ratio of ships over matches in each region during these weeks. To proxy for the unpredictable component of weather, we partition the globe into cells of $9° \times 9°$, and for each cell we collect data on the speed of the horizontal (E/W) and vertical (N/S) component of wind, as well as wave period and height. To control for seasonality, we residualize the weather measurements for each cell on a quarter fixed effect. The potential regressors include one and two weeks lagged values of all the weather measurements for cells in the sea. Finally, we follow Belloni et al. (2012) to select the relevant instruments in each region.

in labor markets, where there is large wage dispersion among workers who are observationally identical. This observation has generated a substantial and influential literature on frictional wage inequality, i.e. wage inequality that is driven by search frictions.[27] Similarly, Table 7 in Appendix D shows that there is substantial price dispersion in shipping contracts. More specifically, at best we can account for about 70% of price variation, controlling for ship size, as well as quarter, origin and destination fixed effects. Moreover, the coefficient of variation of prices within a given quarter, origin and destination triplet is about 30% (23%) on average (median). In the most popular trip, from Australia to China, the *weekly* coefficient of variation is on average 34% and ranges from 15% to 65% across weeks.

In addition, it is worth noting that the type of product carried affects the price paid and overall more valuable goods lead to higher contracted prices, as shown in the same table. In the absence of frictions, if

[27] See for instance Burdett and Mortensen (1998), Postel-Vinay and Robin (2002), Mortensen (2003) and references therein.

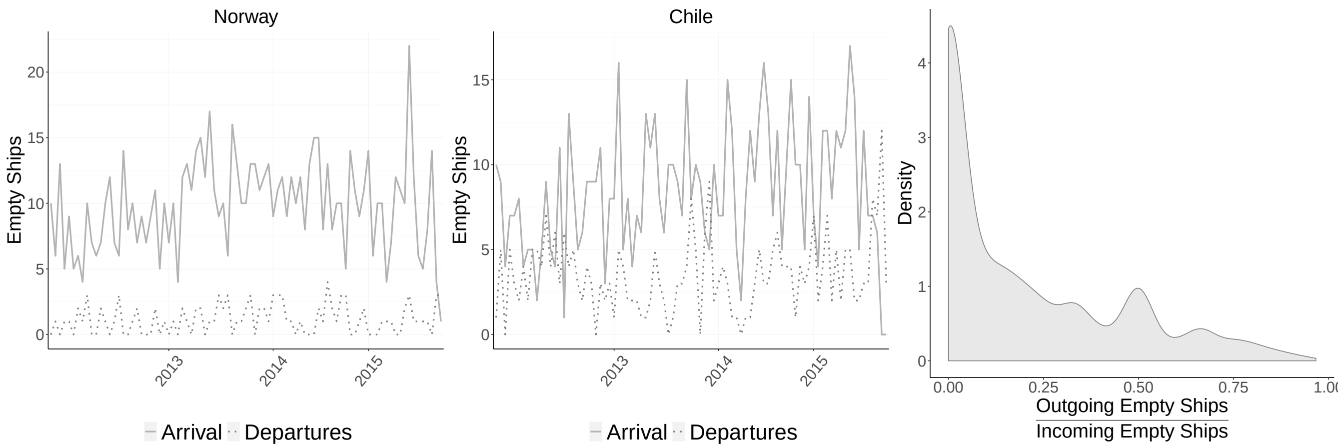


Figure 1: Simultaneous arrivals and departures of empty ships: The first two panels depict the flow of ships arriving empty and loading, and ships leaving empty in two-week intervals in Norway and Chile. The last panel shows the histogram of the ratio of outgoing empty over incoming empty and loading ships across all net exporting countries.

there are more ships than exporters, as is the case during our sample period, we would expect prices to be bid down to the ships' opportunity cost.[28] In contrast, in markets with frictions and bilateral bargaining, since ships now have market power, the price also depends on the exporter's valuation and exporters with higher valuations pay more.

As in labor markets, a multitude of factors can lead to frictions (i.e. unrealized matches) in shipping. First, the decentralized and unconcentrated nature of the market and the mere existence of brokers, suggest that information frictions are present. The meeting process involves a disperse network of brokers; oftentimes more than two brokers intervene to close a deal, suggesting that the ship's and the exporter's brokers do not always find each other, and that an "intermediate broker" was necessary to bring the two together (Panayides, 2016). In interviews, brokers claimed to receive 5,000-7,000 emails per day; sorting through these emails is reminiscent of an unemployed worker sorting through hundreds of vacancy postings. Port infrastructure, congestion or capacity constraints may also hinder matching.

Finally, we discuss two features of this market, ship homogeneity and random search. While in labor markets, as some researchers have argued, observed or unobserved heterogeneity may partly explain the documented facts, in shipping, heterogeneity is much more limited. Indeed, the data suggests that ship heterogeneity alone is not a prominent explanation for search frictions. Ships do not specialize neither

[28]In a frictionless market with more ships than exporters and homogeneous ships, in equilibrium the price from an origin to a destination would be such that ships are indifferent between transporting the cargo and staying unmatched.

geographically, nor in terms of products: during the period of our data ships deliver cargo to 13 out of 15 regions on average and carry at least 2 of the 3 main products (coal, ore and grain). Moreover, neither shipowner characteristics, nor shipowner fixed effects have any explanatory power in price regressions, as shown in Table 8 in Appendix D, while ballast decisions of ships in the same region are concentrated around the same options.[29] Random search is also a reasonable approximation in shipping as meetings occur through an unconcentrated network of brokers. Nonetheless, we examine this more rigorously in Appendix B, where we investigate a standard implication of directed search, whether matching rates differ across destinations from a given origin.

Matching function estimation We close this section by discussing the estimated search frictions implied by the estimated matching function. Here, we follow the same approach to estimate the matching function as BKP; we thus provide a brief overview of the procedure and then present the implications for search frictions.

A sizable literature has estimated matching functions in several different contexts (e.g. labor markets, marriage markets, taxicabs). For instance, in labor markets, one can use data on unemployed workers, job vacancies and matches to recover the underlying matching function. In our data, we observe ships and matches, but not searching exporters; in BKP we simultaneously recovered both exporters, as well as a nonparametric matching function. This approach extends the literature in two dimensions. First, we do not take a stance on the presence of search frictions. When one side of the market (in this case exporters) is unobserved or mismeasured, it is difficult to discern whether search frictions are present. Second, we avoid parametric restrictions on the matching function; this is important, since as shown in Theorem 2, in frictional markets, the conditions for constrained efficiency depend crucially on the elasticity of the matching function with respect to the search input.

Briefly, the estimation draws from the literature on nonparametric identification (Matzkin, 2003) and non-separable instrumental variable techniques (e.g. Imbens and Newey, 2009). We require that $m(s, e)$ is increasing in e, that it exhibits constant returns to scale (although the results are robust to

[29]If heterogeneity were an important driver of ships' ballasting decisions, we would expect ships to choose diverse destinations from a given origin. Yet we find that ballast choices are similar across ships (the CR_3 measure for the chosen destinations, i.e. the concentration ratio of the top 3 destinations, is higher than 70% in most regions). Moreover, homeports are not an important consideration for shipowners, as the crew flies to their home country every 6-8 months.

alternative restrictions; see BKP) and that an instrument that shifts the number of ships exists (the weather shocks). The methodology delivers exporters point-wise and the matching function of each location i nonparametrically. We provide a short description of the approach in Appendix C.1 and refer the reader to BKP for further details, as well as Brancaccio et al. (forthcoming) for a guide on the implementation of this approach in this and other settings.[30]

Figure 5 in Appendix C.1 reports our estimates for search frictions. In particular, to measure the extent of search frictions in different regions, we compute the average percentage of weekly "unrealized" matches; i.e. $(\min\{s_i, e_i\} - m_i) / \min\{s_i, e_i\}$. Search frictions are heterogeneous over space and may be somewhat sizable, with up to 20% of potential matches "unrealized" weekly in regions like South and Central America and Europe. On average, 13.5% of potential matches are "unrealized".[31]

Moreover, we find that the estimated search frictions are positively correlated with the observed within-region price dispersion (0.47), another indicator of search frictions. We also find that frictions are negatively correlated with the Herfindahl-Hirschman Index of charterers (those reported in the Clarksons contract data) in a region (-0.31); this suggests that when the clientele is disperse, frictions are higher. Finally, when we estimate the matching function separately for Capesize (biggest size) and Handysize (smallest size) vessels, we find that for Capesize, where the market is thinner, search frictions are lower.

4.3 Model estimation and results

We make four changes that render the model presented in Section 2 amenable to empirical analysis. First, we impose a specific pricing mechanism, Nash bargaining, with γ_i the ship bargaining coefficient in market i. Second, we add randomness to the discrete choice problem for ships of where to ballast, by adding idiosyncratic shocks to equation (4), so that it becomes,

$$U_i^s = \max_j V_{ij}^s + \sigma\epsilon_{ij} \tag{39}$$

[30]For an application to labor markets see Lange and Papageorgiou (2020).

[31]It is worth noting that this does not imply that in the absence of search frictions there would be 13.5% more matches, as we would need to take into account the optimal response of ships and exporters. This is simply a measure of the severity of search frictions in different regions.

where ϵ_{ij} are drawn i.i.d. from the Type I extreme value distribution with standard deviation σ. Third, we consider the version of the model with $\beta < 1$. In Appendix E we demonstrate that our efficiency results hold in this empirical model with discounting and idiosyncratic shocks. Fourth, we also add randomness to the exporters' problem (14), so that they solve the following discrete choice problem of whether and where to export,

$$\max_j \left\{ U^e_{ij} - \kappa_{ij} + \epsilon_{ij} \right\}$$

with ϵ_{ij} drawn i.i.d. from the Type I extreme value distribution; we normalize $U^e_{ii} - \kappa_{ii} = 0$ and interpret this as the option of not exporting at all. We also assume for simplicity that $w_{ij}(q) = w_{ij}$ for all ij.

The main parameters of interest are: the ship travel and wait costs c^s_{ij}, c^s_i, for all i, j, as well as the standard deviation of the logit shocks σ; the exporter valuations w_{ij}, the exporter waiting costs c^e_i (to gain power, we assume that c^e_{ij} do not vary over j), and entry costs κ_{ij} for all i, j; and the bargaining coefficients γ_i for all i. We present the estimation strategy in Appendix C. Briefly, we use the ship parameter estimates from BKP and estimate the exporter parameters and bargaining coefficients from prices and trade flows. Unlike BKP, we allow the bargaining coefficient to vary by region to allow for flexibility, given the importance of that parameter regarding the thin/thick market externalities. Moreover, we bring in additional data to obtain exporter valuations w_{ij} and as a result we are able to estimate the extra parameters capturing exporter wait costs, c^e_i.

The results are presented in Table 6 in Appendix D. The exporter wait costs, c^e_i, are equal to about 3% of the exporters' valuation on average, but there is substantial heterogeneity over space; the estimated costs are highest in Central and South America, as well as parts of Africa. These parameters capture inventory expenditures, delay costs, risks of damage or theft etc. Consistent with this interpretation, we find that exporter wait costs are positively correlated with the recovered wait costs for ships (0.34), and are negatively correlated with the World Bank index of quality of port infrastructure (-0.50). Finally, the estimates for the bargaining coefficients suggest that the exporters get a larger share of the surplus in almost all regions.

5 Efficiency in dry bulk shipping

In this section we present our welfare results. In Section 5.1 we check whether the efficiency conditions hold in the case of bulk shipping. In Section 5.2 we present our main welfare analysis and in Section 5.3 we discuss policy implementation.

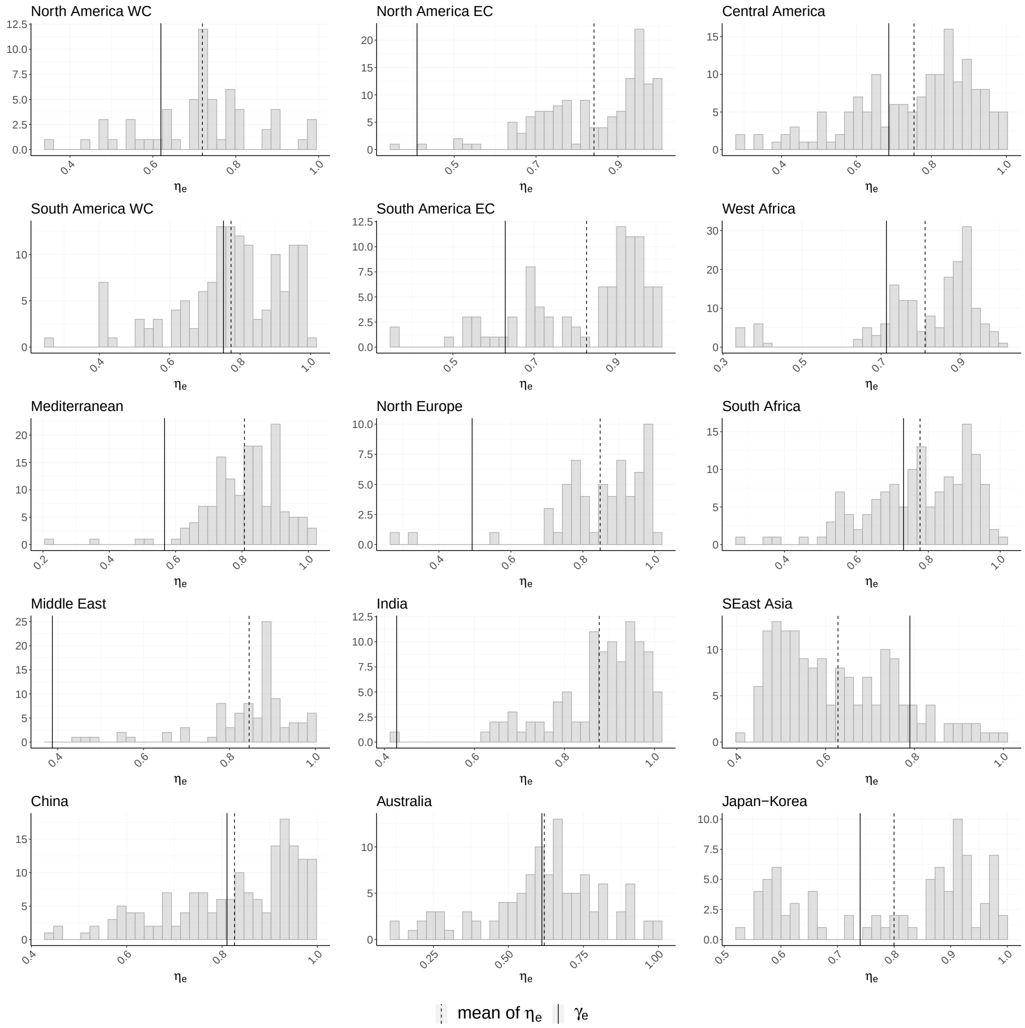
5.1 Is dry bulk shipping efficient?

Efficiency requires that the following conditions are met: (i) the elasticity of the matching function with respect to each input must equal the corresponding bargaining coefficient (thin/thick market externalities); (ii) the ship surplus must equalize across destinations (pooling externalities). We test each of these conditions in the data.

Figure 2 examines whether the thin/thick market externalities are internalized. For each region, the left panel presents the histogram of the estimated matching function elasticity with respect to exporters, as well as the estimated exporter bargaining coefficient. For several regions, as shown in the right panel, we reject that the average elasticity of the matching function η^e is equal to the exporter bargaining coefficient. Although the "knife-edge" nature of these conditions implies that this finding is not particularly surprising, it is worth noting that the difference between the elasticity and the bargaining coefficient is often large, suggesting that the "Hosios conditions" for efficiency (28) and (29) are not satisfied. Moreover, the exporter bargaining coefficient tends to be lower than the matching function elasticity, suggesting that the planner would like to see an increase in the share of the surplus accruing to the exporter.

Figure 3 checks whether the pooling externalities are internalized. For each region i, it plots the coefficient of variation of the ship surplus from traveling to all destinations $j \neq i$. When pooling externalities are internalized, this coefficient of variation should be equal to zero, since the ship is indifferent across destinations. Figure 3 demonstrates that this is not the case in bulk shipping. In all regions the coefficient of variation is significantly different from zero, and larger than 20%, while in several regions it is substantially higher.

We conclude that the market has not internalized neither the pooling externalities, nor the thin/thick market ones.



	t-stat
North America WC	4.900
North America EC	10.155
Central America	3.002
South America WC	3.497
South America EC	4.080
West Africa	1.169
Mediterranean	6.129
North Europe	7.756
South Africa	1.649
Middle East	6.936
India	8.200
South East Asia	0.685
China	1.290
Australia	1.932
Japan-Korea	2.500

Figure 2: The left panel compares the exporter bargaining coefficient γ_i^e and the elasticity of the matching function with respect to exporters, estimated nonparametrically. The histogram corresponds to the estimated elasticity at different points in time. The dotted vertical line is the average elasticity and the solid line is the estimated bargaining coefficient. The right panel presents the t-statistic for the null that the exporter bargaining coefficient γ_i^e coincides with the average elasticity of the matching function with respect to exporters.

5.2 Welfare loss

We now come to our main welfare analysis. We begin by a comparison of (i) the market equilibrium; (ii) the constrained efficient outcome we analyzed in Section 3; (iii) the frictionless equilibrium (first-best), i.e., the outcome in a world without search frictions, so that $m = \min\{s, e\}$. To compute the constrained efficient outcome, we compute the equilibrium under the efficient prices given in equation (31) of Corollary

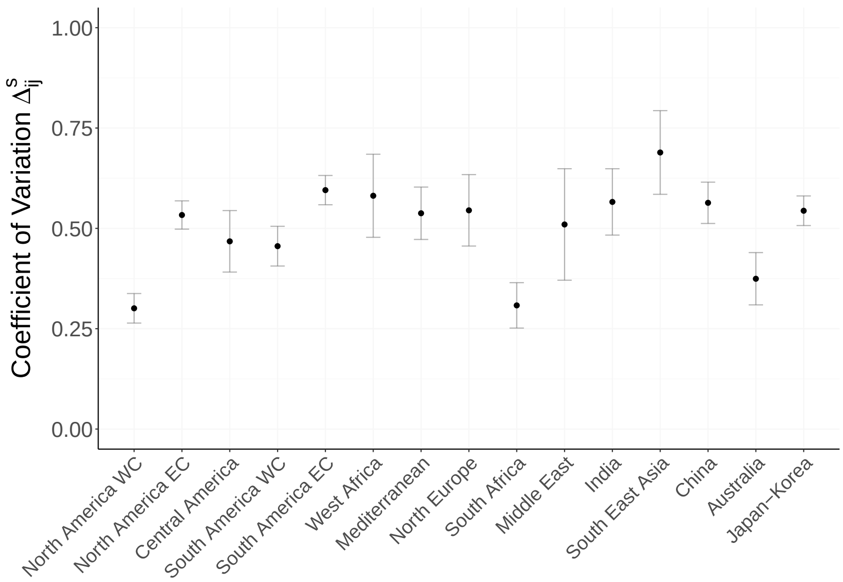


Figure 3: For each region i, we plot the coefficient of variation (standard deviation over mean) of ship surplus for all destinations $j \neq i$. When pooling externalities are internalized, the coefficient of variation should be zero.

1.[32] In terms of policy relevance, one can think of (ii) as what can be achieved by policy makers who are not able to affect the meeting process or the search environment. In contrast, (iii) loosely corresponds to a centralized market; one can think of it as a meeting platform, like Uber, which however does not exercise market power.[33] This three-way comparison allows us to assess both the overall impact of frictions on welfare, as well as the impact of the two externalities under study.

The results are shown in Table 2. As reported in the first three columns, total welfare in the market equilibrium allocation is 6% lower than the constrained efficient allocation and 14% lower than the frictionless equilibrium. Moreover, externalities coming from search frictions have a substantial impact on world trade, both in terms of value and volume. Indeed, trade volume is 13% higher under constrained efficiency and 36% higher under the first-best, while net trade value (i.e. $w_{ij} - \kappa_{ij}$) is 12% higher under constrained efficiency and 43% higher under the first-best. Moreover, ships would ballast 10% and 0.6% less under constrained efficiency and the first-best respectively; this suggests that although the majority of ballast traveling is attributed to the natural imbalance in the supply and demand of commodities rather

[32] Alternatively we can impose the optimal tax/subsidies derived in Corollary 2. The resulting allocation is the same.

[33] Other work has indeed modeled platforms as the eradication of search frictions; e.g. Frechette et al. (2019); Buchholz (2020).

than frictions as expected, some wasteful traveling does exist in the market equilibrium. Finally, ships wait less under constrained efficiency and in the frictionless world (9% and 23% respectively).

These results relay an important message: under the optimal policy, the market is able to achieve about 44% of the first-best welfare gains, which, following the literature (see footnote 33), we interpret as centralization. This is important as, in contrast to policies like taxes/subsidies, centralization may not be feasible in practice, or, may come with substantial market power if provided by private firms. Indeed, platforms that reduce search frictions between agents are emerging in a multitude of markets (for instance, Uber/Lyft in the taxi market, Uber Freight and other entrants in the trucking industry, but also Airbnb in the rental housing market and peer-to-peer lending in financial markets). Yet these platforms are likely exerting market power rather than acting as benevolent planners, so that the 14% welfare gain in the first-best allocation is likely a crude upper bound on the gains from centralization. Hence, the constrained efficient allocation, which at the very least achieves almost half of overall welfare gains, may well be the desirable outcome, and it is attainable by policy.

We now discuss the different role of the two externalities. In the third and fourth columns of Table 2, we compute the welfare loss when only pooling externalities or only thin/thick market externalities are internalized. To do so, we impose the relevant tax derived in Section 3.3 (see Footnotes 18 and 19) and compute the equilibrium. The welfare gains are 3.3% when thin/thick market externalities are internalized and 5.1% when pooling externalities are internalized. This suggests that both externalities introduce substantial distortions in the market equilibrium, with pooling externalities having a larger impact.

Table 2 also reveals that the two externalities have a qualitatively different impact on the economy; this is illustrated by the change in the total trade volume and value. The thin/thick market externalities have a large impact on the volume of trade, as they essentially distort the number of searching agents and therefore the total *number* of matches formed. Indeed, as shown in Table 2, correcting the thin/thick market externalities has a bigger impact on trade volume (which rises by 19%) than correcting both externalities (in which case trade volume rises by 13%). In contrast, pooling externalities have a large impact on trade value, as they distort the composition of exports by favoring destinations with low social value. As shown in Table 2, correcting the pooling externalities leads to a large increase in trade *value* (by 13%), as destinations with high social value are subsidized.

	Frictionless	Constrained Efficient	Pooling	Thin/Thick
Welfare	14.32 %	6.33 %	5.14 %	3.29%
Trade	36.50 %	13.55 %	-13.62 %	19.36 %
Trade value (net)	42.71 %	11.69 %	13.61 %	6.48%
Ballast miles	-0.60 %	-9.68%	-6.43%	-12.12%
Waiting ships	-22.97 %	-9.48%	5.19%	-8.47%

Table 2: Welfare Loss Analysis. The first column presents the frictionless allocation, i.e. the market equilibrium in the absence of search frictions when the matching function is m=min$\{s, e\}$. The second column presents the constrained efficient allocation, i.e. the market equilibrium under the efficient prices. The third and fourth columns present the market equilibrium when only the pooling and only the thin/thick market externalities respectively are internalized. All columns present the percent difference compared to the market equilibrium.

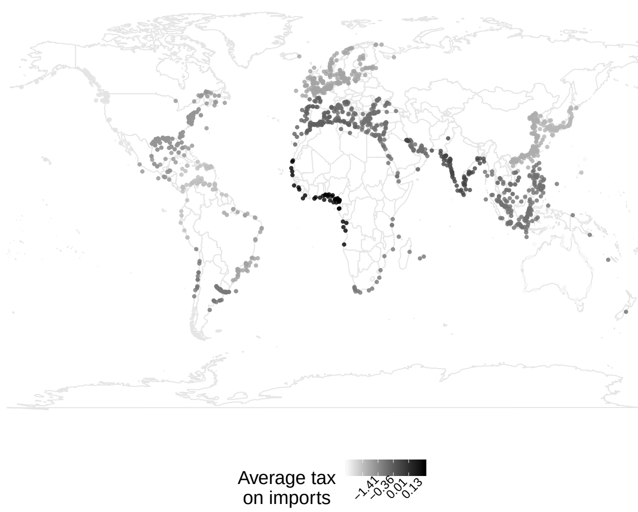
We next delve deeper into each externality in turn. As described in Section 3.2, thin/thick market externalities relate to the efficient entry of searching agents. Based on our estimates, the elasticity of the matching function with respect to exporters, η_i^e , is large, so that an additional exporter has a substantial positive externality on matching rates. However, for most regions, the exporter bargaining coefficient is lower than η_i^e (see Figure 2). Therefore, shipping prices are too high to achieve the socially efficient number of exporters. When thin/thick market externalities are internalized, this imbalance is corrected by lowering prices and increasing exporter entry, as shown in Figure 7 in Appendix D.[34]

Next, we turn to the pooling externalities. The optimal trip tax/subsidy that restores pooling externalities depends on the social surplus, $\bar{\Delta}_{ij}$, which is determined by several factors, such as the exporter valuation, distance, as well as the ship continuation value at the destination. In the right panel of Figure 4, we regress the optimal trip tax, h_{ij}^q, on these features. The planner subsidizes exporters with higher value w_{ij}; this is not surprising, as w_{ij} enters total welfare directly. In addition, the planner taxes distant destinations, as they are associated with high travel costs. Beyond this "direct" benefit (w_{ij}) and cost (c_{ij}^s) however, the planner also values the attractiveness of a destination j for ships. Attractive regions for ships may involve many customers, high value matches, low travel costs to other locations etc. Indeed,

[34] In addition to this "direct" effect, the final allocation is also determined by "general equilibrium" effects. Ships may avoid ballasting to regions with large price declines. As ship supply declines in those regions, prices rise and exporters stop entering, thus dampening the direct effect on prices. This phenomenon is most pronounced in the East Coast of North America and Northern Europe: these regions, which experience a large fall in prices, rely heavily on ships ballasting there as they are big exporters.

the planner subsidizes destinations that are big exporters, implying that the ship can easily reload there, and he taxes destinations that force the ship to ballast afterwards and/or to ballast somewhere far.

The left panel of Figure 4 plots the average import tax for each region (i.e., $\sum_i q_{ij} h^q_{ij} / \sum_i q_{ij}$). The highest subsidy is awarded to trips towards the West Coast of North America, as well Australia: these regions are high-value importers (w_{ij} is high when j corresponds to these regions), but at the same time, they offer high continuation values to ships that arrive there, as they are also big exporters. In other words, both the trip there is valuable, and the ship continuation value is high. In contrast, the highest taxes are levied on West Africa and India, as these are both low value importers and provide poor reloading options to ships. While imports in some regions are taxed, it is worth noting that exporters from all countries end up gaining in our results.[35]



	h^q_{ij}
Net exporter revenues on route ij	-0.09^{**}
	(0.03)
Trip duration (log)	0.007
	(0.23)
Total net value of exports from j	-0.78^{**}
	(0.10)
Probability of ballast from j (log)	2.86^{**}
	(0.89)
Duration of ballast trip from j (log)	3.88^{**}
	(1.12)

Figure 4: Pooling Externalities. The left panel plots the the average import tax for each region, $\sum_i q_{ij} h^q_{ij} / \sum_i q_{ij}$. The right panel regresses the optimal tax h^q_{ij} on features that affect the social surplus of a trip on route ij.

5.3 Policy implementation

Although the prices and optimal taxes that restore (constrained) efficiency have known expressions, they may not be feasible to implement in practice, either because the planner does not have access to all instruments; or because the expressions may be too complex or computationally challenging. As an

[35]In addition, as mentioned in Section 3.3, we restrict the planner budget to be zero at each origin, so that the taxes redistribute exporters only across destinations within each origin.

example, the planner may not be able to set prices. Moreover, he may be able to tax trips, but not searching agents; indeed, it may be difficult to tax hailing passengers and searching exporters, or waiting taxis/ships. Finally, the matrix h^q may be very large, in which case the planner might prefer a simpler tax scheme.

In this section we consider simple policies that are designed to mimic the optimal taxes, but may be more easily implementable. In particular, we consider the following taxes: (i) an origin-specific tax on matches which can be interpreted as a flat tax on exports; (ii) a destination-specific tax on matches which can be interpreted as a customs tax; (iii) a linear in distance tax, resembling the taxi price schedule.

Table 3 reports the maximum welfare gains under these tax schemes. The destination-specific tax works best, as it achieves welfare gains of 2.8%. The origin-specific tax delivers only 0.9% welfare gains. This is consistent with our finding that pooling externalities account for a larger portion of the overall welfare loss. Indeed, to resolve pooling externalities, it is crucial to impose different taxes across destinations. Finally, taxes that are a function of distance achieve no welfare gains. This suggests that the pricing scheme used in taxis is far from efficient and cannot alleviate either externality. This finding is not surprising given the optimal taxes derived in Section 3.3, that explicitly target the origin and the destination to correct the thin/thick and pooling externalities respectively.

Optimal tax h^q_{ij}	Export tax h^q_j	Customs tax h^q_i	Distance αd_{ij}
6.33%	2.82%	0.9%	0%

Table 3: Simple policy instruments. This table reports the maximum welfare gains attained via three simple policy instruments: an origin-specific tax on matches (second column); a destination-specific tax on matches (third column); a linear in distance tax (fourth column).

6 Conclusion

This paper studies efficiency in decentralized transport markets, such as taxis, ships and trucks. In this setup, search frictions create two externalities: thin/thick market and pooling externalities. Because of the latter, we show that the well-known Hosios (1990) conditions are not sufficient to restore efficiency when agents are not homogeneous. We derive explicit and intuitive conditions for efficiency, which lead naturally to the efficient pricing rules. Moreover, we derive the optimal taxes that restore efficiency for a social planner that cannot set prices. Then, using data from dry bulk shipping, we demonstrate that search frictions are present and lead to a sizeable social loss. However, through optimal taxes/subsidies the market can achieve substantial welfare gains. In fact, these policies achieve 44% of the first-best welfare gains, suggesting that they may constitute a good alternative to centralized platforms. Finally, we use the intuition obtained from analyzing the nature of the two externalities to design simple policies that mimic the optimal taxes.

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Online Appendix

A Proofs

A.1 Preliminaries: limit equilibrium outcomes and associated dual variables

In this section we show that every limit equilibrium outcome can be associated with a set of dual variables corresponding to the no-discounting limits of the agents' value function. These variables will be instrumental in the proofs of Theorems 1 and 2 below.

Let (s, E, q, b, τ) be a limit equilibrium outcome and $(s^n, E^n, q^n, b^n, \tau^n, \beta^n)_{n\geq 0}$ be a corresponding sequence of equilibrium outcomes and discount factors, as defined in Definition 2. For each n, let $V^{s,n}, U^{s,n}, \Delta^{s,n}$ and $U^{e,n}, \Delta^{e,n}$ be the corresponding value functions and meeting surpluses for carriers and customers, respectively. Fix an arbitrary reference location i^*.

Lemma 1. *The sequences* $(V_i^{s,n} - V_{i^*}^{s,n})_{n\geq 0}$, $(U_i^{s,n} - V_{i^*}^{s,n})_{n\geq 0}$, $((1-\beta^n) V_{i^*}^{s,n})_{n\geq 0}$ *and* $\left(\Delta_{ij}^{s,n}\right)_{n\geq 0}$ *are bounded for every* i *and* j. $\left(\Delta_{ij}^{e,n}\right)_{n\geq 0}$ *is also bounded provided that* $\lambda_i^e > 0$.

Proof. Taking into account equation (4) we can rewrite the carrier's surplus, $\Delta_{ij}^{s,n}$, defined in equation (2) as $\Delta_{ij}^{s,n} = \max\left\{\tau_{ij}^n + V_{ij}^{s,n} - \max_j V_{ij}^{s,n}, 0\right\}$. Hence $\Delta_{ij}^{s,n}$ is bounded above by τ_{ij}^n and below by zero. Since τ^n converges to τ, it follows that $\Delta_{ij}^{s,n}$ is bounded. Note that in the steady state, equation (1) becomes: $V_{ij}^s = \left(-c_{ij}^s + \beta d_{ij} V_j^s\right) / \left(1 - \beta\left(1 - d_{ij}\right)\right)$. $(1-\beta^n) V_{i^*}^{s,n}$ is bounded as an average of bounded prices and the finite set of all possible per-period search and traveling costs.[36] $V_i^{s,n} - V_{i^*}^{s,n}$ is bounded below, since

[36]In particular, from equation (3), we have that $-c_i^s + \lambda_i^{s,n} \sum_j G_{ij}^n \Delta_{ij}^{s,n} + V_{ij}^{s,n} \leq V_i^{s,n}$ for all ij, so that

$$-c_i^s + \lambda_i^{s,n} \sum_j G_{ij}^n \Delta_{ij}^{s,n} + \frac{-c_{ij}^s + d_{ij}\beta^n V_j^{s,n}}{1-\beta^n\left(1-d_{ij}\right)} - V_i^{s,n} \leq 0 \tag{40}$$

Let $k_{ij}^n = -c_i^s + \lambda_i^{s,n} \sum_j G_{ij}^n \Delta_{ij}^{s,n} + \frac{-c_{ij}^s}{1-\beta^n\left(1-d_{ij}\right)}$. This is a bounded sequence. Hence (40) is written as $\frac{d_{ij}\beta^n V_j^{s,n}}{1-\beta^n\left(1-d_{ij}\right)} \leq V_i^{s,n} - k_{ij}^n$ for all $i \neq j$, or $V_j^{s,n} \leq \frac{1-\beta^n\left(1-d_{ij}\right)}{d_{ij}\beta^n} V_i^{s,n} - \frac{1-\beta^n\left(1-d_{ij}\right)}{d_{ij}\beta^n} k_{ij}^n$. Applying the same inequality on V_i^s and rearranging, we obtain

$$\left(1 - \frac{\left(1-\beta^n+\beta^n d_{ij}\right)\left(1-\beta^n+\beta^n d_{ji}\right)}{d_{ji}d_{ij}\beta^n}\right) V_j^{s,n} \leq -\frac{\left(1-\beta^n+\beta^n d_{ij}\right)}{d_{ij}\beta^n}\frac{\left(1-\beta^n+\beta^n d_{ji}\right)}{d_{ji}\beta^n} k_{ji}^n - \frac{1-\beta^n-\beta^n d_{ij}}{d_{ij}\beta^n} k_{ij}^n$$

It is easy to see that the right-hand-side is bounded. Moreover, the left-hand-side, after straightforward computations, becomes $\frac{1-\left(1-\beta^n\right)-\beta^n\left(d_{ij}+d_{ji}\right)}{d_{ij}d_{ji}\beta^n}\left(1-\beta^n\right) V_j^{s,n}$. But $\lim_{\beta\to 1} \frac{1-\left(1-\beta^n\right)-\beta^n\left(d_{ij}+d_{ji}\right)}{d_{ij}d_{ji}\beta^n} = \frac{\left(d_{ij}+d_{ji}\right)}{d_{ij}d_{ji}}$ and hence $\left(1-\beta^n\right) V_j^{s,n}$ is bounded.

we have

$$
\begin{aligned}
V_i^{s,n} - V_{i^*}^{s,n} &\geq -c_i^s + \lambda_i^{s,n} \sum_{j \neq i} G_{ij}^n \Delta_{ij}^{s,n} + \frac{\beta^n d_{ii^*} V_{i^*}^{s,n} - c_{ii^*}^s}{1 - \beta^n \left(1 - d_{ii^*}\right)} - V_{i^*}^{s,n} \\
&= -c_i^s + \lambda_i^{s,n} \sum_{j \neq i} G_{ij}^n \Delta_{ij}^{s,n} - \frac{c_{ii^*}^s}{1 - \beta^n \left(1 - d_{ii^*}\right)} - \frac{\left(1 - \beta^n\right) V_{i^*}^{s,n}}{1 - \beta^n \left(1 - d_{ii^*}\right)}
\end{aligned}
$$

and all sequences on the right-hand-side are bounded. Reversing the roles of i and i^* it follows that $V_i^{s,n} - V_{i^*}^{s,n}$ is bounded above as well.

Finally, if $\lambda_i^e > 0$, then $\lambda_i^{e,n} > 0$ for n large enough. Hence, based on Equations (10) and (11), for n large enough, we have,

$$
\frac{-c_{ij}^e + \lambda_i^{e,n} \left(w_{ij}\left(q^n\right) - \tau_{ij}^n\right)}{1 - \left(1 - \lambda_i^{e,n}\right) \beta^n} \leq U_{ij}^{e,n} \leq \kappa_{ij}.
$$

Since the left-hand-side converges, $U_{ij}^{e,n}$ is bounded. Finally, by equation (10), this implies that $\Delta_{ij}^{e,n}$ is bounded as well. $\square$

Given Lemma 1, there exists a sequence $\left(n_k\right)_{k \geq 0} \subseteq \mathbb{N}$ such that we can define the limits

$$
\phi_i = \lim_{k \to \infty} V_i^{s,n_k} - V_{i^*}^{s,n_k}
$$

$$
\psi_i = \lim_{k \to \infty} U_i^{s,n_k} - V_{i^*}^{s,n_k}
$$

$$
\upsilon = \lim_{k \to \infty} \left(1 - \beta^{n_k}\right) V_{i^*}^{s,n_k}
$$

$$
\bar{\Delta}_{ij}^s = \lim_{k \to \infty} \Delta_{ij}^{s,n_k}.
$$

If $\lambda_i^e > 0$ then we can also define the limit

$$
\bar{\Delta}_{ij}^e = \lim_{k \to \infty} \Delta_{ij}^{e,n_k}
$$

otherwise we simply define $\bar{\Delta}_{ij}^e = \max\left\{w_{ij}\left(q\right) - \tau_{ij} - \kappa_{ij}, 0\right\}.$

Note that for every i it holds that

$$\lim_{k\to\infty}\left(1-\beta^{n_k}\right)V_i^{s,n_k} = \underbrace{\lim_{k\to\infty}\left(1-\beta^{n_k}\right)\left(V_i^{s,n_k}-V_{i^*}^{s,n_k}\right)}_{=0} + \lim_{k\to\infty}\left(1-\beta^{n_k}\right)V_{i^*}^{s,n_k} = \upsilon.$$

Definition 4. $\left(\phi,\psi,\upsilon,\bar{\Delta}^s,\bar{\Delta}^e\right)$ is a tuple of equilibrium dual variables associated with the limit equilibrium outcome (s,E,q,b,τ).

Lemma 2. *Let (s,E,q,b,τ) be a limit equilibrium outcome and $\left(\phi,\psi,\upsilon,\bar{\Delta}^s,\bar{\Delta}^e\right)$ be a tuple of dual variables associated with it. Then the following conditions hold for every i,j:*

$$\psi_i \geq \phi_j - \frac{c_{ij}^s}{d_{ij}} - \frac{\upsilon}{d_{ij}} \text{ with equality if } b_{ij}>0 \tag{41}$$

$$\bar{\Delta}_{ij}^s \geq 0 \text{ with equality if } q_{ij} < s_i\lambda_i^s G_{ij} \tag{42}$$

$$\phi_i \geq -c_i^s + \lambda_i^s \sum_{j\neq i} G_{ij}\bar{\Delta}_{ij}^s + \psi_i \text{ with equality if } s_i>0 \tag{43}$$

$$\upsilon \geq 0 \text{ with equality if } \sum_{ij}\frac{q_{ij}+b_{ij}}{d_{ij}} < S \tag{44}$$

$$\bar{\Delta}_{ij}^e \geq 0 \text{ with equality if } q_{ij} < \lambda_i^e e_{ij} \tag{45}$$

$$-c_{ij}^e + \lambda_i^e\bar{\Delta}_{ij}^e = 0 \tag{46}$$

$$\bar{\Delta}_{ij}^s \geq \tau_{ij} + \phi_j - \psi_i - \frac{\upsilon}{d_{ij}} \text{ with equality if } q_{ij}>0 \tag{47}$$

$$\bar{\Delta}_{ij}^e \geq w_{ij}(q) - \kappa_{ij} - \tau_{ij} \text{ with equality if } q_{ij}>0. \tag{48}$$

Proof. The reader can verify this by taking the no-discounting limits of the equilibrium conditions (2)-(14). For example, the equilibrium conditions (4) and (8) can be written as (taking into account that in steady state $V_{ij}^s = \left(\beta d_{ij}V_j^s - c_{ij}^s\right)/\left(1-\beta\left(1-d_{ij}\right)\right)$)

$$U_i^{s,n} > \frac{\beta^n d_{ij}V_j^{s,n} - c_{ij}^s}{1-\left(1-d_{ij}\right)\beta^n}, \text{ with equality if } b_{ij}^n > 0.$$

Subtracting $V_{i^*}^{s,n}$ from both sides we obtain,

$$U_i^{s,n} - V_{i^*}^{s,n} > \frac{\beta^n d_{ij}\left(V_j^{s,n} - V_{i^*}^{s,n}\right)}{1-(1-d_{ij})\beta^n} - \frac{c_{ij}^s}{1-(1-d_{ij})\beta^n} - \frac{(1-\beta^n)V_{i^*}^{s,n}}{1-(1-d_{ij})\beta^n}, \quad \text{with equality if } b_{ij}^n{>}0$$

Taking limits of both sides as $n \to \infty$ yields Condition (41).

As another example, notice that the equilibrium conditions (2), (6) and (7) are equivalent to

$$\Delta_{ij}^{s,n} \geq 0, \text{ with equality if } q_{ij}^n < s_i^n \lambda_i^{s,n} G_{ij}^n$$

and

$$\Delta_{ij}^{s,n} \geq \tau_{ij}^n + V_{ij}^{s,n} - U_i^{s,n}, \text{ with equality if } q_{ij}^n > 0.$$

Taking the limit of the first one gives Condition (42). The second condition can be written as,

$$\Delta_{ij}^{s,n} \geq \tau_{ij}^n + V_{ij}^{s,n} - V_{i^*}^{s,n} - (U_i^{s,n} - V_{i^*}^{s,n}), \text{ with equality if } q_{ij}^n > 0.$$

Taking the limits of both sides gives Condition (47).

Analogous arguments establish the remaining conditions. More precisely, Condition (43) is a consequence of the equilibrium conditions (3) and (5); Condition (44) is a consequence of the equilibrium conditions (3) and (9); Condition (45) results from the the equilibrium conditions (10) and (12); Condition (46) is obtained from the equilibrium conditions (11) and (14); and finally, Condition (48) is obtained from the equilibrium conditions (10), (13) and (14). Finally notice that combining conditions (47) and (48) we obtain,

$$\bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e \geq w_{ij}(q) - \kappa_{ij} + \phi_j - \psi_i - \frac{\upsilon}{d_{ij}} \text{ with equality if } q_{ij} > 0. \tag{49}$$

□

A.2 Proof of Theorem 1

Consider Problem (20), and let $\phi, \psi, \upsilon, \bar{\Delta}^s$ and $\bar{\Delta}^e$ be the dual variables associated with constraints (15), (16), (17), (21) and (22), respectively. Since Problem (20) is concave, convex duality implies that

$\left(s, E, q, b, \phi, \psi, \upsilon, \bar{\Delta}^s, \bar{\Delta}^e\right)$ is an optimal dual pair of Problem (20) (that is, (s, E, q, b) is an optimal solution of Problem (20) and $\left(\phi, \psi, \upsilon, \bar{\Delta}^s, \bar{\Delta}^e\right)$ are the multipliers associated with the constraints) if and only if it satisfies the Karush-Kuhn-Tucker conditions (see for example Bertsekas, 2009, Prop. 5.2.2, pg. 167). The reader can verify that these can be written as conditions (41)-(46), as well as condition (49). Hence the result follows by taking $\left(\phi, \psi, \upsilon, \bar{\Delta}^s, \bar{\Delta}^e\right)$ to be a tuple of equilibrium dual variables associated with s, E, q, b and applying Lemma 2.

A.3 Proof of Theorem 2

First, we show that V_p is concave. Then, we characterize its supergradient. Finally, we exploit this characterization to prove the main result.

Lemma 3. *For each $(s, e, G) \geq 0$ such that $\sum_i s_i \leq S$ and $\sum_j G_{ij} = 1\ \forall i$, consider the set $M(s, e, G)$ of all pairs $(q, b) \geq 0$ satisfying constraints (15)-(17) given s, e, G. The multi-valued map $(s, e, G) \to M(s, e, G)$ satisfies*

$$(1-\lambda) M(u) + \lambda M(u') \subseteq M\left((1-\lambda) u + \lambda u'\right)$$

for every $u = (s, e, G)$ and $u' = (s', e', G')$.

Proof. It follows from the concavity of the matching function. □

Next we show that:

Lemma 4. *Let $f(x, u)$ be convex in (x, u) and $M(u)$ satisfies the convexity property of the previous lemma. Then the function:*

$$g(u) = \inf_{x \in M(u)} f(x, u)$$

is convex.

Proof. The above is a well-known result when $M(u)$ is convex and does not vary with u (see for instance Boyd and Vandenberghe, 2004). We adapt the proof to this case. Let u_1, u_2 and $\lambda \in [0, 1]$ and $\epsilon > 0$. Then there exist $x_1 \in M(u_1)$ and $x_2 \in M(u_2)$ such that $f(x_1, u_1) \leq g(u_1) + \epsilon$ and $f(x_2, u_2) \leq g(u_2) + \epsilon$. Then,

$$g\left(\lambda u_1 + (1-\lambda) u_2\right) = \inf_{x \in M(\lambda u_1 + (1-\lambda) u_2)} f\left(x, \lambda u_1 + (1-\lambda) u_2\right)$$

Since $\lambda x_1 + (1-\lambda) x_2 \in \lambda M(u_1) + (1-\lambda) M(u_2) \subseteq M(\lambda u_1 + (1-\lambda) u_2)$, we have,

$$\inf_{x \in M(\lambda u_1 + (1-\lambda) u_2)} f(x, \lambda u_1 + (1-\lambda) u_2) \leq f(\lambda x_1 + (1-\lambda) x_2, \lambda u_1 + (1-\lambda) u_2)$$

Since $f(\cdot)$ is convex in (x,u) we have,

$$\begin{aligned} g(\lambda u_1 + (1-\lambda) u_2) &\leq f(\lambda x_1 + (1-\lambda) x_2, \lambda u_1 + (1-\lambda) u_2) \\ &\leq \lambda f(x_1, u_1) + (1-\lambda) f(x_2, u_2) \\ &\leq \lambda g(u_1) + (1-\lambda) g(u_2) + \epsilon \end{aligned}$$

Since this is true for all ϵ, convexity is established. □

Applying this lemma to the function $-V^p(s,e,G)$, defined in (23), we obtain that $V^p(s,e,G)$ is concave. Hence, it is differentiable almost everywhere in its domain. Denote by $\partial V^p(s,e,G)$ the supergradient of V^p at a search allocation s,e,G, that is, the set of all vectors

$$y = \left(y(s_i)_{i\in I}, y(e_i)_{i\in I}, y(G_{ij})\right)_{i,j\in I} \in \mathbb{R}^I \times \mathbb{R}^I \times \mathbb{R}^{I\times I}$$

such that for every search allocation s', e', G':

$$V^p(s', e', G') - V^p(s,e,G) \leq \sum_i y(s_i)(s_i' - s_i) + \sum_i y(e_i)(e_i' - e_i) + \sum_{ij} y(G_{ij}) \left(G_{ij}' - G_{ij}\right).$$

Similarly, for every i,j, we denote by $\partial_{s_i} V^p(s,e,G)$, $\partial_{e_i} V^p(s,e,G)$ and $\partial_{G_{ij}} V^p(s,e,G)$ the supergradients of V^p at s,e,G with respect to s_i, e_i and G_{ij}, respectively.

Lemma 5. *Take a limit equilibrium allocation (s,e,G,q,b), and let $\left(\phi,\psi,\upsilon,\bar{\Delta}^s,\bar{\Delta}^e\right)$ be a tuple of equilibrium dual variables associated with it. For every i,j define*

$$y(s_i) = -\phi_i - c_i^s + \frac{dm_i(s_i,e_i)}{ds_i} \sum_j G_{ij}\left(\bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e\right) + \psi_i$$

$$y(e_i) = -c_{ij}^e + \frac{dm_i(s_i,e_i)}{de_i} \sum_j G_{ij}\left(\bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e\right)$$

$$y\left(G_{ij}\right)=-e_i c^e_{ij}+m_i\left(s_i,e_i\right)\left(\bar{\Delta}^s_{ij}+\bar{\Delta}^e_{ij}\right).$$

Then $y\in\partial V^p\left(s,e,G\right)$.

Proof. Consider Problem (23) defining $V^p\left(s,e,G\right)$. Its Lagrangian can be written as

$$\begin{aligned}L\left(q',b',\psi',\phi',\bar{\Delta}',\upsilon'|s,e,G\right)=&W\left(q'\right)+\sum_{ij}\left(q'_{ij}+b'_{ij}\right)\left(-\frac{c^s_{ij}}{d_{ij}}+\phi'_j-\psi'_i-\frac{\upsilon}{d_{ij}}\right)-\sum_{ij}q'_{ij}\left(\bar{\Delta}'_{ij}+\kappa_{ij}\right)\\&-\sum_i e_i\sum_j G_{ij}c^e_{ij}-\sum_i s_i\left(\phi'_i-\psi'_i+c^s_i\right)+\sum_i m_i\left(s_i,e_i\right)\sum_j G_{ij}\bar{\Delta}'_{ij}+S\upsilon'\end{aligned}$$

and the Karush-Kuhn-Tucker (K-K-T) conditions as

$$\psi_i\geq\phi_j-\frac{c^s_{ij}}{d_{ij}}-\frac{\upsilon}{d_{ij}}\text{ with equality if }b_{ij}>0$$

$$\bar{\Delta}_{ij}\geq 0\text{ with equality if }q_{ij}<m_i\left(s_i,e_i\right)G_{ij}$$

$$\bar{\Delta}_{ij}\geq\phi_j-\frac{c^s_{ij}}{d_{ij}}-\kappa_{ij}-\psi_i-\frac{\upsilon}{d_{ij}}\text{ with equality if }q_{ij}>0$$

$$\upsilon\geq 0\text{ with equality if }\sum_{ij}\frac{q_{ij}+b_{ij}}{d_{ij}}<S$$

which are equivalent to the set of Conditions (41), (42)/(45), (49) and (44), respectively, taking $\bar{\Delta}_{ij}=\bar{\Delta}^s_{ij}+\bar{\Delta}^e_{ij}$. Since the problem is concave, the K-K-T conditions are necessary and sufficient for optimality. Hence letting $\left(\phi,\psi,\upsilon,\bar{\Delta}^s,\bar{\Delta}^e\right)$ be a tuple of equilibrium dual variables associated with (s,e,G,q,b), it follows that $\left(q,b,\psi,\phi,\bar{\Delta}^s+\bar{\Delta}^e,\upsilon\right)$ is an optimal dual pair for Problem (23). From the assumptions of Theorem 2 , it follows that (q,b) is the unique optimal solution of Problem (23). Hence the result follows from Theorem 2 of Marimon and Werner (2019). □

We now proceed with the proof of the main result. By the previous analysis, Problem (24) is concave, hence optimality is characterized by the K-K-T conditions. Recall that we are assuming that s and e are in the interior of the feasible set ($s_i,e_i>0$ for each i and $\sum_i s_i<S$). Hence conditions (25) and (26) are equivalent to the first order conditions,

$$0\in\partial_{s_i}V^p\left(s,e,G\right)\ \forall i\text{ and }0\in\partial_{e_i}V^p\left(s,e,G\right)\ \forall i$$

respectively. Denoting by ν_{ij} and μ_i the multipliers associated with the constraints $G_{ij} \geq 0$ and $\sum_j G_{ij} = 1$, condition (27) is equivalent to

$$0 - \nu_{ij} - \mu_i \in \partial_{G_{ij}} V^p\left(s, e, G\right)$$

for some $\mu, \nu \in \mathbb{R}^I \times \mathbb{R}_+^{I \times I}$ such that $\nu_{ij} G_{ij} = 0$. It follows from the previous Lemma that conditions

$$\forall i : y\left(s_i\right) = 0 \tag{50}$$

$$\forall i : y\left(e_i\right) = 0 \tag{51}$$

$$\forall i, j : y\left(G_{ij}\right) + \mu_i \leq 0 \text{ with equality if } G_{ij} > 0 \tag{52}$$

are sufficient for conditions (25), (26) and (27), respectively, and they are necessary whenever $V^p\left(s, e, G\right)$ is differentiable. Comparing with condition (43), condition (50) is equivalent to

$$\begin{aligned}
&\forall i : -c_i^s + \frac{dm_i\left(s_i, e_i\right)}{ds_i} \sum_j G_{ij}\left(\bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e\right) + \psi_i = \phi_i = -c_i^s + \lambda_i^s \sum_j G_{ij} \bar{\Delta}_{ij}^s + \psi_i \\
&\quad \Leftrightarrow \eta_i^s \sum_j G_{ij}\left(\bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e\right) = \sum_j G_{ij} \bar{\Delta}_{ij}^s,
\end{aligned}$$

Similarly, comparing with condition (46), condition (51) is equivalent to

$$\begin{aligned}
&\forall i : -c_{ij}^e + \frac{dm_i\left(s_i, e_i\right)}{de_i} \sum_j G_{ij}\left(\bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e\right) = 0 = -c_{ij}^e + \lambda_i^e \sum_j G_{ij} \bar{\Delta}_{ij}^e \\
&\quad \Leftrightarrow \sum_j G_{ij} \bar{\Delta}_{ij}^e = \eta_i^e \sum_j G_{ij}\left(\bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e\right).
\end{aligned}$$

Condition (52) requires

$$-e_i c_{ij}^e + m_i\left(s_i, e_i\right)\left(\bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e\right) + \mu_i \leq 0 \text{ with equality if } G_{ij} > 0. \tag{53}$$

Since $-e_i c_{ij}^e + m_i\left(s_i, e_i\right) \bar{\Delta}_{ij}^e = 0$ from Condition (46), this is equivalent to Condition (iii) in the Theorem 2. This completes the proof of Theorem 2.

A.4 Proof of Corollary 1

Suppose that (s, e, G, q, b, τ) is efficient. Conditions (i) and (ii) of Theorem 2 imply that $\eta_i^s = 1 - \eta_i^e$ for all i. For every ij such that $G_{ij} > 0$, Conditions (i) and (iii) of Theorem 2 imply $\bar{\Delta}_{ij}^s = (1 - \eta_i^e) \sum_j G_{ij} \bar{\Delta}_{ij}$. By Condition (48) we have $\bar{\Delta}_{ij}^e = w_{ij}(q) - \kappa_{ij} - \tau_{ij}$. Substituting $\bar{\Delta}_{ij}^s = \bar{\Delta}_{ij} - \bar{\Delta}_{ij}^e = \bar{\Delta}_{ij} - w_{ij}(q) + \kappa_{ij} + \tau_{ij}$ yields Condition (31).

A.5 Efficiency Without Constant Returns to Scale

In the discussion following Theorem 2 we noted that, unless all matching functions display constant returns to scale, efficiency cannot be achieved. In this section we allow the planner to charge a price to customers, τ^e, that is different from the price paid to carriers, τ^s. The price wedge $\tau^e - \tau^s$ can be interpreted as a tax/subsidy. We show that in this setting the planner can achieve efficiency even when the matching functions do not exhibit constant returns to scale.

The definition of equilibrium can be extended to this case in a straightforward manner: $(s, e, G, q, b, \tau^e, \tau^s)$ is an equilibrium outcome, if carriers behave optimally given τ^s, λ^s and G; customers behave optimally given τ^e and λ^e; the feasibility constraints are satisfied; and λ^s, λ^e and G are consistent with the allocation.

The social planner's problem is the one stated in (24). Notice that the tax revenues do not appear in the social welfare since they are a transfer from the agents to the planner (one can imagine that the tax revenues are paid back to agents by means of a lump sum transfer). On the other hand, the limit social surplus resulting from a match now takes the planner's revenue into account:

$$\bar{\Delta}_{ij} = \bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e + \tau_{ij}^e - \tau_{ij}^s.$$

With these modifications, we can proceed along the lines of the proof of Theorem 2 to show the following:

Theorem 3. *Let $(s, e, G, q, b, \tau^e, \tau^s)$ be a limit equilibrium outcome. Suppose that Problem (23) admits a unique optimal solution. Then:*

(i) Carriers internalize thin/thick market externalities if and only if

$$\forall i \in I : \frac{\sum_j G_{ij} \bar{\Delta}_{ij}^s}{\sum_j G_{ij} \bar{\Delta}_{ij}} = \eta_i^s. \tag{54}$$

(ii) Customers internalize thin/thick market externalities if and only if

$$\forall i \in I: \frac{\sum_j G_{ij} \bar{\Delta}_{ij}^e}{\sum_j G_{ij} \bar{\Delta}_{ij}} = \eta_i^e. \tag{55}$$

(iii) Customers internalize pooling externalities if and only if

$$\bar{\Delta}_{ij}^s + \tau_{ij}^e - \tau_{ij}^s = \max_{k \neq i} \left(\bar{\Delta}_{ik}^s + \tau_{ik}^e - \tau_{ik}^s \right) \tag{56}$$

for all ij such that $G_{ij} > 0$.

Hence, the characterization of efficiency in the economy with a price wedge is the same as the one in the main text except for the last condition, which requires that, at each location i, the sum of the carrier surplus and the planner's revenue is constant across destinations– in other words, customers must fully internalize the differences in the matching surpluses across different destinations. We can then use this result to characterize the optimal pricing rules. Let $(s, e, G, q, b, \tau^e, \tau^s)$ be an efficient limit equilibrium outcome and suppose that (s, e, G, q, b) is efficient. For simplicity, consider the case where $G_{ij} > 0$ for all ij. Condition (55) can be written as

$$\sum_j G_{ij} \left(\bar{\Delta}_{ij}^s + \tau_{ij}^e - \tau_{ij}^s \right) = \sum_j G_{ij} \left(\bar{\Delta}_{ij} - \bar{\Delta}_{ij}^e \right) = (1 - \eta_i^e) \sum_j G_{ij} \bar{\Delta}_{ij}.$$

Hence Condition (56) implies that

$$\bar{\Delta}_{ij} - \bar{\Delta}_{ij}^e = \bar{\Delta}_{ij}^s + \tau_{ij}^e - \tau_{ij}^s = (1 - \eta_i^e) \sum_j G_{ij} \bar{\Delta}_{ij}$$

for every ij. Substituting $\bar{\Delta}_{ij}^e = w_{ij}(q) - \tau_{ij}^e - \kappa_{ij}$ into this equation we find

$$\tau_{ij}^e = w_{ij}(q) - \kappa_{ij} - \bar{\Delta}_{ij} + (1 - \eta_i^e) \sum_j G_{ij} \bar{\Delta}_{ij} \tag{57}$$

which is the pricing rule in equation (31). The average price wedge can be derived by summing conditions

(54) and (55), and recalling that $\tau^e_{ij} - \tau^s_{ij} = \bar{\Delta}_{ij} - \bar{\Delta}^e_{ij} - \bar{\Delta}^s_{ij}$:

$$\sum_j G_{ij} \left(\tau^e_{ij} - \tau^s_{ij} \right) = (1 - \eta^e_i - \eta^s_i) \sum_j G_{ij} \bar{\Delta}_{ij}. \tag{58}$$

This expression can be interpreted as saying that the average price wedge at each location is proportional to the "degree of decreasing returns to scale". Under constant returns to scale the wedge is zero: consistently with our main results, efficiency in this case can be achieved by setting a unique price on every route. If the matching function has decreasing returns to scale then the price wedge is positive, imposing a tax on matches at that location, capturing the social cost of making additional matches harder to form because of decreasing returns. On the contrary, matches are subsidized when the matching functions have increasing returns.

Conversely, it is easy to see that equations (57) and (58) imply equations (54)-(56).

We state the conclusions of this section below:

Corollary 3. *Let $(s, e, G, q, b, \tau^e, \tau^s)$ be a limit equilibrium outcome. Then s, e, G, q, b is efficient if and only if for all i, j*

$$\tau^e_{ij} = w_{ij}(q) - \kappa_{ij} - \bar{\Delta}_{ij} + (1 - \eta^e_i) \sum_j G_{ij} \bar{\Delta}_{ij}$$

and

$$\sum_j G_{ij} \left(\tau^e_{ij} - \tau^s_{ij} \right) = (1 - \eta^e_i - \eta^s_i) \sum_j G_{ij} \bar{\Delta}_{ij}$$

A.6 Proof of Corollary 2

Before proceeding with the proof, we briefly describe how incentives and total welfare are affected by a vector of taxes/subsidies $h = (h^q, h^s, h^e)$. The dynamic problem for customers is the same as in Section 2.2 except that now customers waiting at location i pay the amount h^e_i every period (on top of the private waiting cost c^e_{ij}); carriers searching at location i pay the amount h^s_i every period (on top of their private search cost c^s_i); and every match on route ij is taxed by the amount h^q_{ij}. It does not matter which side pays the trip tax (customers or carriers), so suppose that it is paid by customers (see footnote 16).

Therefore, the only expressions that change compared to Section 2.2 are the carriers' value of searching:

$$V_i^s = \max\left\{-c_i^s - h_i^s + \lambda_i^s \sum_{j\neq i} G_{ij}\Delta_{ij}^s + U_i^s, \beta V_i^s\right\}$$

and the customers' value of waiting and meeting surplus:

$$U_{ij}^e = -c_{ij}^e - h_i^e + \lambda_i^e \Delta_{ij}^e + \beta U_{ij}^e$$

$$\Delta_{ij}^e = \max\left\{w_{ij}(q) - \tau_{ij} - h_{ij}^q - \beta U_{ij}^e, 0\right\}.$$

The definition of equilibrium allocations can be given as in Section 2.2.

The social planner's problem is the one stated in (24). Notice that the tax revenues do not appear in the social welfare since they are a transfer from the agents to the social planner (one can imagine that the tax revenues are paid back to agents by means of a lump sum transfer). On the other hand, the limit social surplus resulting from a match now takes into account the planner's revenue as well:

$$\bar{\Delta}_{ij} = \bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e + h_{ij}^q.$$

Now for the proof of Corollary 2, let q, s, e, G be a limit equilibrium allocation in the economy with taxes. Proceeding as in the proof of Theorem 2, one can show that[37]

[37] In particular, we proceed by showing that the limits of the equilibrium conditions are a slight modification of conditions (41)-(48) in Section A.1

$$w_{ij}(q) - \frac{c_{ij}^s}{d_{ij}} - \kappa_{ij} + \phi_j - \psi_i - h_{ij}^q - \bar{\Delta}_{ij}^e - \bar{\Delta}_{ij}^s - \frac{\upsilon}{d_{ij}} \leq 0 \text{ with equality if } q_{ij} > 0$$

$$-\frac{c_{ij}^s}{d_{ij}} + \phi_j - \psi_i - \frac{\upsilon}{d_{ij}} \leq 0 \text{ with equality if } b_{ij} > 0$$

$$\bar{\Delta}_{ij}^e, \bar{\Delta}_{ij}^s \geq 0 \text{ with equality if } q_{ij} \leq m_i(s_i, e_i) G_{ij}$$

$$\upsilon \geq 0 \text{ with equality if } \sum \frac{q_{ij} + b_{ij}}{d_{ij}} < S$$

$$\phi_i = -c_i^s - h_i^s + \psi_i + \lambda_i^s \sum_j G_{ij}\bar{\Delta}_{ij}^s$$

$$-c_{ij}^e - h_i^e + \lambda_i^e \bar{\Delta}_{ij}^e \leq 0$$

$$w_{ij}(q) - \tau_{ij}^e - \kappa_{ij} - \bar{\Delta}_{ij}^e \leq 0 \text{ with equality if } q_{ij} > 0$$

Moreover, Nash bargaining requires that $(1-\gamma_i)\bar{\Delta}_{ij}^s = \gamma_i \bar{\Delta}_{ij}^e$. On the other hand, the conditions for different externalities to be internalized are unchanged. Comparing the efficiency conditions with the new equilibrium conditions as in the proof

- Thin/thick market externalities are internalized if and only if for all i,

$$\sum_j G_{ij}\bar{\Delta}^s_{ij} = \eta^s_i \sum_j G_{ij}\bar{\Delta}_{ij} + \frac{h^s_i}{\lambda^s_i} \tag{59}$$

  and

$$\sum_j G_{ij}\bar{\Delta}^e_{ij} = \eta^e_i \sum_j G_{ij}\bar{\Delta}_{ij} + \frac{h^e_i}{\lambda^e_i}$$

- Pooling externalities are internalized if and only if for all i, j,

$$\bar{\Delta}^s_{ij} + h^q_{ij} \le L_i \text{ with equality if } G_{ij} > 0$$

  where L_i is an arbitrary constant.

Using the definition $\bar{\Delta}_{ij} = \bar{\Delta}^e_{ij} + \bar{\Delta}^s_{ij} + h^q_{ij}$ and the Nash bargaining condition $(1-\gamma_i)\,\bar{\Delta}^s_{ij} = \gamma_i\bar{\Delta}^e_{ij}$ it follows that $\bar{\Delta}_{ij} = \frac{1}{\gamma_i}\bar{\Delta}^s_{ij} + h^q_{ij}$ or $\bar{\Delta}^s_{ij} = \gamma_i\bar{\Delta}_{ij} - \gamma_i h^q_{ij}$. Substituting $\bar{\Delta}^s_{ij}$ into (59) we obtain the Condition (34). We proceed similarly for customers to obtain Condition (35).

Next, we turn to the relationship $\bar{\Delta}^s_{ij} + h^q_{ij} \le L_i$ with equality if $G_{ij} > 0$. The constant L_i is related to the Lagrange multiplier associated with the constraint $\sum_j G_{ij} = 1$. Consider all j such that $G_{ij} > 0$. Then $\bar{\Delta}^s_{ij} + h^q_{ij} = L_i$. Multiply by G_{ij} and sum over j to obtain, $L_i = \sum_j G_{ij}\bar{\Delta}^s{}_{ij} + \sum_j G_{ij}h^q_{ij}$. Clearly the sums can be extended to all j since the terms with $G_{ij} = 0$ do not contribute to the sum. Thus pooling externalities are internalized if and only if

$$\bar{\Delta}^s_{ij} + h^q_{ij} \le \sum_j G_{ij}\bar{\Delta}^s{}_{ij} + \sum_j G_{ij}h^q_{ij}$$

with equality if $G_{ij} > 0$. We now express $\bar{\Delta}^s_{ij}$ in terms of $\bar{\Delta}_{ij}$ using the surplus sharing condition which yields $\bar{\Delta}^s_{ij} = \gamma_i\left(\bar{\Delta}_{ij} - h^q_{ij}\right)$:

$$\gamma_i\left(\bar{\Delta}_{ij} - h^q_{ij}\right) + h^q_{ij} \le \gamma_i \sum_j G_{ij}\left(\bar{\Delta}_{ij} - h^q_{ij}\right) + \sum_j G_{ij}h^q_{ij}$$

of Theorem 2 we get the result shown below.

$$\gamma_i \bar{\Delta}_{ij} + (1 - \gamma_i)\, h_{ij}^q \le \gamma_i \sum_j G_{ij} \bar{\Delta}_{ij} + (1 - \gamma_i) \sum_j G_{ij} h_{ij}^q$$

which proves (36).

B Random search in bulk shipping

In this section we investigate whether search in bulk shipping is random (or undirected), as assumed in the model of Section 2. We contrast this with the case of directed search (see e.g. Moen, 1997), where carriers choose to search in a specific "market", i.e. a market for customers heading to a specific destination. Under directed search, profitable markets attract more carriers, thereby reducing their matching probabilities compared to less profitable markets. We can directly test this implication of directed search by checking whether in a given origin, i, ships' waiting time is different across destinations j. We use 15 regions, so for a given region there are (up to) 14 possible destinations; therefore there are $\binom{14}{2} = 91$ such equalities to test for every origin i. Using a simple F-test we are only able to reject the null of no difference for 16% of the equalities.

In addition, we examine the coefficient of variation of matching probabilities within a given origin. Weighted by trade shares, the average coefficient of variation is just 8%. In contrast, the coefficient of variation of trip prices from a given origin is substantially higher and equal to 46%, suggesting that differences in the attractiveness of different types of customers is reflected in prices, but not in matching probabilities, as would be the case in directed search.

C Estimation and computation details

C.1 Model estimation and results

In this section we discuss the estimation of the model. The main parameters of interest are: the matching functions $m_i(s_i, e_i)$ for all i, the ship travel and wait costs c_{ij}^s, c_i^s, for all i, j, as well as the standard deviation of the logit shocks σ; the exporter valuations w_{ij}, the exporter waiting costs c_i^e, and entry costs κ_{ij} for all i, j; and the bargaining coefficients γ_i for all i. The available data consist of the matches m_i and ships s_i for all i, the ship ballast choice probabilities P_{ij}, for all ij, the average prices τ_{ij} on all routes

ij, the exporter entry probabilities P^e_{ij}, for all ij as well as total trade values by country pair (Comtrade). We describe the estimation of each object in turn.

Matching function estimation We briefly outline the approach adopted to estimate the matching function in BKP. To illustrate, assume that s and e are independent. We assume that $m(s,e)$ is continuous and strictly increasing in e, that it exhibits constant returns to scale (CRS), so that $m(as, ae) = am(s,e)$ for all $a > 0$, and that there is a known point $\{\bar{s}, \bar{e}, \bar{m}\}$, such that $\bar{m} = m(\bar{s}, \bar{e})$. The intuition behind the identification argument is as follows: the observed correlation between s and m informs us on $\partial m(s,e)/\partial s$, since by assumption s and e are independent; then, due to CRS, this derivative also delivers the derivative $\partial m(s,e)/\partial e$; once these derivatives are known, the matching function is known and can be inverted to provide the number of exporters.

More formally, suppose we have a sample $\{s_{it}, m_{it}\}_{t=0}^{T}$ for each market i. The unknowns of interest are the I matching functions $m_i(\cdot)$ and the exporters e_{it}, for all i, t; henceforth, we suppress the i subscript to ease notation. Let $F_{m|s}$ denote the distribution of matches conditional on ships, and F_e the distribution of exporters, e. Then at a given point $\{s_t, e_t, m_t\}$ we have:

$$\begin{aligned} F_{m|s=s_t}(m_t|s=s_t) &= \Pr(m(s,e) \leq m_t|s=s_t) = \Pr\left(e \leq m^{-1}(s, m_t)\,|s=s_t\right) \\ &= \Pr\left(e \leq m^{-1}(s_t, m_t)\right) = F_e(e_t) \end{aligned}$$

This equation, along with the CRS assumption, allows us to recover the distribution $F_e(e)$, for all e: using the known point $\{\bar{s}, \bar{e}, \bar{m}\}$ and letting $a = e/\bar{e}$, for all e,

$$F_e(a\bar{e}) = F_{m|s=a\bar{s}}(m(a\bar{s}, a\bar{e})\,|s=a\bar{s}) = F_{m|s=a\bar{s}}(a\bar{m}|s=a\bar{s})$$

We use this and vary a to trace out $\hat{F}_e(e)$, relying on a kernel density estimator for the conditional distribution $\hat{F}_{m|s=a\bar{s}}(a\bar{m}|s=a\bar{s})$.[38]

Since it is unlikely that s and e are independent, we employ an instrument, which consists of the ocean

[38] We choose the known point, $\{\bar{s}, \bar{e}, \bar{m}\}$, to be of the form $1 = m(\bar{s}, 1)$, so that one exporter is always matched when there are $\bar{s}$ ships. We set $\bar{s}$ iteratively, to be the lowest value such that $m_t \leq e_t$, for all t, thus obtaining a conservative bound on search frictions.

weather conditions (unpredictable wind at sea) that shift the arrival of ships at a port without affecting the number of exporters (also employed in the search frictions test, see Section 4.2).[39] Table 4 presents the first stage estimates.

Figure 5 reports our estimates for search frictions, along with confidence intervals constructed from 200 bootstrap samples.

	F-stat
North America West Coast	21.132
North America East Coast	18.429
Central America	17.877
South America West Coast	18.671
South America East Coast	16.889
West Africa	16.333
Mediterranean	46.072
North Europe	28.651
South Africa	13.153
Middle East	68.037
India	29.521
South East Asia	34.909
China	28.642
Australia	35.977
Japan-Korea	32.794

Table 4: First Stage, Matching Function Estimation. Regressions of the number of ships in each region on the unpredictable component of weather conditions in the surrounding seas. The table reports the F-statistic. For the construction of the instrument, see Table 1.

Ship parameters We use the estimates for the ship parameters $\left\{c_{ij}^s, c_i^s, \sigma\right\}$ from BKP. To estimate these parameters, we used a Nested Fixed Point Algorithm (Rust, 1987): at every guess of the parameters $\left\{c_{ij}^s, c_i^s, \sigma\right\}$ for all i, j, we employ a fixed point algorithm to solve for the ship value functions V_i^s, V_{ij}^s, U_i^s, for all i, j from equations (1), (3), and (39), using the observed average prices for each route ij and the observed meeting probability λ_i^s (which is set equal to the average m_i/s_i). We then match the ship ballast

[39]Assume that an instrument z exists such that $s = h(z, \eta)$, with z independent of e, η. The approach now has two steps. In the first step, we recover η using the relationship $s = h(z, \eta)$. In the second step, we repeat the above conditioning on both s (as before) and η, $F_{m|s=s_t,\eta}(m_t|s = s_t, \eta) = F_{e|\eta}(e_t|\eta)$. We recover the unknowns of interest e and $m(\cdot)$, by integrating both sides over η.

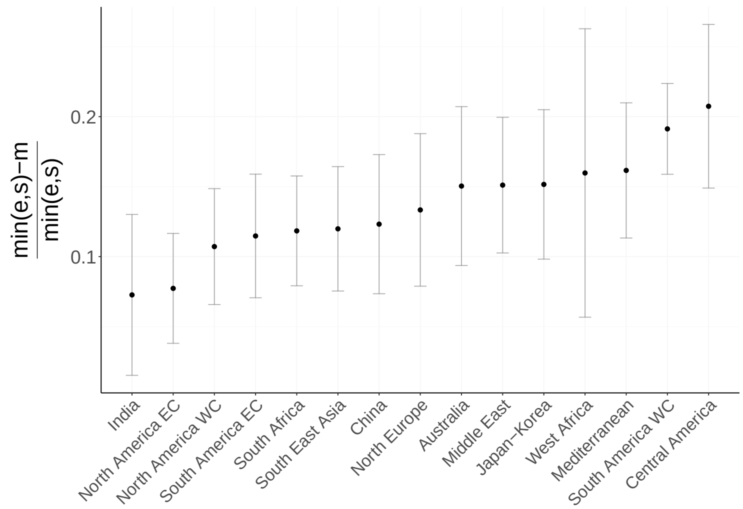

Figure 5: Search Frictions. Average weekly share of unrealized matches, with confidence intervals from 200 bootstrap samples.

choices predicted by our model and given by the logit choice probabilities,

$$P_{ij} = \frac{\exp\left(V_{ij}^s/\sigma\right)}{\sum_l \exp\left(V_{il}^s/\sigma\right)} \tag{60}$$

to the observed ballast choices. We do so by maximizing over the parameters via Maximum Likelihood. See BKP for further details on identification and estimation.[40] Table 5 reports the estimates of the ship cost parameters, along with confidence intervals constructed from 200 bootstrap samples.

Exporter parameters and bargaining coefficients We are left with four sets of parameters: the exporter valuations w_{ij}, the waiting costs c_i^e, the bargaining coefficients γ_i; and the exporter entry costs κ_{ij}, for all i, j.

The valuations w_{ij} are the revenues of exporters in i from selling their commodities to destination j. We compute them using aggregate trade data from Comtrade, which reports product-level export values

[40] In BKP we construct seven groups for the sailing cost c_{ij}^s, roughly based on the continent and coast of the origin; and we estimate wait costs c_i^s, for all i. The seven groups are: (i) Central America, West Coast Americas; (ii) East Coast Americas; (iii) West and South Africa; (iv) Mediterranean, Middle East and North Europe; (v) India; (vi) Australia and Southeast Asia; (vii) China, Japan and Korea. Since c_{ij}^s is the per week sailing cost from i to j, its major component is the cost of fuel. We set this cost for one of the groups (for trips originating from the East Coast of North and South America) equal to the average weekly fuel price (40,000 US dollars); such a restriction is required for identification as is standard in dynamic discrete choice. Moreover, since the fuel cost is paid by the exporter when the ship is loaded, we add it to the observed prices.

and quantities by country pair. We focus on bulk commodities and compute the average value of a cargo of commodities exported from each region i to each j, which forms our direct estimate for w_{ij}; details are provided in the next section.

Next, we turn to c_i^e and γ_i, for all i, which we estimate from observed shipping prices. Nash bargaining implies the surplus sharing condition,

$$(1-\gamma_i)\left(\tau_{ij}+V_{ij}^s-U_i^s\right)=\gamma_i\left[w_{ij}-\tau_{ij}-U_{ij}^e\right]$$

where if we substitute the exporter value U_{ij}^e from its steady state value, $U_{ij}^e=\left(-c_i^e+\lambda_i^e\left(w_{ij}-\tau_{ij}\right)\right)/\left(1-\beta\left(1-\lambda_i^e\right)\right)$, we obtain,

$$\tau_{ij}=\gamma_i\frac{c_i^e+\left(\left(1-\beta\right)\left(1-\lambda_i^e\right)\right)w_{ij}}{1-\beta\left(1-\lambda_i^e\right)-\gamma_i\lambda_i^e}+\frac{\left(1-\gamma_i\right)\left(1-\beta\left(1-\lambda_i^e\right)\right)}{1-\beta\left(1-\lambda_i^e\right)-\gamma_i\lambda_i^e}\left(U_i^s-V_{ij}^s\right)$$

In this equation, the only unknowns are γ_i and c_i^e, for all i; indeed, note that λ_i^e is known from the matching function (set equal to m_i/e_i); U_i^s, V_{ij}^s are known once the ship cost parameters are known; w_{ij} is obtained from Comtrade data as described above; and β is calibrated to 0.995. We thus estimate γ_i and c_i^e via non-linear least squares. Identification results from variation over the regions i, j: intuitively, the identification of the bargaining coefficient γ_i relies on the correlation of prices τ_{ij} and values w_{ij} across destinations j, while the inventory cost c_i^e matches the overall level of prices at origin i. To gain power, we restrict c_i^e to be constant within a continent.

Finally, exporter entry costs κ_{ij} are estimated using the exporter entry probabilities, which are given by

$$P_{ij}^e=\frac{\exp\left(U_{ij}^e-\kappa_{ij}\right)}{1+\sum_{l\neq i}\exp\left(U_{il}^e-\kappa_{il}\right)} \tag{61}$$

for $j\neq i$, where $P_{ii}^e=1/\left(1+\sum_{l\neq i}\exp\left(U_{il}^e-\kappa_{il}\right)\right)$ is interpreted as the option of not exporting at all. Then, following BKP:

$$\ln P_{ij}^e-\ln P_{ii}^e=U_{ij}^e-\kappa_{ij}=\frac{-c_i^e+\lambda_i^e\left(w_{ij}-\tau_{ij}\right)}{1-\beta\left(1-\lambda_i^e\right)}-\kappa_{ij}$$

where κ_{ij} is the only unknown.[41] The results are presented in Table 6.

[41]To recover P_{ii}^e, the share of the "outside good", corresponding to the choice of not exporting, we use the total production

C.2 Exporter valuations

We construct exporter valuations, w_{ij}, from product-level data on export value and quantity by country-pair, obtained from Comtrade. We select bulk commodities among all possible 4-digit HS product codes. The list includes cereals (except rice and barley); oil seeds (which consists of mostly soybeans); cocoa beans; salt and cement; ores; mineral fuels (except petroleum coke); fertilizers; fuel wood and wood pulp; metals; cermets and articles thereof.

To compute the average value of a cargo exported from region i to j, we first compute the average "price" of a ton exported by dividing total export value by total export quantity from i to j. Then, we multiply this price by the average ship tonnage capacity in our sample.[42]

Finally, although most countries belong to one of our regions (depicted in Figure 6), the USA and Canada each belong to two regions (according to the coast). We thus need to split the Comtrade data for the USA and Canada into east and west coast export values. To do so, we employ data on state-level exports from the US Census, as well as on province-level exports from the Canadian International Merchandise Trade Database. In particular, we assign every state (province) to either the east or the west coast and compute, for every product, the share of the total value of trade in that commodity that is exported by east and west coast states (provinces). Then, we compute the total value and quantity of trade for the region East Coast of North America (West Coast of North America) by summing over products the share of the value of east (west) coast trade by the total value of the country's trade for the USA and Canada. Implicitly, this approach assumes that export values from these two regions are only different due to the composition of products, not their prices.

C.3 Algorithm to compute the efficient allocation

Here, we describe the algorithm employed to compute the steady state of our model. In order to simulate both the market equilibrium and the efficient allocation we approximate the matching function that we obtained non-parametrically with a Cobb Douglas. In particular, for each region we impose

of the relevant commodities for each region i.

[42]This is robust to using the average ship tonnage capacity on route ij.

$m_{it} = A_i s_{it}^{1-\alpha_i} e_{it}^{\alpha_i}$, and select the parameters (A_i, α_i) through non-linear least-squares using the non-parametrically estimated exporters.

The algorithm proceeds as follows:

1. Make an initial guess for $\{U^{e,0}, \tau^0, s^0, E^0\}$.

2. At each iteration k, inherit $\left\{U^{e,k-1}, \tau^{k-1}, s^{k-1}, E^{k-1}\right\}$. Let G^{k-1}, e^{k-1}, and q^{k-1} denote the associated destination shares, searching exporters, and matches respectively.[43] Moreover, let $\lambda^{e,k-1}$ and $\lambda^{s,k-1}$ denote the associated matching rates. We update our guess according to the following steps:

   (a) First, in an inner loop we compute the ship optimal policy and value function implied by the matching rates $\lambda^{s,k-1}$, prices τ^{k-1} , and destination shares G^{k-1}. In particular, after initializing $V^{s,0}$, repeat the following steps until convergence

      i. At iteration h, compute the value of traveling $V_{ij}^{s,h}$ from $V_{ij}^{s,h} = \frac{-c_{ij}^s + d_{ij}\beta V_i^{s,h-1}}{1-\beta(1-d_{ij})}$.

      ii. Compute the value $U_i^{s,h}$ from:

$$U_i^{s,h} = \sigma \ln \left(\exp \frac{\beta V_i^{s,h-1}}{\sigma} + \sum_{j \neq i} \exp \frac{V_{ij}^{s,h}}{\sigma} \right) + \sigma \gamma^{euler}$$

      where γ^{euler} is the Euler constant.[44]

      iii. Update $V_i^{s,h}$ from $V_i^{s,h} = -c_i^s + \left(1 - \lambda_i^{s,k-1}\right) U_i^{s,h} + \lambda_i^{s,k-1} \sum_j G_{ij}^{k-1} \left(V_{ij}^{s,h} + \tau_{ij}^{k-1}\right)$

      iv. Upon convergence, we set $V_{ij}^{s,k} = V_{ij}^{s,\infty}$, $V_i^{s,k} = V_i^{s,\infty}$, $U_i^{s,k} = U_i^{s,\infty}$, and compute the ship optimal choice probabilities based on $P_{ij}^k = \exp\left(\frac{V_{ij}^{s,k}}{\sigma}\right) / \left(\sum_{l \neq i} \exp\left(\frac{V_{il}^{s,k}}{\sigma}\right) + \exp\left(\frac{\beta V_i^{s,k}}{\sigma}\right)\right)$ for $i \neq j$ and $P_{ii}^k = \exp\left(\frac{\beta V_i^{s,k}}{\sigma}\right) / \left(\sum_{l \neq i} \exp\left(\frac{V_{il}^{s,k}}{\sigma}\right) + \exp\left(\frac{\beta V_i^{s,k}}{\sigma}\right)\right)$ for $i = j$.

   (b) To update the efficient prices τ^k compute the total surplus from matching as,[45]

$$\Delta_{ij}^k = w_{ij} - \delta \beta U_{ij}^{e,k-1} + V_{ij}^{s,k} - U_i^{s,k},$$

[43] That is, $e_i^{k-1} = \sum_j e_{ij}^{k-1}$, $G_{ij}^{k-1} = \frac{e_{ij}^{k-1}}{e_i^{k-1}}$, and $q_i^{k-1} = m\left(s_i^{k-1}, e_i^{k-1}\right)$

[44] This is the closed form expression for the expectation of the maximum over multiple choices, and is obtained by integrating U_i^s over the distribution of ϵ.

[45] Following BKP we assume that unmatched exporters survive with probability δ so that their effective discount factor is $\beta\delta$ (this is also true in the estimation procedure even though it was omitted there for notational simplicity). We calibrate $\delta = 0.99$. This makes no difference in our theoretical analysis.

and compute the efficient prices based on,

$$\tau_{ij}^{k} = w_{ij} - \delta\beta U_{ij}^{e,k-1} - \Delta_{ij}^{k} + \alpha_i \sum_j G_{ij}^{k-1}\Delta_{ij}^{k},$$

where α_i denotes the elasticity of the matching function with respect to the number of exporters. Similarly, compute prices under Nash bargaining using the surplus sharing condition (33).

(c) Update the exporter value function $U^{e,k}$ based on the efficient prices τ^k and matching rates $\lambda^{e,k-1}$ setting $U_{ij}^{e,k} = \frac{-c_i^e + \lambda_i^{e,k-1}\left(w_{ij} - \tau_{ij}^{k}\right)}{1-\beta\delta\left(1-\lambda_i^{e,k-1}\right)}$.

(d) Finally, update the number of ships and exporters searching $\left\{s^k, E^k\right\}$ according to

$$s_i^k = \sum_j P_{ij}^k \left(s_j^{k-1} - q_j^{k-1}\right) + \sum_j q_{ij}^{k-1},$$

and

$$e_{ij}^{k} = \underbrace{\mathcal{E}_i \frac{\exp\left(U_{ij}^{e,k} - \kappa_{ij}\right)}{1 + \sum_{l\neq i}\exp\left(U_{il}^{e,k} - \kappa_{il}\right)}}_{\text{new entrants}} + \delta \underbrace{\left(e_i^{k-1} - q_i^{k-1}\right)}_{\text{unmatched}}$$

where $\mathcal{E}_i$ is the mass of potential entrants.

3. If $\left\|s^k - s^{k-1}\right\| < \epsilon$, $\left\|E^k - E^{k-1}\right\| < \epsilon$, $\left\|U^{e,k} - U^{e,k-1}\right\| < \epsilon$, and $\left\|\tau^k - \tau^{k-1}\right\| < \epsilon$ stop; otherwise go back to point (a).

	Port Costs	Sailing Costs	Logit Shock
	c_i^s	c_{ij}^s	σ
North America West Coast	227.65	46.75	
	(8.77)	(0.36)	
North America East Coast	272.3	-	
	(4.31)	-	
Central America	175.41	46.75	
	(5.06)	(0.36)	
South America West Coast	265.55	46.75	
	(7.77)	(0.36)	
South America East Coast	292.5	-	
	(5.23)	-	
West Africa	145.3	47.65	
	(4.84)	(0.33)	
Mediterranean	121.89	46.16	
	(3)	(0.28)	
North Europe	122.48	46.16	
	(1.71)	(0.28)	
South Africa	220.11	47.65	
	(7.28)	(0.33)	
Middle East	118.45	46.16	
	(2.14)	(0.28)	
India	97.23	45.93	
	(1.8)	(0.28)	
South East Asia	93.14	40.99	
	(1.02)	(0.28)	
China	91.07	40.89	
	(0.98)	(0.25)	
Australia	193.29	40.99	
	(2.85)	(0.28)	
Japan-Korea	100.41	40.89	
	(1.9)	(0.25)	
			16.53
			(0.1070)

Table 5: Ship cost estimates (from BKP). All parameters in 1,000 USD. Standard errors computed from 200 bootstrap samples. The sailing cost for the East Coast of North and South America is set equal to the weekly fuel cost at 40,000 US dollars.

	Exporter wait costs c_i^e	Ship bargaining coefficient γ_i	Average exporter value $\bar{w}_i$
North America West Coast	83.49	0.384	13,738
	(10.72)	(0.018)	
North America East Coast	83.49	0.585	12,192
	(10.72)	(0.012)	
Central America	302.3	0.344	14,350
	(69.28)	(0.038)	
South America West Coast	302.3	0.259	20,096
	(69.28)	(0.017)	
South America East Coast	302.3	0.371	6,971
	(69.28)	(0.042)	
West Africa	396.9	0.292	4,547
	(512.74)	(0.078)	
Mediterranean	3.44	0.412	10,508
	(8.60)	(0.014)	
North Europe	3.44	0.517	14,577
	(8.60)	(0.014)	
South Africa	396.9	0.24	6,224
	(512.74)	(0.075)	
Middle East	20.53	0.615	7,160
	(9.04)	(0.026)	
India	20.53	0.568	6,305
	(9.04)	(0.022)	
South East Asia	174.01	0.215	4,918
	(44.00)	(0.036)	
China	282.8	0.194	8,231
	(77.86)	(0.028)	
Australia	174.01	0.388	12,475
	(44.00)	(0.037)	
Japan-Korea	282.8	0.265	2,977
	(77.86)	(0.038)	

Table 6: Average exporter valuation (over destinations), wait costs and bargaining coefficients estimates. All the estimates are in 1,000 USD. To gain power, we restrict exporter wait costs to be constant within a continent. Standard errors computed from 200 bootstrap samples.

D Additional figures and tables

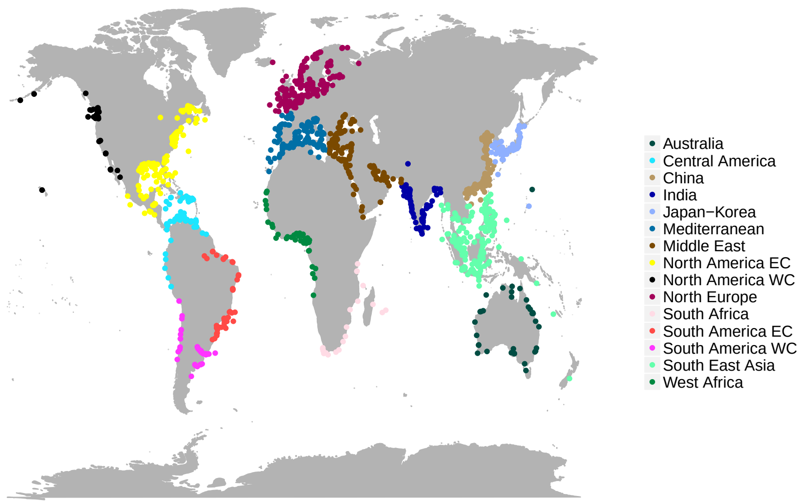

Figure 6: Definition of regions. Each color depicts one of the 15 geographical regions.

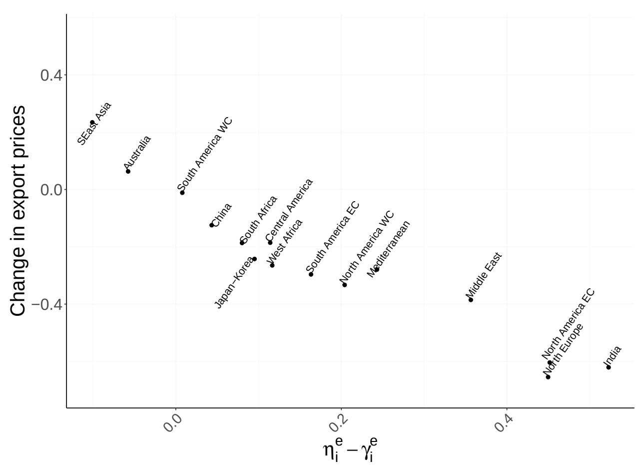

Figure 7: The vertical axis reports the change in prices when only thin/thick market externalities are internalized. The horizontal axis reports the difference between the estimated exporter bargaining coefficient and the estimated elasticity of the matching function with respect to exporters. We do not allow ships to reallocate to capture the direct effect of the thin/thick market externalities. See also discussion in footnote 34.

	I	II	III
	log(price per day)		
Probability of ballast		0.234**	0.556**
		(0.030)	(0.081)
Avg duration of ballast trip (log)		0.166**	0.065**
		(0.014)	(0.032)
Coal			0.088**
			(0.045)
Fertilizer			0.245**
			(0.051)
Grain			0.131**
			(0.048)
Ore			0.124**
			(0.045)
Steel			0.135**
			(0.049)
Constant	10.284**	9.127**	8.915**
	(0.103)	(0.099)	(0.408)
Destination FE	Yes	No	No
Origin FE	Yes	Yes	Yes
Ship type FE	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes
Obs	11,014	11,011	1,662
R^2	0.694	0.674	0.664

$^{**}p < 0.05, ^{*}p < 0.1$

Table 7: Shipping price regressions (Table II in BKP). The dependent variable is the logged price per day in USD. The independent variables include combinations of: the average frequency of ballast traveling after the contract's destination (Probability of ballast), the average logged duration (in days) of the ballast trip after the contract's destination, as well as ship type, origin, destination and quarter FEs. The product is reported in only 20% of the sample, so the regression in column III has substantially fewer observations. The omitted product category is cement.

	log(price per day)			
	I	II	III	IV
$I\{\text{orig.} = \text{home country}\}$			0.004	
			(0.019)	
$I\{\text{dest.} = \text{home country}\}$			−0.012	
			(0.015)	
ln (Number Employees)				0.008
				(0.007)
ln (Operating Revenues)				0.003
				(0.005)
Time FE	Qtr×Yr	Qtr×Yr	Qtr×Yr	Qtr×Yr
Shipowner FE	No	Yes	No	No
Ship characteristics	Yes	Yes	Yes	Yes
Region FE	Orig. & Dest.	Orig. & Dest.	Orig. & Dest.	Orig. & Dest.
Observations	7,263	7,263	7,973	7,973
Adj. R^2	0.530	0.540	0.537	0.537

*p<0.1; **p<0.05; ***p<0.01

Table 8: Regression of shipping prices on shipowner characteristcs and fixed effects (Table SI in Supplement to BKP). Shipping prices, ships' characteristics (age and size), and the identity of the shipowner are obtained from Clarksons. Information on shipowner characteristics is obtained from ORBIS. In particular, we match the shipowners in Clarksons to ORBIS; we do so for two reasons: (i) ORBIS allows us to have reliable firm identities, as shipowners may appear under different names in the contract data; (ii) ORBIS reports additional firm characteristics (e.g. number of employees, revenue, headquarters). Here we identify the shipowner with the global ultimate owner (GUO); results are robust to controlling for the identity of the domestic owner (DUO) and the shipowner as reported in Clarksons. Finally, the data used span the period 2010-2016.

E Supplemental Material: Discounting, preference shocks and out of steady state dynamics

In this section we show that the main results of Section 3.2 are valid in a more general setup. In particular, we extend the model of Section 2 to allow for idiosyncratic preference shocks in carriers relocation choice (relevant in our empirical application), as well as out of steady state dynamics, and we derive an efficiency result analogous to that of Theorem 2.

E.1 Model

We begin by laying out the model focusing on the changes made compared to Section 2.

States and transitions In this Appendix we do not consider the steady state equilibrium. Hence, we now state explicitly the dependence of actions and value functions on the relevant state variables and transitions, which were only implicit in the model of Section 2. At the beginning of a given time period, the state of the economy is described by a vector,

$$z = (x, y) \in \mathbb{R}_+^{I \times I} \times \mathbb{R}_+^{I \times I}.$$

The first element of z, $x = (x_{ij})_{i,j \in I}$, corresponds to the supply at every origin i,

- x_{ii} is the measure of carriers waiting at location i
- x_{ij} is the measure of carriers traveling from i to j, either empty or full, for every destination $j \neq i$.

The second element of z, $y = (y_{ij})_{i,j \in I}$, corresponds to demand. For every origin-destination pair ij, y_{ij} is the measure of customers who are waiting on route ij at the beginning of the current period. These are customers that entered in some previous period and have not yet been matched with a carrier.

At a given state z, the choice sets that agents face, as well as the search and matching process are the same as in Section 2. At each origin i, a measure $s_i \leq x_{ii}$ of carriers choose to search for a customer, while the remaining measure $x_{ii} - s_i$ choose to remain inactive. Similarly, a measure $e_{ij} \geq y_{ij}$ of customers search for a carrier on route ij, so that $e_{ij} - y_{ij}$ is the measure of new customers joining the existing search pool.

Once a customer and a carrier meet, they can choose whether to match or remain unmatched. The outcome of this process is a vector (b, q) describing the measure of carriers that start traveling empty (b_{ij}) or full (q_{ij}) on each route ij. The state transitions as a function of the allocation (s, E, q, b) are as follows for all ij:

$$
\begin{aligned}
x_{ii}^{+1}(s, E, q, b|z) &= x_{ii} - s_i + \sum_j d_{ji}(x_{ji} + q_{ji} + b_{ji}) \\
x_{ij}^{+1}(s, E, q, b|z) &= (1 - d_{ij})(x_{ij} + q_{ij} + b_{ij}) \\
y_{ij}^{+1}(s, E, q, b|z) &= e_{ij} - q_{ij}.
\end{aligned} \tag{62}
$$

The feasibility constraints on the allocation (s, E, q, b) are as follows for all i, j:

$$
\begin{aligned}
x_{ii} \geq s_i, \quad e_{ij} \geq y_{ij}, \quad s_i, e_{ij}, q_{ij}, b_{ij} \geq 0 \\
\sum_j (q_{ij} + b_{ij}) = s_i, \quad m_i(s_i, e_i) G_{ij} \geq q_{ij}
\end{aligned} \tag{63}
$$

Prices, expectations and allocation rules The pricing rule maps each state into the associated vector of transportation prices on each route, $\boldsymbol{\tau} : z \mapsto \boldsymbol{\tau}(z) = (\boldsymbol{\tau}_{ij}(z))_{i,j\in I}$. As in Section 2, we begin by remaining agnostic regarding the structure of the pricing rule, and later we characterize the pricing rules that are consistent with efficient equilibria and compare them to Nash bargaining.

In state z carriers expect to meet customers at rate $\boldsymbol{\lambda}_i^s(z)$ in location i, and customers expect to meet carriers at rate $\boldsymbol{\lambda}_i^e(z)$, where $\boldsymbol{\lambda}^s : z \mapsto \boldsymbol{\lambda}^s(z) = (\boldsymbol{\lambda}_i^s(z))_{i\in I}$, $\boldsymbol{\lambda}^e : z \mapsto \boldsymbol{\lambda}^e(z) = (\boldsymbol{\lambda}_i^e(z))_{i\in I}$. Agents make optimal choices under rational expectations about the state transitions, the matching probabilities and prices at each state, generating an allocation rule $(\boldsymbol{s}, \boldsymbol{E}, \boldsymbol{q}, \boldsymbol{b}) : z \mapsto (\boldsymbol{s}(z), \boldsymbol{E}(z), \boldsymbol{q}(z), \boldsymbol{b}(z))$, mapping states into feasible allocations. That is, for every state z, $(\boldsymbol{s}(z), \boldsymbol{E}(z), \boldsymbol{q}(z), \boldsymbol{b}(z))$ satisfies (63).

Similarly to Section 2, we will sometimes denote an allocation rule by $(\boldsymbol{s}, \boldsymbol{e}, \boldsymbol{G}, \boldsymbol{q}, \boldsymbol{b})$, where $\boldsymbol{e}_i(z) = \sum_j \boldsymbol{e}_{ij}(z)$ and $\boldsymbol{G}_{ij}(z) = \boldsymbol{e}_{ij}(z) / \boldsymbol{e}_i(z)$, and we will often refer to the first triplet $(\boldsymbol{s}, \boldsymbol{e}, \boldsymbol{G})$ as a search rule.

Preference shocks and carrier optimality Carriers' payoff structure is the same as in Section 2. In addition, we allow for stochasticity in carriers' preferences for destinations. The stochastic component at each origin i is represented by a random vector $\epsilon_i = (\epsilon_{ij})_{i,j\in I}$ that enters the carriers' utility of relocating to different destinations additively, is i.i.d. across carriers and satisfies the conditional independence assumption, $\epsilon_i^{+1}|z^{+1} \perp \epsilon_i, z$. To simplify the exposition, we assume that ϵ_i is independent of z and i, so that $\forall i, z : \epsilon_i \sim \mathbb{P} \in \Delta\mathbb{R}^I$, although this assumption is not needed for the results. We assume that $\mathbb{P}$ has full support and that it admits a continuous density.

The value of a carrier that remained unmatched at origin i at state z depends on the particular realization of the shock. We denote its expectation by

$$U_i^s(z) = \mathrm{E}_{\mathbb{P}} \max_j \left(V_{ij}^s(z) + \epsilon_{ij} \right). \tag{64}$$

The values of a carrier traveling from i to j and a carrier waiting in i are given by:

$$\begin{aligned} V_{ij}^s(z) &= -c_{ij}^s + \beta \left[d_{ij} V_j^s \left(z^{+1} \right) + (1 - d_{ij}) V_{ij}^s \left(z^{+1} \right) \right] \\ V_i^s(z) &= \max \left\{ -c_i^s + \boldsymbol{\lambda}_i^s(z) \sum_{j \neq i} G_{ij} \Delta_{ij}^s(z) + U_i^s(z), \beta V_i^s \left(z^{+1} \right) \right\} \end{aligned}$$

as before, where

$$\Delta_{ij}^s(z) = \max \left\{ \boldsymbol{\tau}_{ij}(z) + V_{ij}^s(z) - U_i^s(z), 0 \right\}$$

is the carrier expected surplus of being matched with respect of being unmatched.

Denote by P^b the matrix of carrier relocation choice probabilities associated with $b \in \mathbb{R}^{I \times I}$:

$$P_{ij}^b = b_{ij} / \sum_k b_{ik}.$$

for all ij. Optimality in state z requires that for all ij,

$$P_{ij}^{\boldsymbol{b}(z)} = \mathbb{P} \left[V_{ij}^s(z) + \epsilon_{ij} = \max_k \left(V_{ik}^s(z) + \epsilon_{ik} \right) \right]. \tag{65}$$

The remaining optimality conditions of Section 2 still hold. In particular, carriers search only when it is profitable to do so:

$$\boldsymbol{s}_i(z) > 0 \rightarrow V_i^s(z) = -c_i^s + \boldsymbol{\lambda}_i^s(z) \sum_{j \neq i} G_{ij} \Delta_{ij}^s(z) + U_i^s(z) \tag{66}$$

Moreover, they do not reject any match yielding a strictly positive surplus, and they accept only matches yielding a positive surplus:

$$\boldsymbol{q}_{ij}(z) < \boldsymbol{\lambda}_i^s(z)\,\boldsymbol{s}_i(z)\,G_{ij} \rightarrow \Delta_{ij}^s(z) = 0$$
$$\boldsymbol{q}_{ij}(z) > 0 \rightarrow \Delta_{ij}^s(z) = \boldsymbol{\tau}_{ij}(z) + V_{ij}^s(z) - U_i^s(z)\,. \tag{67}$$

Customer optimality Customer value functions are the same as in Section 2, but we make the dependence on the state of the economy explicit. In state z, the meeting surplus of the marginal customer (with respect to being unmatched) is given by

$$\Delta_{ij}^e(z) = \max\left\{w_{ij}(\boldsymbol{q}(z)) - \boldsymbol{\tau}_{ij}(z) - \beta U_{ij}^e\left(z^{+1}\right), 0\right\},$$

where $U_{ij}^e(z)$ is the value of customer with destination j that is searching for a carrier in location i:

$$U_{ij}^e(z) = -c_{ij}^e + \boldsymbol{\lambda}_i^e(z)\,\Delta_{ij}^e(z) + \beta U_{ij}^e\left(z^{+1}\right). \tag{68}$$

Optimality requires that the marginal customer does not reject a match yielding a strictly positive surplus:

$$\boldsymbol{q}_{ij}(z) < \boldsymbol{\lambda}_i^e(z)_{\,ij}(z) \rightarrow \Delta_{ij}^e(z) = 0. \tag{69}$$

The measure of customers searching on each route ij is pinned down by a free entry condition for the marginal customer:

$$U_{ij}^e(z) - \kappa_{ij} \leq 0, \quad \text{with equality if } \boldsymbol{e}_{ij}(z) > y_{ij}. \tag{70}$$

Equilibrium An outcome is a tuple $(\boldsymbol{s}, \boldsymbol{E}, \boldsymbol{q}, \boldsymbol{b}, \boldsymbol{\tau})$ consisting of an allocation rule and a price rule.

Definition 5. An outcome is a Markovian equilibrium if, for every state z:

1. $(\boldsymbol{s}(z), \boldsymbol{E}(z), \boldsymbol{q}(z), \boldsymbol{b}(z))$ satisfies the feasibility constraints (63).

2. $(\boldsymbol{s}(z), \boldsymbol{q}(z), \boldsymbol{b}(z))$ satisfies the carrier optimality conditions (64)-(67) given $\boldsymbol{\tau}(z), \boldsymbol{\lambda}^s(z), z^{+1}$ and $\boldsymbol{G}(z)$.

3. $(\boldsymbol{E}(z), \boldsymbol{q}(z))$ satisfies the customer optimality and free entry conditions (68)-(70) given $\boldsymbol{\tau}(z), \boldsymbol{\lambda}^e(z)$ and z^{+1}.

4. Expectations are consistent with the realized outcomes:

$$\forall i: \boldsymbol{\lambda}_i^s(z) = m_i(\boldsymbol{s}_i(z), \boldsymbol{e}_i(z)) / \boldsymbol{s}_i(z), \ \boldsymbol{\lambda}_i^e(z) = m_i(\boldsymbol{s}_i(z), \boldsymbol{e}_i(z)) / \boldsymbol{e}_i(z)$$
$$z^{+1} = z^{+1}(\boldsymbol{s}(z), \boldsymbol{E}(z), \boldsymbol{q}(z), \boldsymbol{b}(z)).$$

$(\boldsymbol{s}, \boldsymbol{E}, \boldsymbol{q}, \boldsymbol{b})$ is an equilibrium allocation rule if there exists a price rule $\boldsymbol{\tau}$ such that $(\boldsymbol{s}, \boldsymbol{E}, \boldsymbol{q}, \boldsymbol{b}, \boldsymbol{\tau})$ is a Markovian equilibrium.

E.2 Externalities and efficiency

The social planner solves an infinite horizon constrained Markov decision problem in which, conditional on every initial state z, he chooses a dynamic allocation rule maximizing the discounted sum of future social payoffs.

The social welfare $W^p(s, e, G, q, b; z)$ at each state z and for every allocation (s, e, G, q, b) entails the welfare terms encountered in Section 3.1, but involves an additional term that captures the welfare due to carrier preference shocks. This term is given by $\left(\sum_j b_{ij}\right) f\left(P_i^b\right)$ where $f(P_i^b)$ represents the value associated with the best allocation of shocks to destinations at i conditional on the aggregate choice probabilities being given by P_i^b:[46]

$$W^p(s, e, G, q, b; z) \equiv W(q) - \sum_{ij}(x_{ij} + q_{ij} + b_{ij})\, c_{ij}^s - \sum_i s_i c_i^s - \sum_i e_i \sum_j G_{ij} c_{ij}^e - \sum_{ij}(e_i G_{ij} - y_{ij})\, \kappa_{ij} + \sum_{ij} b_{ij} f\left(P_i^b\right).$$

In what follows, we use the upper bar notation $\bar{a} = (a^t)_{t=0}^{\infty}$ for infinite sequences. When dealing with a sequence of allocations $\left(\bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b}\right)$ and an initial state z^0, unless stated otherwise, it is understood

[46]Formally:

$$f\left(P_i^b\right) = \max_{\pi \in \Pi\left(P_i^b\right)} \mathrm{E}_\pi(\epsilon_j) \tag{71}$$

where the expectation on the right hand side is with respect to a joint realization of the vector $\epsilon = (\epsilon_j)_{j \in I}$ and the destination $j \in I$, and $\Pi\left(P_i^b\right)$ is the set of all probability measures $\pi \in \Delta\left(\mathbb{R}^I \times I\right)$ such that the marginal of π over I is P_i^b and the marginal of π over $\mathbb{R}^I$ is $\mathbb{P}$. For a discussion, the reader is referred to Galichon (2018). For example, if ϵ_i is distributed according to a logit, we have $f\left(P_i^b\right) = -\sum_j P_{ij}^b \ln P_{ij}^b$.

that $\bar{z}$ refers to the sequence of states induced by $\left(\bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b}\right)$ from z^0:

$$z^{t+1} = z^{+1}\left(s^t, e^t, G^t, q^t, b^t; z^t\right).$$

for $t \geq 0$. Moreover, when dealing with a feasible allocation rule $(\boldsymbol{s}, \boldsymbol{e}, \boldsymbol{G}, \boldsymbol{q}, \boldsymbol{b})$ and an initial state z^0, it is understood that $\left(\bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b}\right)$ refers to the sequence of allocations induced by $(\boldsymbol{s}, \boldsymbol{e}, \boldsymbol{G}, \boldsymbol{q}, \boldsymbol{b})$ from z^0:

$$\left(s^t, e^t, G^t, q^t, b^t\right) = \left(\boldsymbol{s}\left(z^t\right), \boldsymbol{e}\left(z^t\right), \boldsymbol{G}\left(z^t\right), \boldsymbol{q}\left(z^t\right), \boldsymbol{b}\left(z^t\right)\right).$$

The planner's dynamic problem at state z^0 is given by

$$\begin{aligned} V^p\left(z^0\right) = \max_{\left(\bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b}\right)} & \sum_{t=0}^{\infty} \beta^t W^p\left(s^t, e^t, G^t, q^t, b^t; z^t\right) \\ \text{s.t. } & x_{ii}^t \geq s_i^t \\ & e_i^t G_{ij}^t \geq y_{ij}^t \\ & \sum_j \left(q_{ij}^t + b_{ij}^t\right) = s_i^t \\ & m_i\left(s_i^t, e_i^t\right) G_{ij}^t \geq q_{ij}^t \\ & \sum_j G_{ij}^t = 1 \\ & s_i^t, e_i^t, G_{ij}^t \geq 0, \quad \forall i, j, t \end{aligned} \tag{72}$$

Definition 6. An allocation rule $(\boldsymbol{s}, \boldsymbol{e}, \boldsymbol{G}, \boldsymbol{q}, \boldsymbol{b})$ is efficient at a state z^0 if $\left(\bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b}\right)$ solves Problem (72).

Similarly to Section 2, we distinguish three different potential sources of inefficiency. To do so, for each state z^0, let $\mathcal{A}\left(z^0\right)$ be the set of allocation sequences $\left(\bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b}\right)$ which are feasible from z^0, that is, they satisfy the constraints of Problem (72). Let also

$$\mathcal{SA}\left(z^0\right) = \left\{\left(\bar{s}, \bar{e}, \bar{G}\right) : \exists\left(\bar{q}, \bar{b}\right), \left(\bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b}\right) \in \mathcal{A}\left(z^0\right)\right\}$$

be the set of feasible sequences of search allocations, and

$$\begin{aligned}
\mathcal{SA}\left(z^0|\bar{e},\bar{G}\right) &= \left\{\bar{s}: \left(\bar{s},\bar{e},\bar{G}\right) \in \mathcal{SA}\left(z^0\right)\right\} \\
\mathcal{SA}\left(z^0|\bar{s},\bar{G}\right) &= \left\{\bar{e}: \left(\bar{s},\bar{e},\bar{G}\right) \in \mathcal{SA}\left(z^0\right)\right\} \\
\mathcal{SA}\left(z^0|\bar{s},\bar{e}\right) &= \left\{\bar{G}: \left(\bar{s},\bar{e},\bar{G}\right) \in \mathcal{SA}\left(z^0\right)\right\}.
\end{aligned}$$

For every $\left(\bar{s},\bar{e},\bar{G}\right) \in \mathcal{SA}\left(z^0\right)$, we define the maximum dynamic welfare attainable by this sequence by:

$$\begin{aligned}
V^p\left(\bar{s},\bar{e},\bar{G},z^0\right) = \max_{\bar{q},\bar{b}} \sum_{t=0}^{\infty} \beta^t W^p\left(s^t,e^t,G^t,q^t,b^t;z^t\right) \\
\text{s.t.} \left(\bar{s},\bar{e},\bar{G},\bar{q},\bar{b}\right) \in \mathcal{A}\left(z^0\right)
\end{aligned} \tag{73}$$

so that we have

$$V^p\left(z^0\right) = \max_{\left(\bar{s},\bar{e},\bar{G}\right)\in\mathcal{SA}\left(z^0\right)} V^p\left(\bar{s},\bar{e},\bar{G},z^0\right).$$

Given an equilibrium allocation rule $(\boldsymbol{s},\boldsymbol{e},\boldsymbol{G},\boldsymbol{q},\boldsymbol{b})$ and an initial state z^0 we say that:

(i) Carriers internalize thin/thick market externalities at z^0 if $\bar{s}$ solves

$$\max_{\bar{s}'\in\mathcal{SA}\left(z^0|\bar{e},\bar{G}\right)} V^p\left(\bar{s}',\bar{e},\bar{G},z^0\right)$$

(ii) Customers internalize thin/thick market externalities at z^0 if $\bar{e}$ solves

$$\max_{\bar{e}'\in\mathcal{SA}\left(z^0|\bar{s},\bar{G}\right)} V^p\left(\bar{s},\bar{e}',\bar{G},z^0\right)$$

(iii) Customers internalize pooling externalities at z^0 if $\bar{G}$ solves

$$\max_{\bar{G}'\in\mathcal{SA}\left(z^0|\bar{s},\bar{e}\right)} V^p\left(\bar{s},\bar{e},\bar{G}',z^0\right).$$

Next we state the equivalent of Theorem 2 in the current framework. Given $(\boldsymbol{s},\boldsymbol{e},\boldsymbol{G})$, we denote by $\eta_i^s(z) = d\ln m_i\left(\boldsymbol{s}_i(z),\boldsymbol{e}_i(z)\right)/d\ln s_i$ and $\eta_i^e(z) = d\ln m_i\left(\boldsymbol{s}_i(z),\boldsymbol{e}_i(z)\right)/d\ln e_i$. For simplicity, in order to avoid delving into corner conditions, in the statement below we assume that the equilibrium path originating from z^0 is such that we have $s_i^t, e_i^t > 0$ for every t, i.

Theorem 4. *Suppose that at a given state z^0, Problem (73) admits a unique optimal solution, and let $(\boldsymbol{s}, \boldsymbol{e}, \boldsymbol{G}, \boldsymbol{q}, \boldsymbol{b})$ be an equilibrium allocation rule. Then the following statements hold:*[47]

(i) Carriers internalize thin/thick market externalities at z^0 if and only if, for every $t \geq 0$:

$$\forall i \in I: \sum_j \boldsymbol{G}_{ij}^t \left(z^t\right) \Delta_{ij}^s \left(z^t\right) = \eta_i^s \left(z^t\right) \sum_j \boldsymbol{G}_{ij} \left(z^t\right) \left(\Delta_{ij}^s \left(z^t\right) + \Delta_{ij}^e \left(z^t\right)\right).$$

(ii) Customers internalize thin/thick market externalities at z^0 if and only if, for every $t \geq 0$:

$$\forall i \in I: \sum_j \boldsymbol{G}_{ij} \left(z^t\right) \Delta_{ij}^e \left(z^t\right) = \eta_i^e \left(z^t\right) \sum_j \boldsymbol{G}_{ij} \left(z^t\right) \left(\Delta_{ij}^s \left(z^t\right) + \Delta_{ij}^e \left(z^t\right)\right)$$

(iii) Customers internalize pooling externalities at z^0 if and only if, for every $t \geq 0$, for each origin i,

$$\Delta_{ij}^s \left(z^t\right) = \max_{k \neq i} \Delta_{ik}^s \left(z^t\right)$$

for every ij such that $\boldsymbol{G}_{ij} \left(z^t\right) > 0$.

The following section provides the proof.

E.3 Proof of Theorem 4

The proof follows the same reasoning as under the steady state assumption, but has to overcome a number of technical difficulties due to the infinite dimensional form of the planner's optimization problem.

E.3.1 Preliminaries

Let X be a compact and convex subset of $\mathbb{R}^N$ for some $N \in \mathbb{N}$, and $\beta \in (0,1)$. For each pair of sequences $\bar{x}, \bar{y} \in X^{\mathbb{N} \cup \{0\}}$ we define the inner product $\langle \bar{x}, \bar{y} \rangle = \sum_{t=0}^{\infty} \beta^t x^t \cdot y^t$ for all $\bar{x}, \bar{y}$, where $\cdot$ denotes the standard

[47] Formally, the only if parts of statements (i) to (iii) hold for almost every sequence $\left(\bar{s}, \bar{e}, \bar{G}\right) \in \mathcal{SA}\left(z^0\right)$. That is, there exists a dense subset D of $\mathcal{SA}\left(z^0\right)$ such that the only if part of the statements hold whenever $\left(\bar{s}, \bar{e}, \bar{G}\right) \in D$. See Section E.3 for details.

inner product on $\mathbb{R}^N$. Define the norm $\|\bar{x}\| = \sqrt{\langle \bar{x}, \bar{x} \rangle}$ and $L^{2,\beta} = \left\{ \bar{x} \in X^{\mathbb{N} \cup \{0\}} : \|\bar{x}\| < \infty \right\}$. Then $\left(L^{2,\beta}, \|\cdot\| \right)$ is a Banach space.

Let $\mathcal{X} \subseteq L^{2,\beta}$ be a convex set and $f : \mathcal{X} \to \mathbb{R}$ be a continuous and concave function.

Definition 7. For every $\bar{x} \in \mathcal{X}$, the super gradient of f at $\bar{x}$, denoted $\partial f(\bar{x})$, is the set of all sequences $\bar{y} \in X^{\mathbb{N} \cup \{0\}}$ such that, for every $\bar{x}' \in \mathcal{X}$:

$$f(\bar{x}') - f(\bar{x}) \leq \langle \bar{x}' - \bar{x}, \bar{y} \rangle .$$

f is differentiable at $\bar{x}$ if its super gradient at $\bar{x}$ contains a unique element.

Lemma 6. *Let $D \subset \mathcal{X}$ be the set of sequences at which f is differentiable. Then D is a dense subset of $\mathcal{X}$.*

Proof. See (Asplund, 1968), Theorem 2. □

Lemma 7. *$\bar{x}$ maximizes f over $\mathcal{X}$ if and only if 0 belongs to $\partial f(\bar{x})$.*

Proof. Immediate from the definition of $\partial f(\bar{x})$. □

The following lemma will be useful in the derivation of each of the three statements internalizing the respective externalities in Theorem 4. As before, z denotes the state. The interpretation of the variables x, θ will change based on each externality considered; for instance, in the case of carrier thin/thick market externalities, x corresponds to s, while θ corresponds to e, G. The function f summarizes all constraints, H defines the state dynamics and u the welfare.

Lemma 8. *Let $L, M, N > 0$ and Z, X, Θ, be compact and convex subsets of $\mathbb{R}^L$, $\mathbb{R}^M$ and $\mathbb{R}^N$, respectively, $u : X \times \Theta \times Z \to \mathbb{R}$ be a concave and continuously differentiable function, $H : X \times \Theta \times Z \to Z$ be a linear function, and for each $k = 1, ..., K > 0$, let $f_k(x, \theta, z)$ be a continuously differentiable and concave* *function. Let* $z^0 \in Z$*, and* $\boldsymbol{\Theta} \subseteq \Theta^{\mathbb{N}\cup\{0\}}$ *be such that for every* $\bar{\theta} \in \boldsymbol{\Theta}$ *problem*

$$P\left(\bar{\theta}\right): \max_{x \in X^{\mathbb{N}\cup\{0\}}} \sum_{t=0}^{\infty} \beta^t u\left(x^t, \theta^t, z^t\right)$$

$$\forall t, k: \; f_k\left(x^t, \theta^t, z^t\right) \geq 0$$

$$\forall t: \; z^{t+1} = H\left(x^t, \theta^t, z^t\right)$$

is feasible, and let $V\left(\bar{\theta}\right)$ *denote its value. Then* V *is concave. Moreover, suppose that* $\bar{\theta} \in \boldsymbol{\Theta}$ *is such that* $P\left(\bar{\theta}\right)$ *admits a unique optimal solution, and let* $\bar{x} \in X^{\mathbb{N}\cup\{0\}}$*,* $\bar{\lambda} \in \left(\mathbb{R}^K\right)^{\mathbb{N}\cup\{0\}}$ *and* $\bar{\phi} \in \left(\mathbb{R}^L\right)^{\mathbb{N}\cup\{0\}}$ *be such that, for every* t, k, l, m*:*

$$\lambda_k^t \geq 0 \text{ with equality if } f_k\left(x^t, \theta^t, z^t\right) > 0 \tag{74}$$

$$\frac{\partial u\left(x^t, \theta^t, z^t\right)}{\partial x_m} + \sum_k \lambda_k^t \frac{\partial f_k\left(x^t, \theta^t, z^t\right)}{\partial x_m} + \beta \sum_l \frac{\partial H_l\left(x^t, \theta^t, z^t\right)}{\partial x_m} \phi_l^{t+1} = 0 \tag{75}$$

where the sequence $\bar{\phi}$ *is defined recursively by:*

$$\phi_l^t = \frac{\partial u\left(x^t, \theta^t, z^t\right)}{\partial z_l} + \sum_k \lambda_k^t \frac{\partial f_k\left(x^t, \theta^t, z^t\right)}{\partial z_l} + \beta \sum_{l'} \frac{\partial H_{l'}\left(x^t, \theta^t, z^t\right)}{\partial z_l} \phi_{l'}^{t+1} \tag{76}$$

$$\lim_{t \to \infty} \beta^t \phi_l^t = 0$$

and the sequence $\bar{z}$ *is defined recursively by*

$$\forall t: \; z^{t+1} = H\left(x^t, \theta^t, z^t\right).$$

Then $\bar{\theta}$ *maximizes* V *over* $\boldsymbol{\Theta}$ *if*

$$\forall t, n: \; \frac{\partial u\left(x^t, \theta^t, z^t\right)}{\partial \theta_n} + \sum_k \lambda_k^t \frac{\partial f_k\left(x^t, \theta^t, z^t\right)}{\partial \theta_n} + \sum_l \frac{\partial H_l\left(x^t, \theta^t, z^t\right)}{\partial \theta_n} \phi_l^{t+1} = 0 \tag{77}$$

and the above condition is also necessary whenever V *is differentiable at* $\bar{\theta}$*.*

Proof. Concavity of V can be proved using an argument analogous to the proof of Lemma 4. For a generic sequence $\bar{a} = \left(a^t\right)_{t=0}^{\infty}$ and for every $T > 0$, we use the notation $\bar{a}^T \equiv \left(a^t\right)_{t=0}^{T}$ to denote the truncation of $\bar{a}$ at T. When dealing with a sequence $\left(\bar{\theta},\bar{x}\right)$ and an initial state z^0, unless stated otherwise, it is understood that $\bar{z}$ refers to the sequence of states induced by $\left(\bar{\theta},\bar{x}\right)$ and the map H from z^0:

$$\forall t\geq 0:\, z^{t+1}=H\left(x^t,\theta^t,z^t\right).$$

Let $\bar{\theta},\bar{x},\bar{\lambda},\bar{\phi}$ be as in the statement. For every $T>0$ consider the finite horizon problem,

$$P\left(T,\bar{\theta}\right):\, V^T\left(\bar{\theta}^T\right)=\max_{\bar{x}^T\in X^{T+1}}\sum_{t=0}^{T}\beta^t u\left(x^t,\theta^t,z'^t\right)+\beta^{T+1}\sum_l z_l^{T+1}\phi_l^{T+1}$$
$$\text{s.t. }\forall t=0,...,T:\forall k:\, f_k\left(x^t,\theta^t,z^t\right)\geq 0.$$

By standard convex optimization theory, Conditions (74), (76) and (75) imply that $\left(\bar{x}^T,\bar{\lambda}^T\right)$ is an optimal dual pair for Problem $P\left(T,\bar{\theta}\right)$. Hence for every feasible sequence $\bar{x}'$ and for every $T>0$ we have

$$\sum_{t=0}^{T}\beta^t u\left(x^t,\theta^t,z^t\right)+\beta^{T+1}\sum_l z_l^{T+1}\phi_l^{T+1}\geq\sum_{t=0}^{T}\beta^t u\left(x'^t,\theta^t,z'^t\right)+\beta^{T+1}\sum_l z_l'^{T+1}\phi_l^{T+1}$$

Sine Z and u are bounded[48], taking limits on both sides implies that $\bar{x}$ is optimal for $P\left(\bar{\theta}\right)$. Hence by our assumptions it must be the unique optimal solution for $P\left(\bar{\theta}\right)$. Define

$$\forall t,n:\, y_n^t=\frac{\partial u\left(x^t,\theta^t,z^t\right)}{\partial\theta_n}+\sum_k\lambda_k^t\frac{\partial f_k\left(x^t,\theta^t,z^t\right)}{\partial\theta_n}+\sum_l\frac{\partial H_l\left(x^t,\theta^t,z^t\right)}{\partial\theta_n}\phi_l^{t+1}.$$

We show that $\bar{y}\in\partial V\left(\bar{\theta}\right)$. From Marimon and Werner (2019) it follows that $\bar{y}^T\in\partial V\left(\bar{\theta}\right)$ for all $T>0$:

$$\forall\bar{\theta}'\in\mathbf{\Theta}:\, V^T\left(\bar{\theta}'\right)-V^T\left(\bar{\theta}\right)\leq\sum_{t=0}^{T}\beta^t\sum_n y_n^t\left(\theta_n'^t-\theta_n^t\right)$$

Pick $\bar{\theta}'\in\Theta$ and let $\bar{x}'$ be an optimal solution for $P\left(\bar{\theta}'\right)$. For each T we have

$$\sum_{t=0}^{T}\beta^t u\left(x'^t,\theta^t,z'^t\right)+\beta^{T+1}\sum_l z_l'^{T+1}\phi_l^{T+1}\leq V^T\left(\bar{\theta}'\right)$$

[48] u is bounded, being a continuous function on a compact space.

and

$$V^T\left(\bar{\theta}\right)=\sum_{t=0}^{T}\beta^t u\left(x^t,\theta^t,z^t\right)+\beta^{T+1}\sum_l z_l^{T+1}\phi_l^{T+1}$$

hence

$$\sum_{t=0}^{T}\beta^t u\left(x'^t,\theta^t,z'^t\right)-\sum_{t=0}^{T}\beta^t u\left(x^t,\theta^t,z^t\right)+\beta^{T+1}\sum_l\left(z_l'^{T+1}-z_l^{T+1}\right)\phi_l^{T+1}\leq\sum_{t=0}^{T}\beta^t\sum_n y_n^t\left(\theta'^t-\theta_n^t\right)$$

Taking limits of both sides we get $V\left(\bar{\theta}'\right)-V\left(\bar{\theta}\right)\leq\sum_{t=0}^{\infty}\beta^t\sum_n y_n^t\left(\theta_n'^t-\theta_n^t\right)$. Since $\bar{\theta}'$ was arbitrary, this implies $y\in\partial V\left(\bar{\theta}\right)$. Hence, by Lemma 7, $\bar{\theta}$ maximizes V over $\mathbf{\Theta}$ if $\bar{y}=0$, and this condition is also necessary whenever V is differentiable at $\bar{\theta}$. This completes the proof. □

E.3.2 Proof of main result

This subsection is devoted to the proof of Theorem 4. We first establish two auxiliary lemmas.

Lemma 9. *The function f defined in equation (71) is continuously differentiable. Moreover, given a vector of choice probabilities $p\in\Delta I$, a vector $\phi\in\mathbb{R}^I$ and a scalar $\psi\in\mathbb{R}$, the following are equivalent:*

(i)

$$\psi=E_{\mathbb{P}}\max\left(\phi_j+\epsilon_j\right)\ \textit{and}\ \forall j:\ p_j=\mathbb{P}\left[\phi_j+\epsilon_j=\max_k\left(\phi_k+\epsilon_k\right)\right].$$

(ii)

$$\forall j:\ f\left(p\right)+\frac{\partial f\left(p\right)}{\partial p_j}-\sum_k p_k\frac{\partial f\left(p\right)}{\partial p_k}+\phi_j-\psi=0$$

Proof. It is well known (Galichon, 2018) that

$$\forall p\in\Delta I:\ -f\left(p\right)=\min_{\phi}\left[\sum_j p_j\phi_j-\mathrm{E}_{\mathbb{P}}\max_j\left(\phi_j+\epsilon_j\right)\right].$$

By our assumptions on $\mathbb{P}$, the objective function of the above problem is continuous and strictly convex, hence the set of its solutions is a singleton. By the envelope theorem, this implies that, $\partial f\left(p\right)=\left\{-\phi^*\left(p\right)\right\}$, where $\phi^*\left(p\right)$ is the unique optimal solution of the above problem. Hence f is differentiable. Moreover, continuity of ∂f follows by noting that ϕ^* is continuous by the Maximum Theorem.

For the second part of the statement, it is known (see Galichon, 2018) that (i) is equivalent to

$$\phi \in \partial\left(-f\left(p\right)\right) \text{ and } -f\left(p\right)+\psi=\sum_{j} p_j \phi_j .$$

When f is differentiable, the condition above is equivalent to (ii). This completes the proof. $\square$

Lemma 10. *Let $\bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b}, \bar{\psi}, \bar{\mu}^e, \bar{\mu}^s, \bar{\Delta}, \bar{\phi}^s, \bar{\phi}^e$ be such that*

$$\lim_{t\to\infty} \beta^t \phi_{ij}^{s,t} = \lim_{t\to\infty} \beta^t \phi_{ij}^{e,t} = 0,$$

$\left(\bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b}\right) \in \mathcal{A}\left(z^0\right)$ *and, for every t, i, j, $s_i^t, e_i^t > 0$ and the following conditions hold:*

$$\mu_i^{s,t} \geq 0 \textit{ with equality if } s_i^t < x_{ii}^t$$

$$\mu_{ij}^{e,t} \geq 0 \textit{ with equality if } e_i^t G_{ij}^t > y_{ij}^t$$

$$\Delta_{ij}^t \geq 0 \textit{ with equality if } q_{ij}^t < m_i\left(s_i^t, e_i^t\right) G_{ij}^t$$

$$\Delta_{ij}^t \geq w_{ij}\left(q^t\right) + \phi_{ij}^{s,t} - \beta\mu_{ij}^{e,t+1} - \psi_i^t, \textit{with equality if } q_{ij}^t{>}0$$

$$\psi_i^t = E_{\mathbb{P}} \max_j \left(\phi_{ij}^{s,t} + \epsilon_{ij}\right)$$

$$P_{ij}^{b^t} = \mathbb{P}\left[\phi_{ij}^{s,t} = \max_k \left(\phi_{ik}^{s,t} + \epsilon_{ik}\right)\right]$$

$$\phi_{ii}^{s,t} = \mu_i^{s,t} + \beta\phi_{ii}^{s,t+1}$$

$$\phi_{ij}^{e,t} = \kappa_{ij} - \mu_{ij}^{e,t}$$

$$\phi_{ij}^{s,t} = -c_{ij}^s + \beta\left[d_{ij}\phi_{jj}^{s,t+1} + \left(1-d_{ij}\right)\phi_{ij}^{s,t+1}\right].$$

Then:

(i) $\bar{s}$ *maximizes the function* $\bar{s}' \mapsto V\left(\bar{s}', \bar{e}, \bar{G}, z^0\right)$ *over* $\mathcal{SA}\left(z^0|\bar{e}, \bar{G}\right)$ *if, for every* i, t*:*

$$-c_i^s + \frac{\partial m_i\left(s_i^t, e_i^t\right)}{\partial s_i} \sum_j G_{ij}^t \Delta_{ij}^t + \psi_i^t - \beta \phi_{ii}^{s,t+1} - \mu_i^{s,t} = 0. \tag{78}$$

This condition is also necessary whenever the function $\bar{s}' \mapsto V\left(\bar{s}', \bar{e}, \bar{G}, z^0\right)$ *is differentiable at* $\bar{s}$*.*

(ii) $\bar{e}$ *maximizes the function* $\bar{e}' \mapsto V\left(\bar{s}, \bar{e}', \bar{G}, z^0\right)$ *over* $\mathcal{SA}\left(z^0|\bar{s}, \bar{G}\right)$ *if, for every* i, t*:*

$$\frac{\partial m_i\left(s_i^t, e_i^t\right)}{\partial e_i} \sum_j G_{ij}^t \Delta_{ij}^t - \sum_j G_{ij}^t \left(c_{ij}^e + \kappa_{ij} - \mu_{ij}^{e,t} - \phi_{ij}^{e,t+1}\right) = 0. \tag{79}$$

This condition is also necessary whenever the function $\bar{e}' \mapsto V\left(\bar{s}, \bar{e}', \bar{G}, z^0\right)$ *is differentiable at* $\bar{e}$*.*

(iii) $\bar{G}$ *maximizes the function* $\bar{G}' \mapsto V\left(\bar{s}, \bar{e}, \bar{G}', z^0\right)$ *over* $\mathcal{SA}\left(z^0|\bar{s}, \bar{e}\right)$ *if there exists a sequence* $\bar{\omega}$ *such that, for every* i, t*:*

$$m_i\left(s_i^t, e_i^t\right) \Delta_{ij}^t - e_i^t \left(c_{ij}^e + \kappa_{ij} - \mu_{ij}^{e,t} - \phi_{ij}^{e,t+1}\right) \le \omega_i^t \tag{80}$$

with equality if $G_{ij}^t > 0$*.*

This condition is also necessary whenever the function $\bar{G}' \mapsto V\left(\bar{s}, \bar{e}, \bar{G}', z^0\right)$ *is differentiable at* $\bar{G}$*.*

Proof. We apply Lemma 8 to Problem (73)

$$P\left(\bar{s}, \bar{e}, \bar{G}\right) : \; V^p\left(\bar{s}, \bar{e}, \bar{G}, z^0\right) = \max_{\bar{q}, \bar{b}} \sum_{t=0}^{\infty} \beta^t W^p\left(s^t, e^t, G^t, q^t, b^t; z^t\right)$$
$$\text{s.t.} \left(\bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b}\right) \in \mathcal{A}\left(z^0\right).$$

In doing so, notice that the assumptions of Lemma 8 are satisfied, since by Lemma 9 the function W^p is continuously differentiable, and we can take feasible allocations and states to live inside a compact set.[49]

We use the following notation for the Lagrangian multipliers:

[49]Indeed, let $M = \sum_{ij} x_{ij}^0$. Then for every $\left(\bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b}\right) \in \mathcal{A}\left(z^0\right)$ we must have

$$\forall t, i, j : \; 0 \le s_i^t, q_{ij}^t, b_{ij}^t, x_{ij}^t \le M.$$

multiplier	constraint
$\mu_{ij}^{e,t}$	$e_i^t G_{ij}^t \geq y_{ij}^t$
$\mu_i^{s,t}$	$x_{ii}^t \geq s_i^t$
ψ_i^t	$s_i^t = \sum_j \left(q_{ij}^t + b_{ij}^t\right)$
Δ_{ij}^t	$q_{ij}^t \leq m_i\left(s_i^t, e_i^t\right) G_{ij}^t$
$-\omega_i^t$	$\sum_j G_{ij}^t = 1$

Moreover, we denote $\bar{\phi} = \left(\bar{\phi}^s, \bar{\phi}^e\right)$, where $\bar{\phi}^s$ is the component of $\bar{\phi}$ associated with the supply component of the state and $\bar{\phi}^e$ is associated with the demand component. With this notation in hand, the set of Conditions (74) is given by

$$
\begin{aligned}
\forall t, i, j : \mu_i^{s,t} &\geq 0 \text{ with equality if } s_i^t < x_{ii}^t \\
\mu_{ij}^{e,t} &\geq 0 \text{ with equality if } e_i^t G_{ij}^t > y_{ij}^t \\
\Delta_{ij}^t &\geq 0 \text{ with equality if } q_{ij}^t < m_i\left(s_i^t, e_i^t\right) G_{ij}^t,
\end{aligned}
$$

the set of Conditions (76) is given by

$$
\begin{aligned}
\forall t, i, j : \phi_{ii}^{s,t} &= \mu_i^{s,t} + \beta \phi_{ii}^{s,t+1} \\
\phi_{ij}^{e,t} &= \kappa_{ij} - \mu_{ij}^{e,t} \\
\phi_{ij}^{s,t} &= -c_{ij}^s + \beta\left[d_{ij}\phi_{jj}^{s,t+1} + \left(1 - d_{ij}\right)\phi_{ij}^{s,t+1}\right]
\end{aligned}
$$

Moreover, letting e^*, q^*, b^* be a solution of

$$
\begin{aligned}
&\max_{e,q,b \geq 0} W(q) - \sum_i e_i \left(\min_j c_{ij}^e + \min_j \kappa_{ij}\right) \\
&\text{s.t.} q_{ij} \leq M \\
&\sum_j q_{ij} \leq m_i\left(M, e_i\right)
\end{aligned}
$$

we have that every sequence $\bar{e}$ such that $e_i^t > e_i^*$ for some t, i is clearly sub optimal, hence without loss of generality we can take

$$0 \leq e_i^t, y_{ij}^t \leq e_i^*.$$

and the set of Conditions (75) is given by

$$\forall t, i, j : \Delta_{ij}^{t} \geq w_{ij}\left(q^{t}\right) + \phi_{ij}^{s,t} - \beta\mu_{ij}^{e,t+1} - \psi_{i}^{t} \text{ with equality if } q_{ij}^{t} > 0$$
$$f\left(P_{i}^{b}\right) + \frac{\partial f\left(P_{i}^{b}\right)}{\partial P_{ij}} - \sum_{k} P_{ik}^{b} \frac{\partial f\left(P_{i}^{b}\right)}{\partial P_{ik}} + \phi_{ij}^{s,t} - \psi_{i}^{t} = 0.$$

By Lemma 9, the set of conditions in the second line above is equivalent to

$$\forall t, i, j: \; P_{ij}^{b^t} = \mathbb{P}\left[\phi_{ij}^{s,t} = \max_{k}\left(\phi_{ik}^{s,t} + \epsilon_{ik}\right)\right] \text{ and } \psi_{i}^{t} = \mathrm{E}_{\mathbb{P}} \max_{j}\left(\phi_{ij}^{s,t} + \epsilon_{ij}\right).$$

To prove Statement (i), we apply Lemma 8 to the function $\bar{s}' \mapsto V\left(\bar{s}', \bar{e}, \bar{G}, z^{0}\right)$. Given our assumption that $s_{i}^{t} > 0$ for every t, i, Condition (77) is given by

$$\forall i, t: \; -c_{i}^{s} + \frac{\partial m_{i}\left(s_{i}^{t}, e_{i}^{t}\right)}{\partial s_{i}} \sum_{j} G_{ij}^{t} \Delta_{ij}^{t} + \psi_{i}^{t} - \beta\phi_{ii}^{s,t+1} - \mu_{i}^{s,t} = 0.$$

To prove Statement (ii), we apply Lemma 8 to the function $\bar{e}' \mapsto V\left(\bar{s}, \bar{e}', \bar{G}, z^{0}\right)$. Given our assumption that $e_{i}^{t} > 0$ for every t, i, Condition (77) is given by

$$\forall i, t: \; \frac{\partial m_{i}\left(s_{i}^{t}, e_{i}^{t}\right)}{\partial e_{i}} \sum_{j} G_{ij}^{t} \Delta_{ij}^{t} - \sum_{j} G_{ij}^{t}\left(c_{ij}^{e} + \kappa_{ij} - \mu_{ij}^{e,t} - \beta\phi_{ij}^{e,t+1}\right) = 0.$$

To prove Statement (iii), we apply Lemma 8 to the function $\bar{G}' \mapsto V\left(\bar{s}, \bar{e}, \bar{G}', z^{0}\right)$. Condition (77) is given by

$$\forall i, j, t: \; m_{i}\left(s_{i}^{t}, e_{i}^{t}\right)\Delta_{ij}^{t} - e_{i}^{t}\left(c_{ij}^{e} + \kappa_{ij} - \mu_{ij}^{e,t} - \beta\phi_{ij}^{e,t+1}\right) - \omega_{i}^{t} + \mathrm{pos}_{ij}^{t} = 0$$

where pos_{ij}^{t} is the multiplier associated to the positivity constraint $G_{ij} \geq 0$, which must satisfy

$$\mathrm{pos}_{ij}^{t} \geq 0 \text{ with equality if } G_{ij}^{t} > 0.$$

This completes the proof. □

Proof of main result In order to prove the main result, let everything be as in the statement. Let $(V^{s,t}, U^{e,t}, \Delta^{s,t}, \Delta^{e,t})_{t=0}^{\infty}$ be the sequence of carriers and customers' value functions and meeting surpluses associated with the sequence $\bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b}$ evaluated at the state trajectory z^t, $t >= 0$. For every $t \geq 0$ define $\phi^{s,t} = V^{s,t}$, $\phi^{e,t} = U^{e,t}$, $\psi^t = \mathrm{E}_{\mathbb{P}} U^{s,t}(\epsilon)$, $\Delta^t = \Delta^{s,t} + \Delta^{e,t}$ and

$$
\begin{aligned}
\mu_i^{s,t} &= \max \left\{ -c_i^s + \boldsymbol{\lambda}_i^s \left(z^t\right) \sum_{j \neq i} G_{ij}^t \Delta_{ij}^{s,t} + U_i^{s,t} - \beta V_i^{s,t+1}, 0 \right\} \\
\mu_{ij}^{e,t} &= \kappa_{ij} - U_{ij}^{e,t}.
\end{aligned}
$$

Then $\bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b}, \bar{\psi}, \bar{\mu}^e, \bar{\mu}^s, \bar{\Delta}, \bar{\phi}^s, \bar{\phi}^e$ satisfies the conditions of Lemma 10. Moreover, notice that:

- Condition (78) can be written as

$$
\forall i, t: \frac{\partial m_i \left(s_i^t, e_i^t\right)}{\partial s_i} \sum_j G_{ij}^t \left(\Delta_{ij}^{s,t} + \Delta_{ij}^{e,t}\right) - \boldsymbol{\lambda}_i^s \left(z^t\right) \sum_{j \neq i} G_{ij}^t \Delta_{ij}^{s,t} = 0.
$$

Using $\boldsymbol{\lambda}_i^s \left(z^t\right) = m_i \left(s_i^t, e_i^t\right) / s_i^t$ and rearranging, this is equivalent to

$$
\eta_i^s \left(z^t\right) \sum_j G_{ij}^t \left(\Delta_{ij}^{s,t} + \Delta_{ij}^{e,t}\right) = \sum_j G_{ij}^t \Delta_{ij}^{s,t}.
$$

- Condition (79) can be written as

$$
\forall i, t: \frac{\partial m_i \left(s_i^t, e_i^t\right)}{\partial e_i} \sum_j G_{ij}^t \left(\Delta_{ij}^{s,t} + \Delta_{ij}^{e,t}\right) - \sum_j G_{ij}^t \left(c_{ij}^e + U_{ij}^{e,t} - \beta U_{ij}^{e,t+1}\right) = 0.
$$

Using $U_{ij}^{e,t} = -c_{ij}^e + \boldsymbol{\lambda}_i^e \left(z^t\right) \Delta_{ij}^{e,t} + \beta U_{ij}^{e,t+1}$, $\boldsymbol{\lambda}_i^e \left(z^t\right) = m_i \left(s_i^t, e_i^t\right) / e_i^t$ and rearranging, this is equivalent to

$$
\eta_i^e \left(z^t\right) \sum_j G_{ij}^t \left(\Delta_{ij}^{s,t} + \Delta_{ij}^{e,t}\right) = \sum_j G_{ij}^t \Delta_{ij}^{e,t}.
$$

- Condition (80) can be written as

$$
\begin{aligned}
\forall i, j, t: m_i \left(s_i^t, e_i^t\right) \Delta_{ij}^t - e_i^t \left(c_{ij}^e + U_{ij}^{e,t} - \beta U_{ij}^{e,t+1}\right) \leq \omega_i^t \\
\text{with equality if } G_{ij}^t > 0.
\end{aligned}
$$

Using $U_{ij}^{e,t} = -c_{ij}^{e} + \boldsymbol{\lambda}_i^e\left(z^t\right)\Delta_{ij}^{e,t} + \beta U_{ij}^{e,t+1}$, $\boldsymbol{\lambda}_i^e\left(z^t\right) = \frac{m_i\left(s_i^t, e_i^t\right)}{e_i}$ and rearranging, this is equivalent to

$$\forall i, j, t : \Delta_{ij}^{s,t} \leq -\frac{\omega_i^t}{\boldsymbol{\lambda}_i^e\left(z^t\right)}.$$

with equality if $G_{ij}^t > 0$.

This completes the proof.